1)

III(5): General Equation for g_{00} in ECE Theory

The general expression for the potential energy from the line element of paper 108 is:

$$V = \frac{m}{2} g_{00} \left(c^2 + \frac{L^2}{r^2} \right) \quad -(1)$$

so:

$$\Phi = \frac{V}{m} = \frac{1}{2} g_{00} \left(c^2 + \frac{L^2}{r^2} \right). \quad -(2)$$

Here the angular momentum:

$$L = r^2 \frac{d\phi}{d\lambda} \quad -(3)$$

is a constant of motion.

From ECE theory:

$$\underline{g} = -\underline{\nabla}\Phi - \underline{\omega}\Phi \quad -(4)$$

and

$$\underline{\nabla} \cdot \underline{g} = c^2 (R - \omega T), \quad -(5)$$

so:

$$\boxed{\underline{\nabla} \cdot (\underline{\nabla}\Phi + \underline{\omega}\Phi) = c^2 (\omega T - R)} \quad -(6)$$

This is a differential equation for g_{00} in terms of ω, T, R and L in general. This equation makes no assumptions other than those of Cartan geometry.

In spherical polar coordinates:

$$\underline{\nabla} \cdot (\underline{\nabla}\Phi) = \frac{\partial^2 \Phi}{\partial r^2} + \frac{2}{r}\frac{\partial \Phi}{\partial r}, \quad -(7)$$

$$\underline{\nabla} \cdot (\underline{\omega}\Phi) = \underline{\omega} \cdot \underline{\nabla}\Phi + (\underline{\nabla} \cdot \underline{\omega})\Phi \quad -(8)$$

2)

So:

$$\frac{\partial^2 \Phi}{\partial r^2} + \left(\frac{2}{r} + \omega\right)\frac{\partial \Phi}{\partial r} + \frac{\partial \omega}{\partial r}\Phi = c^2(\omega T - R) \quad -(9)$$

This equation has resonance solutions under well-defined conditions.

From eqs. (2) and (9):

$$\frac{1}{2}\left(c^2+\frac{L^2}{r^2}\right)\frac{\partial^2 g_{00}}{\partial r^2} + \left(\frac{1}{2}\left(\frac{2}{r}+\omega\right)\left(c^2+\frac{L^2}{r^2}\right) - \frac{L^2}{r^3}\right)\frac{\partial g_{00}}{\partial r}$$
$$+ L^2\left(\frac{6}{r^4} - \left(\frac{2}{r}+\omega\right)\frac{1}{r^3}\right)g_{00} + \frac{\partial \omega}{\partial r}\Phi = c^2(\omega T - R) \quad -(10)$$

i.e.:

$$\boxed{\begin{aligned}&\frac{1}{2}\left(c^2+\frac{L^2}{r^2}\right)\frac{\partial^2 g_{00}}{\partial r^2} + \left(\frac{1}{2}\left(\frac{2}{r}+\omega\right)\left(c^2+\frac{L^2}{r^2}\right) - \frac{L^2}{r^3}\right)\frac{\partial g_{00}}{\partial r}\\ &+ \left(L^2\left(\frac{6}{r^4} - \left(\frac{2}{r}+\omega\right)\frac{1}{r^3}\right) + \frac{1}{2}\left(c^2+\frac{L^2}{r^2}\right)\frac{\partial \omega}{\partial r}\right)g_{00}\\ &= c^2(\omega T - R) \quad -(11)\end{aligned}}$$

Solution

a) Eq. (11) may be integrated by computer algebra to give g_{00}.

b) Empirically, from satellite data, it is

3) Known that:

$$g_{00} = 1 + \frac{\mu}{r} \quad - (12)$$

where

$$\mu = -\frac{2mG}{c^2} \quad - (13)$$

In the limit: $r \rightarrow \infty \quad - (14)$

eq. (12) gives:

$$g_{00} \rightarrow 1, \quad - (15)$$

so

$$\frac{\partial^2 g_{00}}{\partial r^2} \rightarrow 0, \quad \frac{\partial g_{00}}{\partial r} \rightarrow 0. \quad - (16)$$

In this limit: $\omega \rightarrow 0, \quad \partial\omega / \partial r \rightarrow 0 \quad - (17)$

so:

$$\omega T - R \rightarrow \frac{4L^2}{c^2 r^4}\left(1 + \frac{\mu}{r}\right) - (18)$$

A particular solution of eq. (18) is:

$$\omega T = \frac{4L^2}{c^2 r^4}\left(\frac{\mu}{r}\right) \quad - (19)$$

$$R = -\frac{4L^2}{c^2 r^4} \quad - (20)$$

so:

$$\omega T = -R\frac{\mu}{r} \quad - (21)$$

i.e.

$$\underline{\nabla} \cdot \underline{g} = c^2 R\left(1 - \frac{\mu}{r}\right) - (22)$$

4) or, empirically:

$$\underline{\nabla} \cdot \underline{g} = c^2 R \left(1 + \frac{2mG}{rc^2} \right), \quad \text{i.e.}$$

$$\boxed{\underline{\nabla} \cdot \underline{g} = c^2 k \rho_m \left(1 + \frac{2mG}{rc^2} \right)} \quad -(23)$$

in this weak field limit. The Coulomb law is corrected by:

$$\boxed{\underline{\nabla} \cdot \underline{E} = \frac{\rho_e}{\epsilon_0} \left(1 + \frac{2mG}{rc^2} \right)} \quad -(24)$$

Here ρ_m is the mass density and ρ_e is the charge density. In this weak field limit:

$$\underline{E} \sim -\underline{\nabla} \phi \quad -(25)$$

so the Poisson equation is corrected by:

$$\nabla^2 \phi = -\frac{\rho_e}{\epsilon_0} \left(1 + \frac{2mG}{rc^2} \right) \quad -(26)$$

and the scalar potential by:

$$\boxed{\phi = \frac{e}{4\pi \epsilon_0 r} \left(1 + \frac{2mG}{rc^2} \right)} \quad -(27)$$