1)

III(3): Metric from Spherically Symmetric Spacetime

The most general line element for spherically symmetric spacetime has been given by Crothers in paper 93. In this note we illustrate the method with the less general metric given by Carroll:

$$ds^2 = e^{2\alpha} c^2 dt^2 - e^{2\beta} dr^2 - r^2 d\Omega^2 \quad -(1)$$

From paper 105 it is known that:

$$e^{2\alpha} = 1 + \frac{\mu}{r} = e^{-2\beta} \quad -(2)$$

where

$$\mu = \frac{T}{R}. \quad -(3)$$

The experimental satellite data show that to a high precision:

$$\frac{T}{R} = -\frac{2MG}{c^2} \quad -(4)$$

where T/R is a well defined ratio of torsion to curvature, M is the attracting mass, such as the mass of the sun for orbits in the solar system, and r is the distance between the centre of mass of the sun and the centre of mass of the orbiting object.

Therefore a line element of this type reproduces the experimental data to high precision.

Note carefully that this is not the result of the Einstein Hilbert field equation, and there has been no assumption of the Christoffel connection. The equation (3) was derived in paper 105.

Eq. (4) is therefore the result of comparison with experimental data, not a prediction of the EH field equation. The spherical symmetry of space-time has been used with the particular choice:

$$ds^2 = e^{2\alpha} c^2 dt^2 - e^{-2\alpha} dr^2 - r^2 d\Omega^2 \quad -(5)$$

where

$$e^{2\alpha} = 1 + \frac{T}{rR}. \quad -(6)$$

If

$$2\alpha << 1 \quad -(7)$$

then

$$e^{2\alpha} = 1 + 2\alpha + \ldots \quad -(8)$$

so

$$\boxed{\alpha \sim \frac{T}{2rR}} \quad -(9)$$

Therefore in the limit:

$$r \rightarrow \infty, \quad -(10)$$

then

$$\alpha \sim -\frac{MG}{rc^2} \quad -(11)$$

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The Newtonian potential is:

$$\Phi = -\frac{GM}{r} \quad -(12)$$

so:

$$\alpha \sim -\frac{\Phi}{c^2} \quad -(13)$$

Therefore in the limit (10):

$$\boxed{ds^2 = \exp\left(-\frac{\Phi}{c^2}\right)c^2dt^2 - \exp\left(\frac{\Phi}{c^2}\right)dr^2 - r^2 d\Omega^2 \quad -(14)}$$

A line element of type (14) reproduces data very accurately in the limit (10).

From eqs. (12) and (4):

$$\frac{T}{rR} = -2\frac{\Phi}{c^2}, \quad -(15)$$

Thus:

$$\boxed{\Phi = -\frac{c^2}{2}\left(\frac{T}{rR}\right)} \quad -(16)$$

This is a purely geometrical result made without use of the EH equation, Ricci flat condition, or Christoffel connection.

Conclusion

The geometrical property responsible for the orbits of objects is T / rR, where r is the inter centre of mass distance. A spherically symmetric spacetime produces the observed experimental data.

Development of T / rR

In paper 105 it was shown that:

$$\underline{\nabla} \cdot \underline{g} = c^2 k \rho_m \quad -(17)$$

where in the simplest case:

$$g = |\underline{g}| = c^2 T^{030}, \quad \rho_m = \frac{1}{k} R^{0}{}_{3}{}^{30} \quad -(18)$$

Here $\underline{g}$ is the acceleration due to gravity and ρ_m is the mass density. These equations come from the Hodge dual of the Bianchi identity.

If we denote:

$$T = T^{030}, \quad R = R^{0}{}_{3}{}^{30} \quad -(19)$$

Then

$$\frac{T}{rR} = \frac{g}{r c^2 \rho_m k} = \frac{1}{c^2 k}\left(\frac{g}{r \rho_m}\right) \quad -(20)$$

i.e.

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$$\frac{g}{\rho_m} = c^2 k . \frac{T}{R} \quad -(21)$$

Therefore the ratio of g to ρ_m is the ratio of torsion to curvature within the universal constant $\underbrace{c^2 k}$.

If we denote:

$$\rho_m = m / V \quad -(22)$$

then in Newtonian dynamics:

$$\frac{g}{m} = \frac{G}{r^2} \quad -(23)$$

so:

$$\frac{T}{R} = \frac{1}{c^2 k} \frac{V g}{m} = \frac{1}{c^2 k} \frac{V G}{r^2} . \quad -(24)$$

Now assume that $V = \frac{4}{3} \pi r^3 \quad -(25)$

to obtain:

$$\frac{T}{R} = \left(\frac{4}{3} \pi \frac{G}{k c^2} \right) r . \quad -(26)$$

Therefore in Newtonian dynamics T / rR is a universal constant:

$$\frac{T}{rR} = \frac{4}{3} \pi \frac{G}{c^2 k} \quad -(27)$$

Finally we:

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$$k = \frac{8\pi G}{c^2} \quad - (28)$$

to obtain:

$$\boxed{\frac{T}{rR} = \frac{1}{6}.} \quad - (29)$$

More generally:

$$\frac{T}{rR} = \frac{G}{c^2 k} \cdot \left(\frac{V}{r^3}\right) \quad - (30)$$

$$\frac{T}{rR} = \frac{1}{8\pi} \left(\frac{V}{r^3}\right) \quad - (31)$$

and

$$\boxed{\frac{T}{R} = \frac{1}{8\pi} \left(\frac{V}{r^2}\right)} \quad - (32)$$

Finally if

$$\left| \frac{T}{R} \right| = \frac{2mG}{c^2} \quad - (33)$$

then:

$$\frac{2mG}{c^2} = \frac{1}{8\pi} \frac{V}{r^2} \quad - (34)$$

and the mass density is:

$$\boxed{\rho_m = \frac{m}{V} = \frac{c^2}{16\pi G r^2}} \quad - (35)$$

The curvature is therefore:

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$$R = \frac{kc^2}{16\pi G} \cdot \frac{1}{r^2}$$

$$\boxed{R = \frac{1}{2} \cdot \frac{1}{r^2}} \quad — (36)$$

and from eq. (32)

$$\boxed{T = \frac{1}{16\pi} \left(\frac{V}{r^4} \right)} \quad — (37)$$

Therefore for a line element of type (1) the torsion and curvature of spacetime are given experimentally by eqs. (36) and (37). Almost all orbits are described by eqs. (36) and (37), which are therefore general properties of spacetime.

The only known exceptions are the orbits of binary pulsars and perhaps the Pioneer / Cassini anomalies of the solar system.