DERIVATION OF THE $\mathbf{B}^{(3)}$ FIELD FROM THE SACHS EINSTEIN THEORY OF GENERAL RELATIVITY: METRIC FOR CURVED SPACE-TIME

by

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ABSTRACT

A quaternion-valued curved space-time metric is derived for the calculation of the $\mathbf{B}^{(3)}$ field from the Sachs Einstein theory of general relativity.

KEYWORDS

Curved space-time metric, Sachs Einstein theory of general relativity, $\mathbf{B}^{(3)}$ field.

1 INTRODUCTION

The field equations of classical general relativity, the Einstein equations, form a set of ten metric equations in ten unknowns. This set of equations represents a field theory of gravitation, and in the language of group theory is based on reducible representations of the Einstein group, which corresponds to the topological group T, a locally covariant compact connected topological field satisfying the second axiom of countability. The most general mathematical system with which to express the laws of physics in general relativity is then the set of quaternions[1]. We refer to this system as “extended general relativity”.

The field equations of electromagnetism can be derived[1] from extended general relativity. The starting point is the realization that the covariance group underlying the tensor form of Einstein and Maxwell field equations are reducible. This is because they entail[1] reflection symmetry, not required by general relativity, as well as the continuous symmetry of the Einstein group E, a Lie group. The irreducible form of the Einstein field equations is obtained by factorizing the differential line element using the quaternion form:

$$ds = q^\mu(X)dX_\mu \tag{1}$$

where the metric q^μ is a set of quaternion valued components of a four vector.

Therefore the basic variable that represents the generalized spacetime that is appropriate to general relatiivity is a 16-component variable. Such a generalization must then EXTEND the physical predictions of the usual tensor forms of the general relativity of gravitation and the standard vector representation of the Maxwell-Heaviside theory of electromagnetism. This extension gives new physical phenomena such as the $\mathbf{B}^{(3)}$ field[2]-[5] of O(3) electrodynamics. The $\mathbf{B}^{(3)}$ field is obtained in this paper from the quaternion valued component:

$$B^{\mu\nu} = \frac{1}{8}QR(q^\mu q^{\nu*} - q^\nu q^{\mu*}) \tag{2}$$

of the complete electromagnetic field tensor[1] of extended general relativity. In eqn.(2) Q is a constant with the units of weber and R is the scalar curvature. Therefore the metric used to obtain the $B^{\mu\nu}$ field must be one of curved space-time.

In Section 2 we devise a metric that gives the $B^{\mu\nu}$ in curved space-time so that the scalar curvature R remains rigorously non-zero. This metric corresponds to circular polarization, as observed empirically in electromagnetism, and is developed in the complex circular basis ((1),(2),(3))[2]-[5]. The electromagnetic field equations that emerge correspond to an O(3) symmetry gauge field theory of electromagnetism, a theory which is contained within extended general relativity. These are equations of higher symmetry than the Maxwell Heaviside field equations in flat space-time, and produce novel optical phenomena from the first principles of extended general relativity, phenomena such as the inverse Faraday effect (IFE) and its resonance counterpart, radiatively induced fermion resonance (RFR)[2]-[5].

2 DEVELOPMENT OF THE METRIC FOR CIRCULAR POLARIZATION IN CURVED SPACE-TIME

The metric is developed from the first principles of curvilinear coordinate analysis[6] by first considering it in three and four dimensional vector notation and extending it to the four dimensional quaternion form required by the Sachs Einstein theory. We start with basic definitions.

Consider the curve in three dimensional space:

$$\mathbf{r} \;=\; X\mathbf{i} + Y\mathbf{j} + Z\mathbf{k} \tag{3}$$

then the unit vector is defined as:

$$\mathbf{i} \;=\; \frac{\frac{\partial \mathbf{r}}{\partial X}}{\left|\frac{\partial \mathbf{r}}{\partial X}\right|},\; etc. \tag{4}$$

the metric vector is defined as:

$$\mathbf{g}_X \;=\; \left|\frac{\partial \mathbf{r}}{\partial X}\right|\mathbf{i} \;=\; \mathbf{i},\; etc. \tag{5}$$

and the metric element as:

$$g_X \;=\; \left|\frac{\partial \mathbf{r}}{\partial X}\right| \;=\; 1,\; etc \tag{6}$$

The line element is defined as:

$$d\mathbf{r} \;=\; \frac{\partial \mathbf{r}}{\partial X}dX + \frac{\partial \mathbf{r}}{\partial Y}dY + \frac{\partial \mathbf{r}}{\partial Z}dZ \;=\; \mathbf{g}_X dX + \mathbf{g}_Y dY + \mathbf{g}_Z dZ \tag{7}$$

The three metric vectors are defined as:

$$\mathbf{g}_X \;=\; \frac{\partial \mathbf{r}}{\partial X};\quad \mathbf{g}_Y \;=\; \frac{\partial \mathbf{r}}{\partial Y};\quad \mathbf{g}_Z \;=\; \frac{\partial \mathbf{r}}{\partial Z} \tag{8}$$

and the Einstein metric tensor is:

$$g_{ij} \;=\; g_{ji} \;=\; \frac{\partial \mathbf{r}}{\partial X} \cdot \frac{\partial \mathbf{r}}{\partial Y},\ etc. \tag{9}$$

If $g_{ij} = 0$ for $i \neq j$ then the coordinate system is orthogonal.

If we now consider the functional relations that define the complex circular basis[2]-[5]:

$$e^{(1)} \;=\; \frac{1}{\sqrt{2}}(X - iY), \quad e^{(2)} \;=\; \frac{1}{\sqrt{2}}(X + iY), \quad e^{(3)} \;=\; Z \tag{10}$$

then

$$\begin{aligned} X &= \frac{1}{\sqrt{2}}\left(e^{(1)} + e^{(2)}\right) \\ Y &= \frac{1}{\sqrt{2}}\left(e^{(1)} - e^{(2)}\right) \\ Z &= e^{(3)} \end{aligned} \tag{11}$$

are curvilinear coordinate relations in three dimensional space. The curve (3) can therefore be written as:

$$\mathbf{r} \;=\; \frac{1}{\sqrt{2}}\left(e^{(1)} + e^{(2)}\right)\mathbf{i} \;+\; \frac{1}{\sqrt{2}}\left(e^{(1)} - e^{(2)}\right)\mathbf{j} \;+ e^{(3)}\mathbf{k} \tag{12}$$

giving the three unit vectors in the complex circular basis

$$\begin{aligned} \mathbf{e}^{(1)} &= \frac{\frac{\partial \mathbf{r}}{\partial e^{(1)}}}{\left|\frac{\partial \mathbf{r}}{\partial e^{(1)}}\right|} \;=\; \frac{1}{\sqrt{2}}\left(\mathbf{i} + i\mathbf{j}\right) \\ \mathbf{e}^{(2)} &= \frac{\frac{\partial \mathbf{r}}{\partial e^{(2)}}}{\left|\frac{\partial \mathbf{r}}{\partial e^{(2)}}\right|} \;=\; \frac{1}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right) \\ \mathbf{e}^{(3)} &= \mathbf{k} \end{aligned} \tag{13}$$

In this basis the line element is:

$$d\mathbf{r} \;=\; \frac{\partial \mathbf{r}}{\partial e^{(1)}} de^{(1)} + \frac{\partial \mathbf{r}}{\partial e^{(2)}} de^{(2)} + \frac{\partial \mathbf{r}}{\partial e^{(3)}} de^{(3)} \tag{14}$$

and the metric vectors are:

giving the three unit vectors in the complex circular basis

$$\begin{aligned} \mathbf{g}^{(1)} &= \frac{\partial \mathbf{r}}{\partial e^{(1)}} \;=\; \frac{1}{\sqrt{2}}\left(\mathbf{i} + i\mathbf{j}\right) \;=\; \mathbf{q}^{(1)} \\ \mathbf{g}^{(2)} &= \frac{\partial \mathbf{r}}{\partial e^{(2)}} \;=\; \frac{1}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right) \;=\; \mathbf{q}^{(2)} \\ \mathbf{g}^{(3)} &\;=\; \mathbf{k} \;=\; \mathbf{q}^{(3)} \end{aligned} \tag{15}$$

extending to four dimensions produces the metric four vectors:

$$\begin{aligned}
\mathbf{q}^{(1)} &= \left(q_0^{(1)}, \mathbf{q}^{(1)}\right) = \left(0, \frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}, 0\right) \\
\mathbf{q}^{(2)} &= \left(q_0^{(2)}, \mathbf{q}^{(2)}\right) = \left(0, \frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, 0\right) \\
\mathbf{q}^{(3)} &= \left(0,\ 0,\ 0,\ 1\right) \\
\mathbf{q}^{(4)} &= \left(1,\ 0,\ 0,\ 0\right)
\end{aligned} \tag{16}$$

The metric three vectors form an O(3) symmetry cyclic relation:

$$\begin{aligned}
\mathbf{q}^{(1)} \times \mathbf{q}^{(2)} &= i\mathbf{q}^{(3)*} \\
\mathbf{q}^{(2)} \times \mathbf{q}^{(3)} &= i\mathbf{q}^{(1)*} \\
\mathbf{q}^{(3)} \times \mathbf{q}^{(1)} &= i\mathbf{q}^{(2)*}
\end{aligned} \tag{17}$$

where the asterisk denote complex conjugation.

In order to decide whether the metric vectors derived in this way are metrics of curved space-time or flat space-time we calculate the Einstein metric tensor. If this is unit diagonal then the metrics correspond to flat space-time, but otherwise they correspond to curved space-time, as required by the Sachs Einstein theory.

First consider the Cartesian unit vector system $(\mathbf{i}, \mathbf{j}, \mathbf{k})$. The metric tensor is formed from

$$\mathbf{q}_1 = \mathbf{i}; \quad \mathbf{q}_2 = \mathbf{j}; \quad \mathbf{q}_3 = \mathbf{k}; \tag{18}$$

and is given by

The metric three vectors form an O(3) symmetry cyclic relation:

$$\begin{aligned}
g_{11} &= \mathbf{q}_{(1)} \cdot \mathbf{q}_{(1)} = 1 \\
g_{22} &= \mathbf{q}_{(2)} \cdot \mathbf{q}_{(2)} = 1 \\
g_{33} &= \mathbf{q}_{(3)} \cdot \mathbf{q}_{(3)} = 1
\end{aligned} \tag{19}$$

so:

$$g_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{20}$$

representing an orthogonal coordinate system in flat, Euclidean space.

Next consider the complex circular coordinate system $\left(\mathbf{e}^{(1)}, \mathbf{e}^{(2)}, \mathbf{e}^{(3)}\right)$. The metric tensor is formed from the ordinary products of the complex unit vectors:

$$\begin{aligned}
\mathbf{q}^{(1)} &= \frac{1}{\sqrt{2}}(\mathbf{i} - i\mathbf{j}) \\
\mathbf{q}^{(2)} &= \frac{1}{\sqrt{2}}(\mathbf{i} + i\mathbf{j}) \\
\mathbf{q}^{(3)} &= \mathbf{k}
\end{aligned} \tag{21}$$

so

$$\mathbf{q}_1 \cdot \mathbf{q}_2 = \mathbf{q}_2 \cdot \mathbf{q}_1 = \mathbf{q}_3 \cdot \mathbf{q}_3 = 1 \tag{22}$$

all other elements being zero, and

$$g_{ij} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{23}$$

representing a non-orthogonal coordinate system in a non-Euclidean, curved, three dimensional space.

The metric four-vectors (16) are therefore metrics of curved space-time.

To find the quaternion-valued equivalent of eqn. (16) we use the Pauli matrix basis, so:

$$\begin{aligned}
\mathbf{e}^{(1)} &= \frac{1}{\sqrt{2}}(\mathbf{i} - i\mathbf{j}) \leftrightarrow \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - i\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\right) := \sigma^{(1)} \\
\mathbf{e}^{(2)} &= \frac{1}{\sqrt{2}}(\mathbf{i} + i\mathbf{j}) \leftrightarrow \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + i\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\right) := \sigma^{(2)} \\
\mathbf{e}^{(3)} &= \mathbf{k} \leftrightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} := \sigma^{(3)}
\end{aligned} \tag{24}$$

and

$$\begin{aligned}
\left[\sigma^{(2)}, \sigma^{(3)}\right] &= -2\sigma^{(1)*} \\
\left[\sigma^{(3)}, \sigma^{(1)}\right] &= -2\sigma^{(2)*} \\
\left[\sigma^{(1)}, \sigma^{(2)}\right] &= -2\sigma^{(3)*}
\end{aligned} \tag{25}$$

The quaternion-valued metric in the complex circular basis is therefore:

$$q^{\mu} = \left(\sigma_0, \sigma^{(1)}, \sigma^{(2)}, \sigma^{(3)}\right) \tag{26}$$

which is a four-vector with sixteen components in all. The quaternion conjugate of this metric is:

$$q^{\mu *} = \left(\sigma_0, -\sigma^{(1)}, -\sigma^{(2)}, -\sigma^{(3)}\right) \tag{27}$$

and the $\mathbf{B}^{(3)}$ field is obtained from:

$$B^{12} \;=\; -\frac{1}{8}QR\big(\sigma^{(1)}\sigma^{(2)}\cdot\sigma^{(2)}\sigma^{(1)}\big) \;=\; \frac{1}{4}QR\sigma^{(3)} \tag{28}$$

In vector notation this result is:

$$\mathbf{B}^{(3)} \;=\; \frac{1}{8}QR\mathbf{k} \tag{29}$$

To check that q^{μ} represents a metric of curved space-time we use the relation given by Sachs[1]:

$$-\frac{1}{2}\big(q^{\mu}q^{\nu *} + q^{\nu}q^{\mu *}\big) \;\leftrightarrow\; \sigma_0 g^{\mu\nu} \tag{30}$$

i.e. the symmetric second rank metric tensor $g^{\mu\nu}$ of Einstein's formulation of general relativity corresponds to the symmetric sum from the quaternion theory: In the flat space-time represented by:

$$\sigma^{\mu} \;=\; \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\right) \tag{31}$$

eqn.(30) gives the result:

$$g^{\mu\nu} \;=\; \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{32}$$

the metric of flat space-time in Einstein's general relativity.

The use of the metric:

$$q^{\mu} \;=\; \big(\sigma_0,\, \sigma^{(1)},\, \sigma^{(2)},\, \sigma^{(3)}\big) \tag{33}$$

gives:

$$g^{\mu\nu} \;=\; \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{34}$$

a metric tensor of curved space-time.

To represent the circularly polarized electromagnetic field in the curved space-time described by eqn.(34), the electromagnetic phase, ϕ, must be incorporated in eqns.(10), which become:

$$\begin{aligned} e^{(1)} &= \frac{1}{\sqrt{2}}(X - iY)e^{i\phi} \\ e^{(2)} &= \frac{1}{\sqrt{2}}(X + iY)e^{-i\phi} \\ e^{(3)} &= Z \\ \phi &= \omega t - \kappa Z \end{aligned} \tag{35}$$

Here ω is the angular frequency, t the time, κ the wave-vector, Z the position vector coordinate (not to be confused with Z). The phase is therefore generally covariant, because it is a number:

$$\phi = \phi' = \omega' t' - \kappa' Z' \tag{36}$$

From eqn.(35),

$$\begin{aligned} X &= \frac{1}{\sqrt{2}}(e^{(1)}e^{-i\phi} + e^{(2)}e^{i\phi}) \\ Y &= \frac{-i}{\sqrt{2}}(e^{(2)}e^{i\phi} - e^{(1)}e^{-i\phi}) \\ Z &= e^{(3)} \end{aligned} \tag{37}$$

and we obtain the metric vectors:

$$\begin{aligned} \mathbf{q}^{(1)} &= \frac{\frac{\partial \mathbf{r}}{\partial e^{(1)}}}{\left|\frac{\partial \mathbf{r}}{\partial e^{(1)}}\right|} = \frac{1}{\sqrt{2}}(\mathbf{i} + i\mathbf{j})e^{-i\phi} \\ \mathbf{q}^{(2)} &= \frac{\frac{\partial \mathbf{r}}{\partial e^{(2)}}}{\left|\frac{\partial \mathbf{r}}{\partial e^{(2)}}\right|} = \frac{1}{\sqrt{2}}(\mathbf{i} - i\mathbf{j})e^{i\phi} \\ \mathbf{q}^{(3)} &= \mathbf{k} \end{aligned} \tag{38}$$

These form an O(3) symmetry cyclic relation, eqn.(17), and the metric tensor from eqn. (38) takes the same form as eqn.(23), i.e. the tensor is one of curved space-time.

The quaternion valued metric four vector from eqn. (38) is:

$$q^{\mu} = \left(q^{\mu(0)},\, q^{\mu(1)},\, q^{\mu(2)},\, q^{\mu(3)}\right) \tag{39}$$

where

$$\begin{aligned} q^{\mu(0)} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \sigma_0 \\ q^{\mu(1)} &= \sigma^{(1)}e^{i\phi} \\ q^{\mu(2)} &= \sigma^{(2)}e^{-i\phi} \\ q^{\mu(3)} &= \sigma^{(3)} \end{aligned} \tag{40}$$

Using eqn.(40) in eqn.(30) gives the curved space-time metric tensor (34).

The metric vector $\mathbf{q}^{(1)}$ in parametric form is the equation:

$$\mathbf{q}^{(1)} = \frac{1}{\sqrt{2}}e^{-i\phi}\left(1,\, i,\, 0\right) \tag{41}$$

whose scalar curvature in inverse square meters is[2]-[5]:

$$R = \left|\frac{\partial^2 \mathbf{q}^{(1)}}{\partial Z^2}\right| = \kappa^2 = \frac{\omega^2}{c^2} \tag{42}$$

From eqn.(40) we can define the three magnetic fields associated with circular polarization in the Sachs Einstein theory:

$$\begin{aligned}\mathbf{B}^{(1)} &= B^{(0)}\mathbf{q}^{(1)}\\ \mathbf{B}^{(2)} &= B^{(0)}\mathbf{q}^{(2)}\\ \mathbf{B}^{(3)} &= B^{(0)}\mathbf{q}^{(3)}\end{aligned} \tag{43}$$

These three fields form the B Cyclic Theorem of O(3) electrodynamics[2]-[5]:

$$\begin{aligned}\mathbf{B}^{(1)}\times\mathbf{B}^{(2)} &= iB^{(0)}\mathbf{B}^{(3)*}\\ \mathbf{B}^{(2)}\times\mathbf{B}^{(3)} &= iB^{(0)}\mathbf{B}^{(1)*}\\ \mathbf{B}^{(3)}\times\mathbf{B}^{(1)} &= iB^{(0)}\mathbf{B}^{(2)*}\end{aligned} \tag{44}$$

3 DISCUSSION

The $\mathbf{B}^{(3)}$ field is obtained from the term (2) of the Sachs-Einstein electromagnetic field tensor through a choice of metric corresponding to circular polarization in electromagnetism, eqns.(40). In vector notation, and with this choice of metric, the $\mathbf{B}^{(3)}$ field[2]-[5] is given by:

$$\begin{aligned}\mathbf{B}^{(3)*} &= -\frac{1}{8}QR\,\mathbf{q}^{(1)}\times\mathbf{q}^{(2)} \;=\; B^{(0)}\mathbf{q}^{(3)}\\ \mathbf{k} &= \mathbf{q}^{(3)}\\ B^{(0)} &= \frac{1}{8}QR\end{aligned} \tag{45}$$

and is therefore derivable from extended general relativity[1]. The transverse $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ are given by

$$\begin{aligned}\mathbf{B}^{(1)*} &= -i\frac{QR}{8}\,\mathbf{q}^{(2)}\times\mathbf{q}^{(3}\\ \mathbf{B}^{(2)*} &= -i\frac{QR}{8}\,\mathbf{q}^{(3)}\times\mathbf{q}^{(1}\end{aligned} \tag{46}$$

The electromagnetic field is therefore described directly in terms of the metric, as required for any field in general relativity; the field is the frame itself, and the frame is one of curved space-time, as required for a finite scalar curvature R. The electromagnetic fields $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ have a finite radius, the Thompson radius κ^{-1}, and the tip of the vectors $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ draw out a circular helix of this radius.

The depiction of circular polarization in special relativity on the other hand is one where the electromagnetic field is an entity distinct from the frame, a frame of flat space-time in which $\mathbf{B}^{(1)}$

and $\mathbf{B}^{(2)}$ are solutions to the Maxwell Heaviside equations and where $\mathbf{B}^{(3)}$ does not exist[2]-[5]. In the Sachs Einstein theory of electromagnetism the depiction of circular polarization is one in curved space-time in which $\mathbf{B}^{(3)}$ is identically non-zero and related to $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ through the B Cyclic Theorem(44)[2]-[5]. To distinquish between the two theories the inverse Faraday effect[2]-[5] is used to observe $\mathbf{B}^{(3)}$ directly in plasma, liquids and solids. The resonance equivalent of the inverse Faraday effect is radiatively induced fermion resonance (RFR), which is a novel resonance spectroscopy that leads to ESR and NMR without permanent magnets[2]-[5].

References

[1] M. Sachs, in M. W. Evans, I. Prigogine and S. A. Rice, eds., "Modern Non-linear Optics", second edition, "Advances in Chemical Physics", (Wiley, New York, 2001), vol. 119(1).

[2] several review articles on O(3) electrodynamics in ref. (1).

[3] M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the $\mathbf{B}^{(3)}$ Field", (World Scientific, Singapore, 2001).

[4] M. W. Evans, J. P. Vigier, et alia, "The Enigmatic Photon" series (Kluwer, Dordrecht, 1994 to 1999, softback, 2002), vols. One to five.

[5] M. W. Evans, Physica B, 182,227,237 (1992).

[6] E. G. Milewski (Ed.), "The Vector Analysis Problem Solver" (REA, New Yorkm 1987).