ULTRA HIGH RESOLUTION FERMION RESONANCE AND MAGNETIC RESONANCE IMAGING

ABSTRACT

The Sakurai equation is used to show the existence in principle of ultra high resolution fermion resonance and ultra high resolution, low field, magnetic resonance imaging.

INTRODUCTION

In this paper, the Sakurai equation {1} is used to show the existence in theory of ultra high resolution fermion resonance and magnetic resonance imaging (MRI). The fermion resonance is caused by circularly polarized radio frequency electromagnetic radiation, and so the new technique is similar in principle to contemporary optical NMR {2-6}, (ONMR), in which a circularly polarized laser is used to enhance NMR sensitivity and spatial resolution and to produce low field MRI {7}. It is shown that, in theory, a circularly polarized radio frequency field interacting with an electron or proton produces resonance at a much higher frequency than that attainable even with the most powerful of contemporary superconducting magnets. This implies that the spectral resolution of ESR, NMR and MRI apparatus can be greatly enhanced in principle using a circularly polarized radio or microwave frequency pump. The operational principle is similar therefore to contemporary ONMR, where the pump is a circularly polarized visible frequency laser.

DISCUSSION

To describe the intrinsic spin of the fermion in non-relativistic quantum mechanics, the standard method ascribed to Sakurai {1} is used. The Hamiltonian in the absence of the radio frequency field is, as usual:

$$H = \frac{1}{2m}(\sigma \cdot p)^2 + V \tag{1}$$

in the SU(2) basis where σ is the Pauli matrix, p the fermion momentum, m the fermion mass and V the potential energy. The Sakurai equation of motion is therefore:

$$H\psi = En\psi \tag{2}$$

where E_n is the eigen-energy and ψ the wave-function. In the presence of an ordinary static magnetic field (B), the minimal prescription replaces p by $p + eA$, where $B = \nabla \times A$. It can be shown that, in this case, the Sakurai equation gives the famous half integral spin term:

$$H\psi = \frac{e\hbar}{2m}\sigma \cdot B\psi \tag{3}$$

usually ascribed to the relativistic Dirac equation. This term is of course the basis of all contemporary ESR, NMR, ONMR and MRI. Note carefully that the existence of all these techniques depends on the quantum ansatz, $p \rightarrow -i\hbar\nabla$, and they are pure quantum effects. They are not however, relativistic effects.

If, however, A is complex valued, as in an electromagnetic field {8-10}, it can be shown straightforwardly that the Sakurai equation contains the additional classical, real valued and physical Hamiltonian:

$$H(RFR) = i\frac{e^2}{2m}\sigma \cdot \mathrm{A} \times \mathrm{A}^*. \tag{4}$$

Using the S.I. relation between beam intensity I and A^2:

$$H(RFR) = \left(\frac{\mu_0 c e^2}{2m}\right)\frac{I}{\omega^2}\sigma_Z, \tag{5}$$

where μ_0 is the vacuum permeability. Fermion resonance occurs between the states of the Pauli matrix σ_Z. For one electron, the resonance frequency is:

$$\omega_{res\,(\mathrm{electron})} = 1.007 \times 10^{28}\frac{I}{\omega^2} \tag{6}$$

and for one proton (allowing for the proton nuclear g factor):

$$\omega_{res\,(\mathrm{proton})} = 1.532 \times 10^{25}\frac{I}{\omega^2}. \tag{7}$$

Under the conditions of contemporary ONMR {2-7}, eqn. (7) gives very small shifts, for example 0.12 Hz for a 528.7 nm laser of 10 watts/cm^2. intensity. However, if the frequency ω in eqn. (7) is reduced to the microwave or radio frequency range, the resonance frequency can occur in the infrared or visible frequencies for easily accessible I. For example, if I is tuned to 10 watts per square centimetre, and $f = \omega/2\pi$ to 100 MHz, the resonance frequency from eqn. (7) occurs at 20.6 cm^{-1} in the far infrared. This is much higher than any nuclear resonance attainable with contemporary superconducting magnets. The resolution of the NMR spectrum is increased dramatically, and the same applies in principle to ESR and MRI, allowing much sharper imaging.

There is a considerable amount of empirical evidence available for the existence of the term H(RFR), starting with its demonstration in the paramagnetic inverse Faraday effect by Pershan et alia {11}. The interaction Hamiltonian (5) is observable under ideal conditions when a microwave beam from a circular waveguide interacts with a fermion beam. These experiments are not available to date, but should be straightforward to carry out. The ONMR technique {2-7} shows clear advantages over conventional NMR but works on a slightly different principle from eqn. (5), which is radiatively induced fermion resonance (RFR) in the sense that the radiation **itself** produces fermion resonance.

If combined with contemporary ONMR and low field MRI technology, {2-7} it should be possible to build a low field MRI instrument {7} which produces ultra high resolution (utilizing the ultra high resonance frequencies obtainable from eqn. (7)). The Hamiltonian (4), finally, can be expressed as:

$$H = -\frac{e\hbar}{2m}\sigma \cdot \boldsymbol{B}^{(3)} \tag{8}$$

where $\boldsymbol{B}^{(3)} = -i(e/\hbar)\mathrm{A} \times \mathrm{A}^*$ is a **topological magnetic field** {12} of contemporary non-Abelian gauge field theory applied to electrodynamics.

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