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ONE-PHOTON SAGNAC EFFECT: CORRESPONDENCE TO O(3) ELECTRODYNAMICS

ABSTRACT

It is shown that the Sagnac effect exists in O(3) electrodynamics in the one photon limit, both with platform at rest and in motion. In the U(1) Yang-Mills gauge field theory (Maxwell-Heaviside theory), there is no Sagnac effect in either case on the classical level, and therefore no one photon Sagnac effect. In other words, the holonomy difference for clockwise and anti-clockwise motion in U(1) electrodynamics is zero and is non-zero in O(3) electrodynamics. The effect is the same in O(3) electrodynamics for an observer on and off the platform, and is a consequence of an O(3) gauge transformation.

INTRODUCTION

There have been many attempts during the last eighty years to explain the Sagnac effect, or Michelson-Gale effect. These attempts have been reviewed recently by Barrett {1}, Fleming {2}, Kelly {3} and Vigier {4}. It is well known that in Maxwell-Heaviside electrodynamics, the effect does not exist, either with platform at rest or in motion {1}. This is a simple consequence of the fact that the free space Maxwell-Heaviside equations and d'Alembert wave equation are invariant under motion reversal symmetry. As demonstrated by Post {5}, the Lorentz invariance of these equations in free space forbids the existence of the Sagnac effect. A well known attempt was made by Post {5} to modify the free space constitutive relation to account for the Sagnac effect, but there is a flaw in this treatment, as pointed out by Barrett {1}. Anandan {5a} has attempted to explain the effect in relativistic physics using the holonomy difference generated by a Wu-Yang phase using U(1) covariant derivatives, but as shown in section 2, there is a flaw in this argument. In section 3, the classical Sagnac effect with platform at rest and in motion is explained using the holonomy difference generated using O(3) covariant derivatives. The correct experimental result is obtained with platform at rest and in motion. The motion of the platform is assumed to produce an O(3) gauge transformation and the holonomy difference is a phase shift in the internal gauge space of the theory. In section 4, the theory is taken to one photon limit, and compared with a recent explanation by Vigier {4} in term of photon mass.

LACK OF U(1) HOLONOMY DIFFERENCE

It can be shown straightforwardly that there is no holonomy difference if the Wu-Yang phase is calculated with U(1) covariant derivatives. For circular clockwise (*C*) and anti-clockwise (*A*) paths, consider the boundary:

$$X^2 + Y^2 = 1 \tag{1}$$

of the path of the light beam in the Sagnac effect. The line integral vanishes around this boundary:

$$\begin{aligned}\oint dr &= \int_0^{2\pi} dX + \int_0^{2\pi} dY = -\int_0^{2\pi} \sin\phi \, d\phi + \int_0^{2\pi} \cos\phi \, d\phi \\ &= 0 = -\oint dr\end{aligned} \tag{2}$$

Therefore, since κ is not a function of r in the U(1) phase:

$$\phi \equiv \omega t - \kappa \cdot r, \tag{3}$$

then

$$\oint \kappa \cdot dr = -\oint \kappa \cdot dr = = 0 \tag{4}$$

Therefore the holonomy in the A and C directions is the same:

$$\exp\left(i\oint_C \kappa \cdot dr \right) = \exp\left(-i\oint_A \kappa \cdot dr \right) = 1 \tag{5}$$

The holonomy difference is zero and there is no Sagnac effect with platform at rest, contrary to observation. Furthermore, in the U(1) Yang-Mills gauge field theory (Maxwell-Heaviside theory), the only electromagnetic vector potential present in free space

$$A^{(1)} = A^{(2)*} = \frac{A^{(0)}}{\sqrt{2}}\left(ii + j \right) e^{i(\omega t - \kappa \cdot r)} \tag{6}$$

is always transverse to the path of the light beam, therefore

$$A^{(1)} \cdot r = 0 \tag{7}$$

and the Wu-Yang holonomy is the same for A and C loops:

$$\exp\left(i\oint_C A^{(1)} \cdot dr \right) = \exp\left(-i\oint_A A^{(1)} \cdot dr \right) = 1 \tag{8}$$

Again, there is no Sagnac effect with platform at rest. Owing to the gauge invariance of the U(1) theory, there is no Sagnac effect with platform in motion. This can be seen from the fact that the Maxwell-Heaviside and d'Alembert equations are gauge invariant constructs in free space, and do not change with frame rotation. The same equations are also invariant under motion reversal symmetry, which generates the A path from the C path, so there is no Sagnac effect with platform at rest by fundamental symmetry. This is contrary to observation {1-4}.

O(3) HOLONOMY DIFFERENCE

In the O(3) Yang-Mills theory of electrodynamics, there is an internal gauge space {6-14} which is a complex representation of physical three dimensional space using a basis ((1), (2), (3)) well founded on the empirical existence of circular polarization. Holonomy differences in the internal space are measurable phase differences {1}. The Sagnac effect with platform at rest and in motion is one of these, and is an optical Aharonov-Bohm effect {14}. The vector potential and field tensor are also vectors in the physical space ((1), (2), (3)), and the theory has been well developed and tested {6-14}. Therefore:

$$A^{\mu} = A^{\mu(1)} e^{(1)} + A^{\mu(2)} e^{(2)} + A^{\mu(3)} e^{(3)} \tag{9}$$

$$G^{\mu\nu} = G^{\mu\nu(1)}e^{(1)} + G^{\mu\nu(2)}e^{(2)} + G^{\mu\nu(3)}e^{(3)} \tag{10}$$

where $e^{(1)}$, $e^{(2)}$, and $e^{(3)}$ are unit vectors of the basis ((1), (2), (3)) {6-14}. This is straightforward Yang-Mills theory {1,14} applied to electrodynamics using the O(3) group as the internal gauge symmetry. The holonomy for this gauge theory is given by {1, 12, 14}:

$$\gamma = \exp\left(\iint\left[D_\mu, D_\nu\right]d\sigma^{\mu\nu}\right) \tag{11}$$

$$\gamma = \exp\left(-ig\iint\partial_\mu A_\nu - \partial_\nu A_\mu d\sigma^{\mu\nu} - g^2\iint\left[A_\mu, A_\nu\right]d\sigma^{\mu}\right) \tag{12}$$

but as shown in section (1), U(1) type integrals such as:

$$I\left(U(1)\right) = \iint\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)d\sigma^{\mu\nu} \tag{13}$$

vanish for both A and C loops, leaving the non-zero holonomy

$$\gamma = \exp\left(-i\kappa^2 Ar\right) = \exp\left(-ig\iint \boldsymbol{B}^{(3)}\cdot d\boldsymbol{Ar}\right) \tag{14}$$

with platform at rest. Here, κ is the wavenumber, $\boldsymbol{B}^{(3)}$ is the Evans-Vigier field, and Ar is the area of the Sagnac loop {1-4, 15}. Under motion reversal symmetry:

$$\hat{T}\left(\boldsymbol{B}^{(3)}\right) = -\boldsymbol{B}^{(3)} \tag{15}$$

so there is a holonomy difference as measured empirically. The real part of this is the observable phase difference, or fringe pattern with platform at rest:

$$\phi = \cos\left(2\kappa^2 Ar \pm 2\pi n\right). \tag{16}$$

This derivation can be self-checked by using a closed loop in Minkowski space-time with O(3) covariant derivatives. If $G_{\mu\nu}$ represents the O(3) field tensor in condensed notation {16}, the holonomy generated by the closed loop or round trip in one direction is:

$$\gamma_A = \exp\left(-ig\iint G_{\mu\nu}dS^{\mu\nu}\right) \tag{17}$$

and in the other direction:

$$\gamma_C = \exp\left(ig\iint G_{\mu\nu}dS^{\mu\nu}\right) \tag{18}$$

where $S^{\mu\nu}$ is the area enclosed by the loop. The holonomy represents a rotation in the internal space. This is a general result for all internal gauge group symmetries and the holonomy can be expressed in the general internal space (a, b, c) as:

$$\gamma = \exp\left(\mp ig\iint\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - ig\varepsilon_{abc}A_\mu^b A_\nu^c\right)dS^{\mu\nu}\right) \tag{19}$$

If the internal symmetry is U(1), the holonomy in either direction is:

$$\gamma(U(1)) = \exp\left(\mp ig\iint\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)dS^{\mu\nu}\right) = \exp\left(\mp ig\oint A_\mu dx^\mu\right) = 1 \qquad (20)$$

and the ordinary Stokes theorem can be used to show that there is no holonomy difference, checking the results of section 2.

If the internal group symmetry is O(3) in the basis ((1), (2), (3)), we obtain

$$\begin{aligned}
&\exp\left(\mp ig\iint\left(\partial_\mu A_\nu^{(1)} - \partial_\nu A_\mu^{(1)}\right)dS^{\mu\nu}\right) = 1 \\
&\exp\left(\mp ig\iint\left(\partial_\mu A_\nu^{(2)} - \partial_\nu A_\mu^{(2)}\right)dS^{\mu\nu}\right) = 1 \\
&\exp\left(\mp ig\iint\left(\partial_\mu A_\nu^{(3)} - \partial_\nu A_\mu^{(3)}\right)dS^{\mu\nu}\right) = 1
\end{aligned} \qquad (21)$$

and the only source of holonomy difference is the commutator term, which is written in general as {16}:

$$\gamma = \exp\left(\mp g^2\iint \varepsilon_{abc}A_\mu^a A_\nu^c dS^{\mu\nu}\right). \qquad (22)$$

Consider the special case:

$$\gamma = \exp\left(\mp g^2\iint\left(A_X^{(1)}A_Y^{(2)} - A_X^{(2)}A_Y^{(1)}\right)dS^{XY}\right) \qquad (23)$$

By definition

$$A^{(1)} = A^{(2)*} = \frac{A^{(0)}}{\sqrt{2}}\left(ii + j\right)e^{i(\omega t - \kappa Z)} \qquad (24)$$

so

$$A_X^{(1)}A_Y^{(2)} - A_X^{(2)}A_Y^{(1)} = iA^{(0)2} \qquad (25)$$

and by definition:

$$B^{(3)} = -igiA^{(0)2} = \kappa A^{(0)} = B^{(0)} \qquad (26)$$

because

$$g = \frac{\kappa}{A^{(0)}}. \qquad (27)$$

The holonomy is therefore:

$$\begin{aligned}
\gamma &= \exp\left(\mp ig\iint B^{(3)}dS^{XY}\right) \equiv \exp\left(\mp igB^{(3)}Ar\right) \\
&= \exp\left(\mp i\kappa^2 Ar\right)
\end{aligned} \qquad (28)$$

and the difference in holonomy for A and C loop is:

$$\Delta\gamma = \exp\left(2i\kappa^2 Ar\right) \tag{29}$$

giving the observable phase difference:

$$\phi = \cos\left(2\kappa^2 Ar \pm 2\pi n\right) \tag{30}$$

with the platform at rest.

In the O(3) theory, there is a further change of orientation in the internal physical space if the platform is spun around the (3) or Z axis by the rotation generator {1, 14}:

$$S = \exp\left(iJ_Z\alpha\left(x^\mu\right)\right). \tag{31}$$

The effect on the wave four-vector, in condensed notation, {16} is a non-Abelian gauge transform:

$$\kappa_\mu \rightarrow S\kappa_\mu S^{-1} - i\left(\partial_\mu S\right)S^{-1} \tag{32}$$

where

$$\kappa^\mu = \kappa^{\mu(1)}e^{(1)} + \kappa^{\mu(2)}e^{(2)} + \kappa^{\mu(3)}e^{(3)} \tag{33}$$

We are interested in the effect on $\kappa^{\mu(3)}$, which is the wave-vector of the light propagating around the Sagnac loops A and C. The effect is, from eqn. (32):

$$\kappa_\mu^{(3)} \rightarrow \kappa_\mu^{(3)} \pm \partial_\mu\alpha \tag{34}$$

where α is the angle in the plane of the platform such that the angular velocity of the platform is:

$$\Omega = \frac{\partial\alpha}{\partial t}. \tag{35}$$

Considering the index $\mu = 0$, we arrive at:

$$\omega \rightarrow \omega \pm \Omega. \tag{36}$$

The extra effect of the rotation of the platform is therefore to shift the observable phase by:

$$\Delta\Delta\xi = \frac{Ar}{c^2}\left(\left(\omega+\Omega\right)^2 - \left(\omega-\Omega\right)^2\right) = \frac{4\omega\Omega Ar}{c^2} \tag{37}$$

$$\Delta\Delta\phi = \cos\left(\frac{4\omega\Omega Ar}{c^2}\right) \tag{38}$$

which is precisely the observed result {1-4}.

This holonomy difference can be expressed through a non-Abelian Stokes theorem {17} from:

$$\gamma = \exp\left(\mp i\oint \kappa_Z^{(3)} dZ\right) \tag{39}$$

where

$$\oint dZ = 2\pi\, Rn \tag{40}$$

i.e. is some multiple of the circumference $2\pi R$ of the Sagnac loop. The same result is obtained by Barrett {1} and expressed as his eqn. (51). The quantity

$$g_m = \frac{1}{V}\iint B^{(3)} dAr = \frac{1}{V}\iint \kappa\, A^{(0)} dAr \tag{41}$$

has the units of "magnetic charge", but is topological in origin.

The phase shift $\Delta\Delta\phi = 4\omega\Omega\, Ar/c^2$ corresponds to the well known time shift $\Delta\Delta t = 4\Omega\, Ar/c^2$ first derived by Sagnac, and quoted by Vigier {4} as his eqn. (1). For all practical purposes, (FAPP), this result is the same as that obtained with finite photon mass theory {4}. It is accurate to one part in 10^{23} as reported by Bilger et al. {18}. However, O(3) electrodynamics is unmodified special relativity. The photon mass theory used by Vigier {4} reduces to:

$$\Delta\Delta t = \frac{4Ar\Omega}{c^2 - v^2} \sim \frac{4Ar\Omega}{c^2} \tag{42}$$

where $v = R\Omega$ for $Ar = \pi R^2$. Therefore Vigier's result is approximate whereas the topological result, eqn (38), is exact for a platform spinning about the Z axis. In order to incorporate photon mass into O(3) electrodynamics, it is necessary to modify the basic equations {6-14} to Proca equations. The exact agreement between O(3) electrodynamics and the experimental results seems to indicate that photon mass is very small, in accord with the estimates given by Einstein, de Broglie and Vigier of less than 10^{-68} kg of magnitude.

The result $\Delta\Delta t = 4\Omega\, Ar/c^2$ is frame invariant experimentally, i.e. is the same to an observer on and off the spinning Sagnac platform. This is in exact agreement with standard special relativity, where c is frame invariant. In photon mass special relativity {4}, c is constant FAPP in the laboratory. The area Ar is frame invariant, and Ω is topological in origin, coming from the inhomogeneous term in eqn. (32). It also represents the frame invariant angular frequency of one set of axes with respect to another. In O(3) electrodynamics, the Sagnac effect is represented by a gauge transformation $\omega \rightarrow \omega \pm \Omega$. In the sum or difference $\omega \pm \Omega$, neither ω nor Ω depends on the rotation generator S, and Ω appears through the functional dependence of α on x^μ in special relativity. The Sagnac effect is therefore topological in nature, depending ultimately on the structure of the vacuum. It is a form of optical Aharonov-Bohm effect {1,14}. The same result is obtained for particles such as the electron and neutron {4}, again emphasizing the topological nature of the effect.

ONE PHOTON SAGNAC EFFECT

In the one photon limit, O(3) electrodynamics produces the result:

$$\frac{\kappa}{A^{(0)}} = \frac{e}{\hbar}. \tag{43}$$

Substituting this into

$$\gamma = \exp\left(\mp i \frac{e}{\hbar} B^{(3)} Ar \right) \tag{44}$$

for a beam made up of one photon, the flux $B^{(3)}Ar$ becomes $\hbar/e$, and so in the one photon limit:

$$\gamma = \exp(\mp i). \tag{45}$$

The observable phase difference is therefore non-zero for one photon in O(3) electrodynamics. The effect with the platform in rotation is the same as eqn. (38) for one photon.

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