1) 117(7): ECE Calculation of the Earth's Gravitomagnetic Angular Frequency.

The earth's gravitomagnetic angular frequency according to ECE theory is:

$$\Omega = \frac{h}{c} = \frac{G}{c^2 r^3} \left(L - \frac{3(\underline{L} \cdot \underline{r})\underline{r}}{r^2} \right) \quad -(1)$$

in the weak field dipole approximation. Here:

$$L = \frac{2}{5} m R^2 \omega \quad -(2)$$

is the angular momentum of the earth's spin about its own axis. Gravity probe B orbits at 650 kilometres above the poles. The data are:

$$\left.\begin{aligned} R &= 6.37 \times 10^{6} \text{ metres} \\ r &= 7.02 \times 10^{6} \text{ metres} \\ m &= 5.98 \times 10^{24} \text{ kilograms} \\ c &= 2.998 \times 10^{8} \text{ m s}^{-1} \\ G &= 6.67 \times 10^{-11} \text{ m}^3 \text{ kgm}^{-1} \text{ s}^{-2} \\ \omega &= 7.29 \times 10^{-5} \text{ rad s}^{-1} \end{aligned}\right\} \quad -(3)$$

Therefore:

$$L = 7.076 \times 10^{33} \text{ kgm m}^2 \text{ s}^{-1} \quad -(4)$$

so:

$$\boxed{\Omega \sim \frac{GL}{c^2 r^3} = 1.52 \times 10^{-14} \text{ rad s}^{-1}} \quad -(5)$$

If we take one year as being 3600 x 24 x 365.25 seconds in the first approximation then:

2)

One year = 3.156×10^{7} seconds — (6)

The change in orientation of the satellite over one year is:

$$\Delta\theta = \Omega \Delta t \quad — (7)$$

$$= 1.52 \times 3.156 \times 10^{-7}$$

$$\boxed{\Delta\theta = 4.80 \times 10^{-7} \text{ radians}} \quad — (8)$$

Finally:

1 radian = 206,265 arcseconds — (9)

So $\Delta\theta = 4.80 \times 10^{-7} \times 2.06265 \times 10^{5}$

$$= 9.90 \times 10^{-2} \text{ arcseconds}$$

$$\boxed{\Delta\theta \doteq 0.099 \text{ arc seconds}} \quad — (10)$$

This result is calculated from:

$$\underline{\nabla} \times \underline{h} = 4\pi G \underline{J} \quad — (11)$$

which is written in analogy to the Newton law:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho \quad — (12)$$

for a mass density ρ in kilograms per metres^3 cubed and a mass current density $\underline{J}$ in kilograms per metres squared per second.

3) The four current is therefore:

$$J^\mu = (c\rho, \underline{J}) \quad -(13)$$

The analogous equations in electrodynamics are:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} \quad -(14)$$

and

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(15)$$

with

$$J^\mu(e/m) = (c\rho, \underline{J})(e/m). \quad -(16)$$

The extra factor 4π occurs from the fact that the proportionality constant in the Coulomb law is defined in SI by $1/(4\pi\epsilon_0)$, but in the Newton law by G (without a 4π).

Remarks

1) The result (1) is exactly the same as given by Pfister.

2) In some sites there is an extra factor of two in the denominator of eq (1).

The discrepancy between (1) and (2) is probably due to confusion in standard physics of calculation.