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Paper 14

The Evans-Vigier Field, $\boldsymbol{B}^{(3)}$, in Dirac's Original Electron Theory: a New Theorem of Field-Fermion Interaction

Dirac's original electron theory is used to show that a classical electromagnetic field interacts with quantized fermion half integral spin through the Evans-Vigier field, $\boldsymbol{B}^{(3)} = -i(e/\hbar)\boldsymbol{A}\times\boldsymbol{A}^*$, where $\boldsymbol{A}\times\boldsymbol{A}^*$ is the conjugate product of field vector potential, $\boldsymbol{A}$, with its own complex conjugate $\boldsymbol{A}^*$; and where $e/\hbar$ is the ratio of elementary charge to Dirac constant. Dirac's theory of the electron is recovered when $\boldsymbol{A}^*$ is replaced by $\boldsymbol{A}$. However, since $\boldsymbol{A}$ is complex from d'Alembert's equation in vacuo, $\boldsymbol{B}^{(3)}$ is always non-zero. It becomes very large at low frequencies for moderate field intensity, and has several important practical applications.

14.1 Introduction

The original description by Dirac [1] of his famous theory of the electron is used in this communication to show that the classical electromagnetic field interacts with quantized fermion spin through the Evans-Vigier field [2—10],

$$B^{(3)} = -i\frac{e}{\hbar}A \times A^* , \tag{2.14.1}$$

where $A \times A^*$ is the conjugate product of the field vector potential with its own complex conjugate A^*. Here e is elementary charge and $\hbar$ the Dirac constant, and $A \times A^*$ is pure imaginary, so that $B^{(3)}$ is real and physical. Equation (2.14.1) represents a new fundamental theorem in field theory, and can be generalized within quantum electrodynamics [11—14]. The demonstration of Eq. (2.14.1) is given in Sec 14.2, and is based closely on the original description by Dirac [1]. The latter's theory is recovered exactly if A^* in Eq. (2.14.1) and related equations is replaced by A. In other words, Dirac assumes [1] that A is pure real, so that $A \times A^*$ (and $B^{(3)}$) is zero in his theory. More generally however, the d'Alembert equation in vacuo [11—14] shows that A is complex, with a real and imaginary part. In consequence the cross product [2—10] $A \times A^*$ is not zero. For a transverse plane wave A the conjugate product $A \times A^*$ is pure imaginary and free of the electromagnetic phase. It is, furthermore, directly proportional to beam power density I (W m^{-2}) and inversely proportional to the square of beam angular frequency (ω). At low frequencies $B^{(3)}$ becomes very large (megatesla) for moderate I (of the order ten watts per square centimeter). This property is potentially of great practical utility, the existence of $B^{(3)}$ being the result of Dirac's fundamental theory of the electron. In Sec. 14.3 we indicate avenues of generalization of this result within contemporary quantum electrodynamics.

14.2 The Conjugate Product in Dirac's Original Electron Theory

Dirac has given a clear description of his own theory of the electron interacting with a classical electromagnetic field in chapter eleven of Ref. 1. In this section Dirac's description is followed closely to show the existence of the field $B^{(3)}$ of Eq. (2.14.1) for a complex electromagnetic potential four-vector, A_μ, the general solution of the vacuum d'Alembert equation

[15]. The space part of A_μ is denoted A, and its complex conjugate by A^*. In his original development [1], Dirac assumed that A is classical and real, so that $A \times A^*$ is zero, because he was aiming at a theory of the anomalous Zeeman effect in a *static* magnetic field. Empirical evidence is now available [16—20], however, to show that for electromagnetic waves, $A \times A^*$ is experimentally observable in magneto-optical phenomena, and is non-zero experimentally.

Dirac's development is based [1] on the quantum relativistic wave equation,

$$\left(p_0 + eA_0 - \rho_1(\sigma \cdot (p + eA)) - \rho_3 mc\right)\psi = 0, \tag{2.14.2}$$

for an electron (or more generally a fermion) in a classical electromagnetic field. Equation (2.14.2) is written here in contemporary standard *(S.I.)* Units, (whereas Dirac uses Gaussian units). The energy momentum four-vector in *S.I.* units in Dirac's notation is

$$p_\mu := (p_0 , p), \tag{2.14.3}$$

and the potential four-vector is

$$A_\mu := (A_0 , A). \tag{2.14.4}$$

The matrices ρ_1, and ρ_3 are [1]

$$\rho_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \rho_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \tag{2.14.5}$$

and ψ is a column four-vector, described in contemporary terms as the Dirac four-spinor. (In his original account [1] Dirac does not use parity inversion to interrelate spinor components, as is the contemporary practice [11—14].

The classical Hamiltonian for an electron in a classical electromagnetic field is now used by Dirac [1] as a guideline to the properties of Eq. (2.14.2). We proceed here by following this method closely, but by indicating at each stage the modifications which enter into the Dirac theory of the electron when $\boldsymbol{A}$ is complex rather than real. The wave equation expected from analogy with the classical theory is Eq. (2.14.30) of chapter eleven of Dirac [1],

$$\left((p_0 + eA_0)^2 - (\boldsymbol{p} + e\boldsymbol{A})^2 - m^2c^2\right)\psi = 0, \tag{2.14.6}$$

and is written for real $\boldsymbol{A}$. For complex A_μ Eq. (2.14.6) becomes

$$\begin{aligned}&\Big((p_0 + eA_0)(p_0 + eA_0^*) - \\ &(\boldsymbol{p} + e\boldsymbol{A})\cdot(\boldsymbol{p} + e\boldsymbol{A}^*) - m^2c^2\Big)\psi = 0.\end{aligned} \tag{2.14.7}$$

In order to make his theory of the electron resemble Eq. (2.14.2) as closely as possible, Dirac multiplies Eq. (2.14.2) by the factor $p_0 + eA_0 + \rho_1(\sigma\cdot(\boldsymbol{p} + e\boldsymbol{A})) + \rho_3 mc$, which for a complex potential four-vector becomes $p_0 + eA_0^* + \rho_1\left(\sigma\cdot(\boldsymbol{p} + e\boldsymbol{A}^*)\right) + \rho_3 mc$, giving the product

$$\begin{aligned}&\Big((p_0 + eA_0^*)(p_0 + eA_0) - (\sigma\cdot(\boldsymbol{p} + e\boldsymbol{A}^*))\\ &\times(\sigma\cdot(\boldsymbol{p} + e\boldsymbol{A})) - m^2c^2\\ &- \rho_1\Big((p_0 + eA_0^*)(\sigma\cdot(\boldsymbol{p} + e\boldsymbol{A}))\\ &- (\sigma\cdot(\boldsymbol{p} + e\boldsymbol{A}^*))(p_0 + eA_0)\Big)\Big)\psi = 0.\end{aligned} \tag{2.14.8}$$

This replaces Eq. (2.14.31) of Dirac's original theory [1]. The product (2.14.8) contains several terms which are developed as follows. The conjugate product term leading to Eq. (2.14.1) originates in $e^2(\sigma\cdot\boldsymbol{A}^*)(\sigma\cdot\boldsymbol{A})\psi$. As shown by Dirac [1], if $\boldsymbol{B}$ and $\boldsymbol{C}$ are any two three-dimensional vectors that commute with σ, then

$$(\sigma\cdot\boldsymbol{B})(\sigma\cdot\boldsymbol{C}) = \boldsymbol{B}\cdot\boldsymbol{C} + i(\sigma\cdot\boldsymbol{B}\times\boldsymbol{C}). \tag{2.14.9}$$

In contemporary terms σ is known as the Pauli matrix [11—14]. For a pure real $\boldsymbol{A}$, there is only one term on the right hand side of Eq. (2.14.9), but for a complex $\boldsymbol{A}$, there enters into the Dirac theory of the electron a new term, which describes the interaction of the conjugate product $\boldsymbol{A}\times\boldsymbol{A}^*$ of the classical electromagnetic field with the non-classical matrix σ. This appears to be an exceedingly useful new result, because resonance can be induced between the two spin states of the fermion, in direct analogy with *NMR* or *ESR*, but instead of using a cumbersome permanent magnet (static magnetic field) to do this we may now use an ordinary radio frequency electromagnetic field generator.

Following the development by Dirac [1], but allowing now for complex $\boldsymbol{A}$, we set

$$\boldsymbol{p} \to -i\hbar\nabla, \tag{2.14.10}$$

and obtain the terms

1) $\boldsymbol{p} \times \boldsymbol{p} = \boldsymbol{0}$, (2.14.11a)

2) $e\boldsymbol{A} \times \boldsymbol{p} = \boldsymbol{0}$, (2.14.11b)

3) $e\boldsymbol{p} \times \boldsymbol{A}^* = -i\hbar e \nabla \times \boldsymbol{A}^* = -i\hbar e \boldsymbol{B}^*$. (2.14.11c)

The last of these uses the quantum prescription Eq. (2.14.10) to describe the interaction of the intrinsic spin of the fermion with the magnetic component of the electromagnetic field,

$$\boldsymbol{B}^* = \nabla \times \boldsymbol{A}^*, \tag{2.14.12}$$

and for a static magnetic field, leads to the famous result [1] that the intrinsic spin angular momentum of a fermion is half integral in the non-relativistic, non-classical limit. In our case, $\boldsymbol{B}^*$ is an electromagnetic plane wave and averages to zero over many cycles of the field. Term three is therefore of no further interest in our analysis. Similarly, we follow Dirac in discarding term one, the cross product $\boldsymbol{p} \times \boldsymbol{p}$. The term $e\boldsymbol{A} \times \boldsymbol{p}$ is also discarded, as usual, because it is a classical quantity multiplying a del operator, $\boldsymbol{p} = -i\hbar\nabla$ [1], operating on zero. (In contrast, the term $e\boldsymbol{p} \times \boldsymbol{A}^*$ becomes $-ie\hbar\nabla \times \boldsymbol{A}^*$, which is a del operator on $\boldsymbol{A}^*$, and this is non-zero.)

Continuing in this way we find that Dirac's original equation (2.14.34) of chapter eleven of his classic text [1] is replaced for complex $\boldsymbol{A}$ by

$$\Big((p_0 + eA_0^*)(p_0 + eA_0) - (\boldsymbol{p} + e\boldsymbol{A}^*) \cdot (\boldsymbol{p} + e\boldsymbol{A}) - ie^2 \sigma \cdot \boldsymbol{A}^* \times \boldsymbol{A} - m^2c^2 - e\hbar\sigma \cdot \boldsymbol{B}^* + i\rho_1 e\hbar\sigma \cdot \boldsymbol{E} \Big)\psi = 0, \tag{2.14.13}$$

in which there appears the conjugate product term, and in which products involving real $\boldsymbol{A}$ are suitably modified using elementary complex algebra (i.e., real, physical, quantities are obtained by multiplying complex ones by their complex conjugates). The term in $\sigma \cdot \boldsymbol{e}$ in our Eq. (2.14.13) can contain a real part if $\boldsymbol{E}$ is a complex plane wave, but the symmetry of this real part is $\hat{T}$ negative, $\hat{P}$ negative [21], and this is not physical. (This is the same kind of reasoning used to argue that there is no electric counterpart of the Faraday effect, or inverse Faraday effect [16—20].) For this reason, and also because $\boldsymbol{E}$ is oscillatory and averages to zero over many field cycles, we take no further interest here in this term. In so doing it is assumed that the term,

$$\text{Real}\ (A_0^* \boldsymbol{A} - \boldsymbol{A}^* A_0) = \boldsymbol{0}. \tag{2.14.14}$$

In the transverse gauge, $A_0 = 0$, and in the Coulomb gauge, A_0 is a real constant which may be zero, so Eq. (2.14.14) is satisfied in both gauges. There may, conceivably, be a gauge in which $A^{(0)}$ is complex, but the physical results of Dirac's electron theory must always be gauge independent.

So the final result of our calculation is

$$\Big((p_0 + eA_0^*)(p_0 + eA_0) - (\boldsymbol{p} + e\boldsymbol{A}^*) \cdot (\boldsymbol{p} + e\boldsymbol{A}) - m^2c^2 - ie^2\sigma \cdot \boldsymbol{A}^* \times \boldsymbol{A} \Big)\psi = 0, \tag{2.14.15}$$

in which there appears the gauge independent term $-ie^2\sigma \cdot \boldsymbol{A}^* \times \boldsymbol{A}$ which allows resonance to occur between the two energy states of the matrix σ, the conjugate product $\boldsymbol{A}^* \times \boldsymbol{A}$ playing the role of a magnetic field defined by Eq. (2.14.1) [2—10]. This is a result of Dirac's relativistic quantum theory of the electron in a classical electromagnetic field. More rigorously, it can also be obtained with contemporary quantum electrodynamics [11—14], in which there are small radiative corrections leading to phenomena such as the Lamb shift and the anomalous magnetic moment of the electron [11—14].

The non-relativistic limit of Eq. (2.14.15) can be obtained using the standard approximations [1],

$$En \sim mc^2, \qquad \boldsymbol{p} \sim \mathbf{0}, \tag{2.14.16}$$

in which case we obtain the interaction energy eigenvalue [1, 2, 14],

$$W := En - mc^2 = \frac{e^2c^2(\sigma\cdot\boldsymbol{A})(\sigma\cdot\boldsymbol{A}^*)}{En + mc^2 + ecA_0} - ecA_0 . \tag{2.14.17}$$

In the radiation gauge, $A_0 = 0$, and this result reduces to

$$W := En - mc^2 = \frac{e^2}{2m}(\boldsymbol{A}\cdot\boldsymbol{A}^* + i\sigma\cdot\boldsymbol{A}\times\boldsymbol{A}^*) . \tag{2.14.18}$$

Using Eq. (2.14.1), the resonance term becomes the familiar equation for the anomalous Zeeman effect,

$$W_R = -\frac{e}{m}\frac{\hbar}{2}\sigma\cdot\boldsymbol{B}^{(3)}, \tag{2.14.19}$$

a result which shows that the Evans-Vigier field $\boldsymbol{B}^{(3)}$ always exists in field-fermion interaction. In other words, the classical electromagnetic field acts on the fundamental half integral spin of a fermion as if it were a magnetic field, $\boldsymbol{B}^{(3)}$. We shall see in Sec. 14.3 that this field has very useful properties.

14.3 Properties of $\boldsymbol{B}^{(3)}$ in Classical and Quantum Electrodynamics

In terms of intensity (I) and angular frequency (ω), $\boldsymbol{B}^{(3)}$ is [22,23],

$$\boldsymbol{B}^{(3)} = \frac{e\mu_0 c}{\hbar}\frac{I}{\omega^2}\boldsymbol{e}^{(3)} = 5.723\times 10^{16}\frac{I}{\omega^2}\boldsymbol{e}^{(3)}, \tag{2.14.20}$$

where $\boldsymbol{e}^{(3)}$ is a unit vector in the propagation axis (3) of the radiation, and μ_0 the permeability in vacuo in *S.I.* units [24]. For a given intensity, therefore, $\boldsymbol{B}^{(3)}$ is inversely proportional to the square of field angular frequency (radians $s^{-1} = 2\pi$ f where f is in hertz, or cycles per second). Whenever the classical electromagnetic field interacts with a fermion of quantum mechanics the $\boldsymbol{B}^{(3)}$ field generates a resonance effect between the two states of the non-classical spinor. The resonance occurs, as usual [25—28], when a photon of probe radiation, $\hbar\omega_{res}$, is absorbed to induce a change between the lower and upper energy states defined by the mathematical properties of the spinor, in this case the third Pauli matrix [1, 2, 14],

$$\sigma_Z = \sigma^{(3)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.14.21}$$

In contemporary terms this is referred to as a *spin flip* between the half integral spin (angular momentum) states of the fermion. In *NMR* the fermion may be a proton, or a neutron, in *ESR* an electron. The factor two which gives rise to the everyday term *half integral fermion spin* is a consequence, however, of an approximation, (demonstrated in Sec. 14.2), and the fundamental reason for the existence of *NMR* and *ESR* can be traced to topology [11—14], in that the group space of a fermion is different from that of a boson. Thus *NMR* and *ESR* are examples of absorption spectroscopies [29] based on the Dirac equation, which is solved in a non-relativistic limit. It is advantageous to bear in mind that Dirac derived the equation purely from the general principles of quantum mechanics and special relativity [1]. These considerations (e.g. that the wave equation must be linear in p_0 and $\boldsymbol{p}$) force the use of anti-commuting 4 x 4 matrices, of which the Pauli matrix are component 2 x 2 matrices. Therefore the fermion intrinsic spin has a deeper meaning than angular momentum, and it is well known that the fermion spin cannot be pictured classically (e.g. as a spinning object in space).

There is no reason, therefore, to assume that *NMR* and /or *ESR* must always be practiced with static magnetic fields, (or that a Pauli matrix must always interact with a static magnetic field) and Sec. 14.2 has shown that the conjugate product $\boldsymbol{A} \times \boldsymbol{A}^*$ is a result of the Dirac equation of a fermion in a classical electromagnetic field, using only the standard *minimal prescription* [11—14],

$$p_\mu \rightarrow p_\mu + eA_\mu. \tag{2.14.22}$$

This prescription of relativistic quantum field theory [11—14] is well known to be the result of type two (local) gauge invariance, which is a fundamental assumption in contemporary orthodoxy. It is also well established that the conjugate product produces various observable magneto-optic effects, prominent among which is the inverse Faraday effect [16—18]. Therefore Sec. 14.2 shows (as far as we are aware, for the first time) that the Dirac equation produces the inverse Faraday effect. This is a reassuring result both for field theory and experimental magneto-optics.

For our purposes the Evans-Vigier field, $\boldsymbol{B}^{(3)}$, from Eq. (2.14.1) is orders of magnitude more intense for a given I at radio frequencies (MHz) rather than at visible frequencies (100 to 1000 THz). This is simply the result of its inverse square dependence on field frequency. For I, for example, of 10.0 watts per square centimeter, the $\boldsymbol{B}^{(3)}$ field reaches an order of magnitude of nanotesla at 5,000 cm^{-1} in the visible, (a hundred thousand times weaker than the Earth's mean magnetic field), but for a 10.0 *MHz* radio frequency field it becomes 1.45 *mega*tesla, causing proton resonance in the *infra-red* at about two thousand wavenumbers [22,23]. This proton resonance frequency (a spectral absorption feature) is of course observable

with infra red radiation in a contemporary spectrometer [29]. (It is actually much easier to observe in the infra red, because the need for the resonance causing, rotating, radio frequency field [25—28] in ordinary *NMR* spectrometers is removed.) These simple calculations therefore reveal an *enormous* potential resolution advantage over contemporary *NMR* (of all varieties) because the latter is practiced with a static magnetic field [25—28] of the order 10 tesla maximum. Much of the effort in contemporary *NMR* practice [25—28] revolves around the (expensive) technology of superconducting magnets, whose fields reach, perhaps, 25 tesla [25—28], with great effort and ingenuity. (World records for these fields are claimed regularly.) This results in proton resonance at, say 0.5 *GHz*, around which the chemical shift structure is observed as fine detail, in one, two, or three correlation dimensions [25—28] with many more or less exotic variations in pulse sequences. With the use of $\boldsymbol{A} \times \boldsymbol{A}^*$, and ordinary radio frequency generators, it appears perfectly feasible to advance this 0.5 *GHz* resonance frequency into the infra red (*THz* range) as just described, making superconducting magnets unnecessary. The advantages of such a technology, if realized, are bounded only by the imagination and art of the spectroscopist.

The theory in Sec. 14.2 is based on a classical electromagnetic field, whereas more rigorously, there are radiative corrections due to quantum electrodynamics (*QED*) [11—14], in which there is an extensive late twentieth century literature. In respect of electron resonance, *QED* leads, as is well known [11—14], to a 1% correction to the factor 2 in Eq. (2.14.19). Therefore in practical *NMR* and *ESR*, *QED* does not play a central role. In the delicate interplay between electron and photon however, *QED* is all-important, and future developments in $\boldsymbol{B}^{(3)}$ theory should aim to quantize the electromagnetic field as it interacts with the already quantized fermion. The anomalous Landé factor of the electron, first discussed by Schwinger [30], should in theory become observable with $\boldsymbol{A} \times \boldsymbol{A}^*$ rather than with a static magnetic field, and the original experimental measurements, ably described again by Dirac [1,31], should be repeatable with an optically generated $\boldsymbol{A} \times \boldsymbol{A}^*$. This line of reasoning can clearly be extended to all magnetic effects, of which there are many now known. In each case, the static magnetic field is replaced by the Evans-Vigier field,

$\boldsymbol{B}^{(3)}$, of the incoming electromagnetic beam, or photon beam. There should probably be non-classical photon statistical effects akin to light squeezing and a whole variety of new optical resonance phenomena should eventually emerge now that the existence of $\boldsymbol{A} \times \boldsymbol{A}^*$ is proven from the Dirac equation.

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Paper 15

The Microwave Optical Zeeman Effect Due to $\boldsymbol{B}^{(3)}$

The optical conjugate product of a circularly polarized laser is used in the Dirac equation to show the presence of a microwave frequency optical Zeeman effect which is proportional at a given angular frequency to the Evans-Vigier field $\boldsymbol{B}^{(3)}$ of the microwave radiation. An experimental arrangement to detect this effect is proposed, using ESR technique.

Key words: Microwave optical Zeeman effect, $\boldsymbol{B}^{(3)}$ field.

15.1 Introduction

Recently, it has been demonstrated that the Dirac equation of one electron in a circularly polarized electromagnetic field can be solved to show the existence of the $\boldsymbol{B}^{(3)}$ (Evans-Vigier) field, a magnetic flux density whose classical source is the conjugate product $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ of plane wave solutions of Maxwell's equations in the vacuum. In the appropriate circular basis [1—5] there exist the cyclically symmetric relations between fields,

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \text{ et cyclicum,} \tag{2.15.1}$$

so that $\boldsymbol{B}^{(3)}$ is phase free. Here $B^{(0)}$ is a scalar amplitude (tesla). Whenever radiation magnetizes matter, the effect depends on $\boldsymbol{B}^{(3)}$ at first and second