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On the representation of the Maxwell Heaviside equations in terms of the Barut field four vector

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Abstract: It is shown that the Maxwell Heaviside equations can be written simply in terms of a field four vector $G^\mu = (0, cB + iE)$, where B is magnetic flux density and E is electric field strength. The method given is much simpler that than used by Barut [1] in expressing the Maxwell Heaviside equations as neutrino equations. Under Lorentz transformation G^μ transforms as a four vector, leading to novel interpretation. The field four vector G^μ leads directly to cyclical relations between magnetic fields in the vacuum.

Key words: Maxwell Heaviside equations – Barut field vector – Lorentz transformation

1. Introduction

It is not well known that electric and magnetic fields can be expressed in terms of four vectors. However, Barut [1] used such a four vector to express the Maxwell Heaviside equations as neutrino equations. In this communication a much simpler expression is found for the Maxwell Heaviside equations in terms of the four vectors:

$$G^\mu = (0, cB + iE) \tag{1}$$

and

$$H^\mu = (0, H + icD) \tag{2}$$

where E is electric field strength, D is electric displacement: B is magnetic flux density and H is magnetic field strength in S. I. units. The Lorentz transform of these field four vectors leads to different expressions for the field components than the standard transformation of the field four tensor $F^{\mu\nu}$ [2, 3], even though the basic equations are the same, the Maxwell Heaviside equations. This shows that it is necessary but not sufficient merely to consider the Lorentz transform of field components. It is necessary and sufficient to consider the Lorentz transform of field *equations*. The Maxwell Heaviside equations can be expressed either in terms of field four vectors or as usual in terms of field four tensors, yet the field components behave differently under Lorentz transformation. The only possible way out of this paradox is to always consider the Lorentz transformation of the underlying field equations.

2. The Maxwell Heaviside equations in terms of field four vectors

Firstly consider the free space Maxwell Heaviside equations using the four vector [1]:

$$G^\mu = (0, cB + iE) \tag{3}$$

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Clearly:

$$G^{\mu} G_{\mu} = G^{\mu'} G'_{\mu} \tag{4}$$

is a Lorentz invariant proportional to the field energy. The free space field equations are simply:

$$\partial_{\mu} G^{\mu} = 0 \tag{5}$$

$$[\partial_i, G_j] + i\,[\partial_0, G_k] = 0. \tag{6}$$

In vector notation and S. I. units these give:

$$\nabla \cdot B = 0; \quad \nabla \cdot E = 0; \tag{7}$$

$$\nabla \times E + \frac{\partial B}{\partial t} = 0; \quad \nabla \times B - \frac{1}{c^2}\frac{\partial E}{\partial t} = 0. \tag{8}$$

For field matter interaction we use the field four vector:

$$H^{\mu} = (0, H + icD) \tag{9}$$

and the Maxwell Heaviside equations are simply:

$$\partial_{\mu} H^{\mu} = i\varrho c$$

$$[\partial_i, H_j] + i\,[\partial_0, H_k] = J_k. \tag{10}$$

In vector notation and S. I. units these are:

$$\nabla \cdot B = 0; \quad \nabla \cdot D = \varrho;$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J; \quad \nabla \times E + \frac{\partial B}{\partial t} = 0. \tag{11}$$

Here ϱ is scalar charge density and J is vector current density. Using the well known fact that ϱ and J themselves form the components of a four vector the Maxwell Heaviside equations for field matter interaction can be combined into just one relation between four vectors:

$$\frac{1}{c}(-i\,\partial_{\mu} H^{\mu},\ ([\partial_i, H_j] + i\,[\partial_0, H_k])) = \left(\varrho, \frac{1}{c} J_k\right) \tag{12}$$

and the free space equivalent is:

$$(\partial_{\mu} G^{\mu},\ [\partial_i, G_j] + i\,[\partial_0, G_k]) = 0. \tag{13}$$

3. Discussion

It is well known that the free space Maxwell Heaviside equations can be written in tensor form as [2, 3]:

$$\partial_{\mu} \tilde{F}^{\mu\nu} := 0 \tag{14}$$

$$\partial_{\mu} F^{\mu\nu} = 0 \tag{15}$$

where the tensor $\tilde{F}^{\mu\nu}$ is the dual of $F_{\varrho\sigma}$. A Lorentz boost in the z direction of G^{μ} produces:

$$\begin{aligned} cB'_x + iE'_x &= cB_x + iE_x \\ cB'_y + iE'_y &= cB_y + iE_y \\ cB'_z + iE'_z &= \gamma(cB'_z + iE'_z) \\ cB'_0 + iE'_0 &= -\gamma\beta(cB'_z + iE'_z) \end{aligned} \tag{16}$$

but a Lorentz boost in the z direction applied to $F^{\mu\nu}$ produces

$$\begin{aligned} cB'_x &= \gamma(cB_x + \beta E_y) \\ cB'_y &= \gamma(cB_y + \beta E_x) \\ cB'_z &= cB'_z \\ B'_0 &= 0 \end{aligned} \tag{17}$$

a completely different result, even though eq. (13) and eqs. (14) and (15) are both precisely equivalent to eqs. (7) and (8). The only common factor is that the charge current four tensor transforms in the same way for vector representation (12) and its equivalent tensor representation, which is:

$$\partial_{\mu} \tilde{F}^{\mu\nu} := 0 \tag{18}$$

$$\partial_{\mu} H^{\mu\nu} := J^{\nu}. \tag{19}$$

The vector representation develops a timelike component under a Lorentz boost in z, while the tensor representation does not. However, the underlying equation in both cases are the Maxwell Heaviside equations, which transform covariantly in both cases, and, obviously, in the same way for both the vector and tensor representations of the field components.

If we define the vectors

$$a := \frac{1}{2}(cB + iE) \tag{20}$$

$$b := \frac{1}{2}(cB - iE) \tag{21}$$

then

$$\begin{aligned} &[a_x, a_y] = i\,a_z \quad \text{et cyclicum} \\ &[b_x, b_y] = i\,b_z \quad \text{et cyclicum} \\ &[a_i, b_j] = 0 \quad (i, j = x, y, z). \end{aligned} \tag{22}$$

Thus a and b each generate a group SU(2), and the two groups commute. The Lorentz group is then SU(2) $\otimes$ SU(2) and transforms in a well defined way labelled by two angular momenta (j, j'), the first corresponding to a and the second to b. Thus a and b are generators of the Lorentz group.

The vector G^{μ} also transforms as the Pauli Lubanski vector in particle rest frame [4], for example a photon with a very tiny mass. This strongly suggests that the vector representation (1) is for the intrinsic spin of the electromagnetic field at a fundamental level (one photon level), while the tensor representation is for orbital angular momentum. This is also suggested by O(3) electrodynamics [5–12], where the fundamental intrinsic spin of the field is given by the Evans Vigier vector $B^{(3)}$ obeying the cyclic relations:

$$B^{(1)} \times B^{(2)} = iB^{(0)} B^{(3)*} \quad \text{et cyclicum.} \tag{23}$$

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