1) Note 108(13): Recalculation of Appendix.

The correct potential checked by computer algebra is:

$$V = \frac{1}{2} m \left(1 - \frac{r_s}{r}\right)\left(c^2 + \frac{L^2}{r^2}\right) \quad -(1)$$

so:

$$\frac{\partial V}{\partial r_s} = -\frac{1}{2}\frac{m}{r}\left(c^2 + \frac{L^2}{r^2}\right) \quad -(2)$$

and

$$\frac{\partial V}{\partial r} = \frac{1}{2} m \left(\frac{r_s}{r^2}\left(c^2 + \frac{L^2}{r^2}\right) - \frac{2L^2}{r^3}\left(1 - \frac{r_s}{r}\right)\right). \quad -(3)$$

So

$$\frac{\partial V}{\partial r_s} = \frac{\partial V}{\partial r}\frac{\partial r}{\partial r_s} \quad -(4)$$

and

$$\frac{\partial r_s}{\partial r} = \left(\frac{\partial V}{\partial r}\right) \Big/ \left(\frac{\partial V}{\partial r_s}\right), \quad -(5)$$

$$\frac{\partial r_s}{\partial r} = \left(\frac{2L^2}{r^3}\left(1 - \frac{r_s}{r}\right) - \frac{r_s}{r^2}\left(c^2 + \frac{L^2}{r^2}\right)\right)\left(c^2 + \frac{L^2}{r^2}\right)^{-1}$$

$$\boxed{\begin{aligned} r_s &= \int \left(\frac{2L^2}{r^3}\left(1 - \frac{r_s}{r}\right) - \frac{r_s}{r^2}\left(c^2 + \frac{L^2}{r^2}\right)\right)\left(c^2 + \frac{L^2}{r^2}\right)^{-1} dr \\ &= \frac{2GM}{c^2} + \frac{a}{r} \end{aligned}} \quad -(6)$$

Therefore we can find the parameter a in terms of L and r by integrating eq. (6) and solving for r_s.