1) 117(2): Magnetic ~~Current~~ Flux Density due to a Localized Current Distribution

A complete treatment of this problem requires vector spherical harmonics. In the standard treatment of electrodynamics:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} \quad -(1)$$

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(2)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad -(3)$$

and it is assumed that:

$$\underline{\nabla} \cdot \underline{A} = 0 \quad -(4)$$

so:

$$\nabla^2 \underline{A} = -\mu_0 \underline{J} \quad -(5)$$

In ECE electrodynamics:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(6)$$

so:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A}) = \mu_0 \underline{J} \quad -(7)$$

and it is not assumed that there is gauge freedom, so in general, eq. (4) is not true in ECE.

In the standard electrodynamics:

$$\underline{A}(\underline{r}) = \frac{\mu_0}{4\pi} \int \frac{\underline{J}(r')}{|r - \underline{r}'|} d^3 r' \quad -(8)$$

The procedure then used in standard electrodynamics is:

$$\frac{1}{|\underline{r} - r'|} = \frac{1}{|\underline{r}|} + \frac{\underline{r} \cdot \underline{r}'}{|\underline{r}|^3} + \cdots \quad (9)$$

2) from which, at dipole order of approximation:

$$\underline{B}(\underline{r}) = \frac{\mu_0}{4\pi}\left(\frac{3\underline{n}(\underline{n}\cdot\underline{m}) - \underline{m}}{|\underline{r}|^3}\right) \quad -(10)$$

where:

$$\underline{n} = \underline{r} / |\underline{r}|. \quad -(11)$$

Eq. (10) is also the field of an electric dipole as shown by Jackson.

<u>ECE Equations of Dynamics</u>

These are:

$$\underline{\nabla} \times \underline{h} = \frac{4\pi G}{c}\,\underline{J} \quad -(12)$$

$$\underline{\nabla} \cdot \underline{h} = 0 \quad -(13)$$

$$\underline{h} = \underline{\nabla} \times \underline{v} - \underline{\omega} \times \underline{v} \quad -(14)$$

In eq. (13) it has been assumed that $\underline{\nabla} \cdot \underline{h}$ is zero in analogy with the observation that $\underline{\nabla} \cdot \underline{B}$ is zero. However, the field equations of ECE allow a more general result. In Gravity Probe B, what is measured is:

$$\underline{h}_1 = \frac{\underline{h}}{c} \text{ in units of rad. } s^{-1}. \quad -(15)$$

So:

3)

$$\underline{\nabla} \times \underline{h}_1 = \left(\frac{4\pi G}{c^2}\right) \underline{J} \quad - (16)$$

It is found experimentally that:

$$h_1 \sim 10^{-14} \text{ radians } s^{-1} \quad - (17)$$

for the Earth.

In the weak field limit of the Earth's gravitational field it is assumed that the spin connection goes to zero, and it is further assumed that:

$$\underline{\nabla} \cdot \underline{q} = 0 \quad - (18)$$

The results of note 117(1) are converted to:

$$\boxed{\underline{h}_1 = \frac{G}{c^2}\left(\frac{\underline{L}}{r^3} - \frac{3\underline{n}(\underline{n}\cdot\underline{L})}{r^3}\right)} \quad - (19)$$

This result is a rough first approximation to the $|\underline{h}_1|$ found by Gravity Probe B.