LORENTZ INVARIANCE OF $A_\mu A_\mu = A_\mu{}' A_\mu{}'$

For a Z boost in Jackson's notation:

$$A_\mu{}' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & i\gamma\beta \\ 0 & 0 & -i\gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} A_X \\ A_Y \\ A_Z \\ iA_0 \end{bmatrix} = \begin{bmatrix} A_X \\ A_Y \\ \gamma A_Z - \gamma\beta A_0 \\ -i\gamma\beta A_Z + i\gamma A_0 \end{bmatrix}$$

$$A_\mu A_\mu = A_X^2 + A_Y^2 + A_Z^2 - A_0^2$$

$$\begin{aligned} A_\mu{}' A_\mu{}' &= A_X^2 + A_Y^2 + \left(\gamma A_Z - \gamma\beta A_0\right)^2 - \left(\gamma A_0 - \gamma\beta A_Z\right)^2 \\ &= A_X^2 + A_Y^2 + \gamma^2 A_Z^2 - 2\gamma^2\beta A_0 A_Z + \gamma^2\beta^2 A_0^2 - \left(\gamma^2 A_0^2 - 2\gamma^2\beta A_0 A_Z + \gamma^2\beta^2 A_Z^2\right) \\ &= A_X^2 + A_Y^2 + \left(\gamma^2 - \gamma^2\beta^2\right) A_Z^2 - \left(\gamma^2 - \gamma^2\beta^2\right) A_0^2 \end{aligned}$$

We know that $A_\mu A_\mu = A_\mu{}' A_\mu{}'$, so this can be true for all v because $\gamma^2\left(1 - \beta^2\right) = 1$.