1) 115(10): Lorentz Transform of the Inhomogeneous Field Equation of Classical Electrodynamics.

These are given in tensor notation by:

$$\partial_\mu F^{\mu\nu} = J^\nu / \epsilon_0 \quad -(1)$$

and in vector notation by:

$$\underline{\nabla} \cdot \underline{D} = \rho \quad -(2)$$

$$\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J} . \quad -(3)$$

as is well known. Eq. (2) is the Coulomb law and eq. (3) is the Ampère Maxwell law. Here $\underline{D}$ is the electric displacement, ρ is the charge density, $\underline{H}$ is the magnetic field strength, $\underline{J}$ is the current density and ϵ_0 the vacuum permittivity.

As for notes 115(8) and 115(9) the relevant four vector is:

$$V^\nu = J^\nu = \left(c\,\underline{\nabla} \cdot \underline{D} ,\ \underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} \right) \quad -(4)$$

Under a Lorentz boost in the X axis:

$$\left.\begin{aligned} V^{0\prime} &= V^0 \cosh\phi - V^1 \sinh\phi \\ V^{1\prime} &= -V^0 \sinh\phi + V^1 \cosh\phi \\ V^{2\prime} &= V^2 \\ V^{3\prime} &= V^3 \end{aligned}\right\} \quad -(5)$$

2)
$$\left.\begin{aligned} J^{0\prime} &= J^0 \cosh\phi - J^1 \sinh\phi \\ J^{1\prime} &= -J^0 \sinh\phi + J^1 \cosh\phi \\ J^{2\prime} &= J^2 \\ J^{3\prime} &= J^3 \end{aligned}\right\} \quad -(6)$$

<u>Coulomb Law in Frame K'</u>

$$c\,\underline{\nabla}\cdot\underline{D}\cosh\phi - \left(\underline{\nabla}\times\underline{H} - \frac{\partial\underline{D}}{\partial t}\right)_x \sinh\phi$$

$$= c\rho\cosh\phi - J_x \sinh\phi \quad -(7)$$

i.e.

$$(\underline{\nabla}\cdot\underline{D})' = \underline{\nabla}\cdot\underline{D}\cosh\phi - \frac{1}{c}\left(\underline{\nabla}\times\underline{H} - \frac{\partial\underline{D}}{\partial t}\right)_x \sinh\phi$$

$$= \rho' = \rho\cosh\phi - \frac{J_x}{c}\sinh\phi \quad -(8)$$

However, it is known that eqns. (2) and (3) are true, so:

$$\boxed{(\underline{\nabla}\cdot\underline{D})' = \rho'} \quad -(9)$$

Similarly:
$$\boxed{\left(\underline{\nabla}\times\underline{H} - \frac{\partial\underline{D}}{\partial t}\right)' = \underline{J}'} \quad -(10)$$

3) Transformed Current and Charge Densities

In the inhomogeneous laws the transformed charge density is:

$$\rho' = \rho \cosh \phi - \frac{J_x}{c} \sinh \phi \quad - (11)$$

for a Lorentz boost in X. Similarly the transformed J_x is:

$$J_x' = -\rho \sinh \phi + \frac{J_x}{c} \cosh \phi \quad - (12)$$

with:

$$J_y' = J_y \quad - (13)$$

$$J_z' = J_z \quad - (14)$$

The laws are invariant but the charge and current densities are covariant. This covariance gives rise to observable effects. So in summary the K' laws are:

$$\boxed{\begin{aligned} (\underline{\nabla} \cdot \underline{B})' &= 0 \\ \left(\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t}\right)' &= \underline{0} \\ (\underline{\nabla} \cdot \underline{D})' &= \rho' \\ \left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t}\right)' &= \underline{J}' \end{aligned}}$$