1)

108(11): Effect of Additional Attraction on Initially Circular Orbits

We consider a potential:

$$V = mc^2\left(\frac{1}{2} - \frac{r_s}{r}\right) + \frac{mL^2}{2r^2} - \frac{mL^2 r_s}{r^3} \quad -(1)$$

with:

$$r_s = \frac{2MG}{c^2} + \frac{a}{r} \quad -(2)$$

The extra term is the attraction:

$$\Delta V = -\frac{am}{r^2}\left(c^2 + \frac{L^2}{r^2}\right) \quad -(3)$$

producing the extra force of attraction:

$$\Delta F = -\frac{\partial \Delta V}{\partial r} = -\frac{2am}{r^3}\left(c^2 + \frac{2L^2}{r^2}\right) \quad -(4)$$

The total potential is

$$V = V_0 + \Delta V \quad -(5)$$

For a circular orbit:

$$\frac{\partial V_0}{\partial r} = 0 \quad -(6)$$

producing:

$$r_0 = \frac{1}{2c^2 r_s}\left(L^2 \pm \left(L^4 - 12L^2c^2r_s^2\right)^{1/2}\right) \quad -(7)$$

2)

An initially stable r_0 will decrease because of the extra attraction force (4).

The equation describing the orbital decay from circular is eq. (7.21) of Marion & Thornton:

$$\frac{d^2 u}{d\theta^2} + u = \frac{\mu r}{l^2} F(r) \quad — (8)$$

where:

$$l = \mu L, \quad — (9)$$

$$u = 1/r \quad — (9a)$$

A logarithmic spiral orbit:

$$r = k \exp(\alpha\theta) \quad — (10)$$

is described by a force law:

$$F(r) = -\frac{l^2}{\mu r^3}\left(\alpha^2 + 1\right) \quad — (11)$$

A spiral orbit is:

$$r = k\theta \quad — (12)$$

is described by:

$$F(r) = -\frac{l^2}{\mu}\left(\frac{6k}{r^4} + \frac{1}{r^3}\right) \quad — (13)$$

So use eq. (4) in eq. (8) to show that the r_s of type (2) results in a spiral-like decay of an initially circular orbit.