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DERIVATION OF THE $B^{(3)}$ FIELD AND CONCOMITANT VACUUM ENERGY DENSITY FROM THE SACHS THEORY OF ELECTRODYNAMICS

P. K. Anastasovski (1), T. E. Bearden (2), C. Ciubotariu (3), W. T. Coffey (4), L. B. Crowell (5), G. J. Evans (6), M. W. Evans (7, 8), R. Flower (9), A. Labounsky (10), B. Lehnert (11), M. Mészáros (12), P. R. Molnár (12), J. K. Moscicki (13), S. Roy (14), and J. P. Vigier (15)

Institute for Advanced Study, Alpha Foundation
Institute of Physics, 11 Rutafa Street, Building H
Budapest, H-1165, Hungary

Also at:

(1) Faculty of Technology and Metallurgy, Department of Physics, University of Skopje, Republic of Macedonia; (2) CTEC Inc, Huntsville, Alabama; (3) Institute for Information Technology, Stuttgart University, Stuttgart, Germany; (4) Department of Microelectronics and Electrical Engineering, Trinity College, Dublin 2, Ireland; (5) Department of Physics and Astronomy University of New Mexico, Albuquerque, New Mexico; (6) Ceredigion County Council, Aberaeron, Wales, United Kingdom; (7) former Edward Davies Chemical Laboratories, University College of Wales, Aberystwyth SY32 1NE, Wales, United Kingdom; (8) sometime JRF, Wolfson College, Oxford, United Kingdom; (9) Applied Science Associates and Temple University Center for Frontier Sciences, Philadelphia, Pennsylvania; (10) The Boeing Company, Huntington Beach, California; (11) Alfvén Laboratory, Royal Institute of Technology, Stockholm, S-100 44, Sweden; (12) Alpha Foundation, Institute of Physics, 11 Rutafa Street, Building H, Budapest, H-1165, Hungary; (13) Smoluchowski Institute of Physics, Jagiellonian University, ul Reymonta 4, Krakow, Poland; (14) Indian Statistical Institute, Calcutta, India; (15) Laboratoire de Gravitation et Cosmologie Relativistes, Université Pierre et Marie Curie, Tour 22-12, 4 ème étage, 4 Place Jussieu, 7525 Paris, Cedex 05, France.

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The archetypical and phaseless vacuum magnetic flux density of O(3) electrodynamics, the $\mathbf{B}^{(3)}$ field, is derived from the irreducible representation of the Einstein group and is shown to be accompanied by a vacuum energy density which depends directly on the square of the scalar curvature R of curved spacetime. The $\mathbf{B}^{(3)}$ field and the vacuum energy density are obtained respectively from the non-Abelian part of the field tensor $F_{\mu\nu}$ and the non-Abelian part of the metrical field equation. Both of these terms are given by Sachs [5].

Key words: $\mathbf{B}^{(3)}$ fields, vacuum energy density, Sachs's electrodynamics, motionless electromagnetic generator.

1. INTRODUCTION

In higher symmetry electrodynamics, the electromagnetic sector is considered to have a symmetry different from the received U(1). In O(3) electrodynamics [1-3], the sector symmetry is described by the O(3) group, and field components appear which do not exist in U(1) symmetry electrodynamics. One of these components is the phase $\mathbf{B}^{(3)}$ field, first introduced in 1992 [4] and developed since then [1-3]. The $\mathbf{B}^{(3)}$ component is defined by the cross product of two circularly polarized and complex conjugate potentials $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$ and through the proportionality factor $g = K/A^{(0)}$:

$$\mathbf{B}^{(3)*} = -ig\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \tag{1}$$

where K is the wavenumber and $A^{(1)}$ the magnitude of $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$. In this Letter, the fundamental definition of $\mathbf{B}^{(3)}$ in vacuum is derived from the non-Abelian component of the field tensor in Sachs theory of electrodynamics [5], which theory is based on a consideration of irreducible representation of Einstein group in general relativity and therefore is the most general theory of electrodynamics currently available, and the fundamental definition used to produce the electromagnetic energy in vacuum due to $\mathbf{B}^{(3)}$, and observable in a reproducible and repeatable device such as the motionless electromagnetic generator (MEG) [6].

2. NON-ABELIAN TERM IN THE FIELD TENSOR OF SACHS'S THEORY

This term is

$$F_{\mu\nu} = \frac{1}{8}\left(q_\mu q_\nu^* - q_\nu q_\mu^*\right) RQ, \tag{2}$$

where q_μ, etc. are quaternion-valued metric components, where R is the scalar curvature and Q is the charge [5]. In the flat spacetime limit, the quaternion-valued components q_μ, etc. reduce to σ_μ, etc., the relation between which is:

$$\begin{aligned}
\sigma^0\sigma^1 - \sigma^1\sigma^0 &= 0,\\
\sigma^0\sigma^2 - \sigma^2\sigma^0 &= 0,\\
\sigma^0\sigma^3 - \sigma^3\sigma^0 &= 0,\\
\sigma^1\sigma^2 - \sigma^2\sigma^1 &= \partial i\sigma^3,\\
\sigma^2\sigma^3 - \sigma^3\sigma^2 &= \partial i\sigma^1,\\
\sigma^3\sigma^1 - \sigma^1\sigma^3 &= \partial i\sigma^2.
\end{aligned} \tag{3}$$

A product such as $q^\mu q^{\mu *}$ is non-commutative but non-antisymmetric in μ and ν. The asterisk here denotes quaternion conjugation, which changes the sign of the time component. There are generally covariant components such as:

$$\begin{aligned}
q_x &= q^1 = (q^{10}, q^{11}, q^{12}, q^{13}),\\
q_y &= q^2 = (q^{20}, q^{21}, q^{22}, q^{23}),\\
q_z &= q^3 = (q^{30}, q^{31}, q^{32}, q^{33}),\\
q_0 &= q^0 = (q^{00}, q^{01}, q^{02}, q^{03}),
\end{aligned} \tag{4}$$

and the row-vector column-vector product

$$[0, \sigma_x, 0, 0]\begin{bmatrix} 0 \\ 0 \\ \sigma_y \\ 0 \end{bmatrix} = 0 \tag{5}$$

results in zero. The quaternion product therefore cannot be interpreted in this way. This result leaves possibilities such as:

$$\begin{aligned}
q_x^1 q_y^{2*} - q_y^2 q_x^{1*} &= \sigma i q^3,\\
\sigma_x\sigma_y - \sigma_y\sigma_x &= \partial i\sigma^3.
\end{aligned} \tag{6}$$

Also, q_x^1 and q_y^{2*} must be one-valued and be 2×2 matrices; thus

$$q_x^1 = \begin{bmatrix} 0 & q_x^1 \\ q_x^1 & 0 \end{bmatrix}, q_y^{2*} = \begin{bmatrix} 0 & -iq_y^2 \\ iq_y^2 & 0 \end{bmatrix}. \tag{7}$$

The metric q^μ must be a function of n^μ, whose space part is represented by ((1), (2), (3)); therefore it is possible to define scalar components:

$$q_x^{(1)} := A_x^{(1)}/A^{(0)},\ q_y^{(2)} := A_y^{(2)*}/A^{(0)}. \tag{8}$$

Accordingly,

$$\mathbf{q}^{(3)*} = -i\mathbf{q}^{(1)} \times \mathbf{q}^{(2)} = \mathbf{q}^{(0)} \tag{9}$$

represents O(3) electrodynamics in this vector basis, and

$$q_x^{(1)} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}}e^{i\phi} \\ \frac{i}{\sqrt{2}}e^{i\phi} & 0 \end{bmatrix}, q_y^{(2)*} = \begin{bmatrix} 0 & \frac{-1}{\sqrt{2}}e^{i\phi} \\ \frac{ie^{-i\phi}}{\sqrt{2}}e^{i\phi} & 0 \end{bmatrix}, \tag{10}$$

where ϕ is the (Wu-Yang) electromagnetic phase. The $\mathbf{B}^{(3)}$ field is therefore defined by

$$\mathbf{B}^{(3)*} = -iqA^{(0)2}\mathbf{q}_x^{(1)} \times \mathbf{q}_y^{(2)*} = -i\mathbf{A}_x^{(1)} \times \mathbf{A}_y^{(2)*}, \tag{11}$$

with metric components

$$\mathbf{q}^{(1)} = \frac{1}{\sqrt{2}}(i\mathbf{i} + \mathbf{j})e^{i\phi} = \mathbf{q}^{(2)*}. \tag{12}$$

3. FIELD EQUATIONS AND ENERGY IN THE VACUUM

The definition of $\mathbf{B}^{(3)}$ is therefore obtained from the field equations given by Sachs [5], in particular from the non-Abelian part of

$$\frac{1}{4}(\kappa_{\rho\lambda}q^\lambda + q^\lambda\kappa_{\rho\lambda}^+) + \frac{1}{8}Rq = kT_\rho, \tag{13}$$

a part which can be written in the circular bases ((1), (2), (3)) as

$$k\mathbf{T}^{(3)} = \frac{1}{8}\mathbf{R}q^{(3)}. \tag{14}$$

This vanishes in flat spacetime, where R is zero. So the energy concomitant with the $\mathbf{B}^{(3)}$ field also vanishes in flat spacetime and is given by

$$En = \int B^{(3)2}dV = \int \left(\frac{1}{8}QR\right)^2 dV = \int (Qk)^2\mathbf{T}^{(3)} \cdot \mathbf{T}^{(3)*}dV. \tag{15}$$

Here $\kappa_{\mu\nu}$ is the curvature tensor [5] and k the Einstein constant. It is concluded that the energy due to $\mathbf{B}^{(3)}$ can be expressed in terms of components of the energy momentum tensor or in terms of the scalar curvature. In both cases, the energy is expressed in terms of fundamental quantities in general relativity and, from the irreducible representations of the Einstein group, is fundamentally non-zero.

4. MOTIONLESS ELECTROMAGNETIC GENERATOR

This result can be applied to explain the basic principle of the motionless electromagnetic generator (MEG) [6], in that the energy (1), i.e.,

$$En = \int \mathbf{B}^{(3)} \cdot \mathbf{B}^{(3)*} dV, \tag{16}$$

is taken from curved spacetime and changed into the energy

$$En_s = \frac{1}{\mu_0} \int \mathbf{B} \cdot \mathbf{B} dV, \tag{17}$$

where $\mathbf{B}$ is a static magnetic field component of the core of the MEG. The latter is a considerably more complicated device [6] that this simple principle would seem to allow, but nevertheless the principle applies to all static magnetic field components of any repeatable and reproducible device. The basic idea is that electromagnetic energy is taken from Riemannian spacetime and changed into electromagnetic energy in usable form. This procedure is sometimes known as "free energy" or "taking energy from the vacuum."

In summary, we have derived and defined the $\mathbf{B}^{(3)}$ component of O(3) electrodynamics from the Sachs theory of electrodynamics, which considers irreducible representations of the Einstein group of general relativity. This $\mathbf{B}^{(3)}$ component is then used to explain the working principles of the motionless electromagnetic generator (MEG).

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