1) Paper 108, Notes 2. Orbital Equation in the non-Relativistic Limit.

The basic equation is:

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\phi^2 \quad -(1).$$

Rewrite this as:

$$\frac{r^2}{c^2}\left(\frac{d\phi}{d\tau}\right)^2 = \left(1 - \frac{r_s}{r}\right)\left(\frac{dt}{d\tau}\right)^2 - \frac{1}{c^2}\left(1 - \frac{r_s}{r}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2 \quad -(2)$$

The constants of motion are:

$$E = mc^2\left(1 - \frac{r_s}{r}\right)\frac{dt}{d\tau}, \quad -(3)$$

$$L = mr^2 \frac{d\phi}{d\tau}. \quad -(4)$$

In the non-relativistic limit:

$$r_s << r \quad -(5)$$

So:

$$\frac{dt}{d\tau} = \frac{E}{mc^2} \quad -(6)$$

From eqns (2) to (4):

$$\boxed{\left(\frac{dr}{d\tau}\right)^2 = \left(\frac{E}{mc}\right)^2 - \left(\frac{cL}{mr}\right)^2\left(1 - \frac{r_s}{r}\right)} \quad -(7)$$

So:

$$r = \int \left[\left(\frac{E}{mc}\right)^2 - \left(\frac{cL}{mr}\right)^2 \left(1 - \frac{r_s}{r}\right) \right]^{1/2} d\tau \quad — (8)$$

$$\tau = \int \left[\left(\frac{E}{mc}\right)^2 - \left(\frac{cL}{mr}\right)^2 \left(1 - \frac{r_s}{r}\right) \right]^{-1/2} dr \quad — (9)$$

In the non-relativistic limit:

$$\tau \rightarrow \int \left[\left(\frac{E}{mc}\right)^2 - \left(\frac{cL}{mr}\right)^2 \right]^{-1/2} dr \quad — (10)$$

and in the non-relativistic limit:

$$\left(\frac{dr}{d\tau}\right)^2 \rightarrow v^2 \quad — (11)$$

Therefore:

$$\boxed{v^2 = \left(\frac{E}{mc}\right)^2 - \left(\frac{cL}{mr}\right)^2 \left(1 - \frac{r_s}{r}\right)} \quad — (12)$$

In the solar system:

$$r_s \rightarrow \frac{2GM}{c^2} \quad — (13)$$

and r is measured from the centre of mass of the sun.

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In the ECE theory in general:

$$v = \frac{dr}{d\tau} \quad -(14)$$

and

$$r_s = -\frac{T}{R}. \quad -(15)$$

In the solar system, the proper time τ, a characteristic time of the orbit, therefore ~~depends~~ varies only ~~on~~ with r, because E and L are constants of motion. This characteristic time is obtained by integrating eq. (9). It is related to the time taken for one revolution. If the orbit is stable then this time is constant. If for example the Earth is an average distance r_0 from the sun, then:

$$\tau = \int_0^{r_0} \left[\left(\frac{E}{mc}\right)^2 - \left(\frac{cL}{mr}\right)^2 \left(1 - \frac{r_s}{r}\right) \right]^{-1/2} dr \quad -(16)$$

and this is <u>one year</u>.

The velocity v from eq (12) is related to the orbital velocity at a given r.

<u>In a binary pulsar</u>, τ is de<u>creasing</u>, because the orbit shrinks by 3.1 mm a ~~year~~ revolution. So in eq. (16), for a given r_0, r_s must

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be modelled to give this result. In a first approximation, E and L can be regarded as constants of the motion, because the decrease of 3.1 mm a revolution is very small compared with the mean separation of the two stars of the binary pulsar.

The distance of closest approach of the two stars is:

$$\boxed{r(\min) = \frac{cL}{E}} \quad -(17)$$

and this can be found experimentally. For each revolution this is gradually decreasing per revolution in a binary pulsar, so L and E are not quite constant.

Numerical Method

Integrate eq. (16) to give τ as a function of r_0, L and E. Find E in the non-relativistic limit from eqns. (17) and (3), with:

$$\frac{dt}{d\tau} \rightarrow 1 \quad -(18)$$

Therefore: $$E \rightarrow mc^2\left(1 + \frac{T}{rR}\right) -(19)$$

5) and :

$$L = \frac{E}{c} r(\text{min}) \quad — (20)$$

By choosing T/R to fit the experimental data, show that τ decreases every revolution, and when the two stars collide, then

$$\tau = 0. \quad — (21)$$

Graph and animate the orbit from eq. (7)

P. S. (22).

In ECE theory :

$$\boxed{v^2 = \left(\frac{dr}{d\tau}\right)^2 = \left(\frac{E}{mc}\right)^2 - \left(\frac{cL}{mr}\right)^2 \left(1 + \frac{T}{rR}\right)}$$

so graph and animate v as a function of r and T/R, assuming that E and L are constants of the motion. As the orbit shrinks, $v \longrightarrow \infty$.

In the solar system :

$$\frac{T}{R} \longrightarrow -\frac{2MG}{c^2}$$