Paper 1

Ultra Relativistic Inverse Faraday Effect

In the ultra relativistic limit of the inverse Faraday effect it is shown that the magnetization of the sample by the circularly polarized electromagnetic field becomes directly proportional to the $\boldsymbol{B}^{(3)}$ field of the radiation. Observation of such an effect is direct observation of the $\boldsymbol{B}^{(3)}$ field.

Key words: Inverse Faraday effect; Ultra relativistic limit; $\boldsymbol{B}^{(3)}$ field.

1.1 Introduction

The simple geometrical hypothesis of a $\boldsymbol{B}^{(3)}$ field of electromagnetic radiation is supported by the inverse Faraday effect, which to date has been verified experimentally in the non-relativistic limit, defined by relatively low intensity and relatively high frequency [1-5]. In such a limit the observable magnetization is proportional to beam intensity through the factor $B^{(0)}\boldsymbol{B}^{(3)}$, where $\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{k}$. The effect is conventionally interpreted through the conjugate product of plane waves in the vacuum, within the traditional framework of Maxwell-Lorentz theory. The *U(1)* constraint imposed on such a theory implies that there is no $\boldsymbol{B}^{(3)}$ field in the vacuum [6—10]. However, if this group constraint is removed, the B cyclic theorem shows that the conjugate product is proportional to the recently inferred $\boldsymbol{B}^{(3)}$ field by ordinary three dimensional geometry [11]. Such group constraints reduce the generality even of the restricted, linear Maxwell-Lorentz-Cartan electrodynamics, and are, in the last analysis, subjective [12—15]. There is no reason therefore to reject the existence of the $\boldsymbol{B}^{(3)}$ field on the basis of the *U(1)* group restriction applied to a linear theory of electrodynamics. By hypothesis, $\boldsymbol{B}^{(3)}$ is a field of non-linear electrodynamics, and is not hypothesized in Maxwell-Lorentz electrodynamics.

Furthermore, the B Cyclic theorem is by its very nature a non-Maxwellian construct, which shows geometrically that the curl of the $\boldsymbol{B}^{(3)}$ field is zero in the vacuum. This prediction has recently been confirmed experimentally [16]. The theorem has been shown to be rigorously Lorentz covariant [17], and therefore quantizes to a $\hat{C}\hat{P}\hat{T}$ conserving field theory. It therefore has merit in special relativity and quantum mechanics. On these criteria, the $\boldsymbol{B}^{(3)}$ field is as valid as any other field component in theories that are Lorentz covariant. There is therefore no reason to assert that $\boldsymbol{B}^{(3)} = \boldsymbol{0}$, and no reason to assert that it is a field of Maxwell-Lorentz electrodynamics. If observed experimentally therefore, it signifies an advance in the basic structure of electrodynamic theory. The B Cyclic theorem is more fundamental in nature than the Maxwell-Lorentz-Cartan theory, because the theorem is, tautologically, an angular momentum operator relation, i.e., within $\hbar$, a relation between rotation generators of space itself [18]. It is therefore as fundamental as a geometrical hypothesis such as the Pythagorean theorem. For the first time, it applies relativistically correct, Lorentz covariant, geometry to *three* magnetic field (or rotation generator) components in vacuo interlinked by the structure of space-time. If we break this link, we automatically impose a subjective constraint, and change the ordinary topology of space-time. There is no way of arguing against the $\boldsymbol{B}^{(3)}$ field using a *model* of electrodynamics, especially a linear model, because the latter is inevitably constructed in the same space-time. The B Cyclic theorem is as fundamental as the Noether

theorem. Space-time geometry, and the concomitant relation between fields, is valid irrespective of any model of electrodynamics, such as that of Maxwell. The B Cyclic theorem is a geometrical relation between three magnetic field components, all three of which are propagating at c through the vacuum. Such a concept obviously does not exist in Maxwell-Lorentz electrodynamics, yet is Lorentz covariant [17] and $\hat{C}\hat{P}\hat{T}$ conserving. Any criticism of $\boldsymbol{B}^{(3)}$ based on Maxwell-Lorentz electrodynamics is therefore misplaced from the beginning [19]. Such criticism applies an inadequate linear model to a fundamental, topological, non-linear and very fundamental new theorem.

In this paper it is shown that in the ultra relativistic limit of the inverse Faraday effect, the magnetization observable in the sample is proportional directly to $\boldsymbol{B}^{(3)}$, to no other magnetic field component, and is a direct demonstration of its existence. Due to the relation between $\boldsymbol{B}^{(3)}$ and the transverse plane waves [20] this result can be obtained from the relativistic Hamilton-Jacobi equation [21] in a limit of very low frequency and very high intensity. Furthermore, this limit is experimentally accessible [22]. In Sec. 1.2, the reasoning leading to this result is reviewed in terms of delayed action at a distance theory, which was shown by Schwarzschild [23] to be fully equivalent to Maxwell-Lorentz theory. This picture is not adequate for the interpretation of the inverse Faraday effect (*IFE*) however, because as we have seen, $\boldsymbol{B}^{(3)}$ is not defined in Maxwell-Lorentz theory, but is a very useful way of thinking of the inverse Faraday effect reduced to its essence. To properly define $\boldsymbol{B}^{(3)}$, a non-linear theory of electrodynamics is the minimum requirement. In Sec. 1.3, the ultra-relativistic limit is developed for one electron, and the equation given showing the direct relation between $\boldsymbol{B}^{(3)}$ and magnetization. The latter is longitudinally directed and can be proportional only to a longitudinally directed magnetic flux density propagating through a vacuum. This is the $\boldsymbol{B}^{(3)}$ field of the radiation.

1.2 Delayed Action at a Distance

As pointed out by Ritz [23], in an elegant criticism of Maxwell-Lorentz electrodynamics, the latter is exactly equivalent to delayed elementary action at a distance. The inverse Faraday effect can be understood in terms of elementary actions without the use of intervening fields. In the simplest case a circling electron in a transmitter radiates into the vacuum and a time t later an electron in a receiver is set into circular motion. Magnetization in the transmitter becomes magnetization in the receiver, both vectors being longitudinally directed in the Z axis. To describe this process mathematically requires the use of the elementary actions in a relativistic equation of motion, the most convenient one for this purpose is the relativistic Hamilton-Jacobi equation, as pointed out by Landau and Lifshitz [24]. The electronic motion set up in the receiver by the field is circular motion, so the elementary actions must be introduced in such a way as to reproduce this experimental fact, which can be inferred from the observation of magnetization in the Z axis of the receiver due to the electron circling about the Z axis.

When the calculation is carried out [25], the final result can be expressed in terms of a magnetic field $\boldsymbol{B}^{(3)}$, but it can also be expressed as an angular momentum due to delayed elementary action at a distance. In the last analysis it is simply a transfer of angular momentum from the transmitter to the receiver. The intervening agent is postulated to be a field, whose mathematical structure is determined by the partial differential equations known as Maxwell's equations. However, as shown by Ritz [23], these equations are no more than model relations between space-time components, whereas the B Cyclic theorem depends on no model.

The ultra relativistic limit being considered here is one in which the observed magnetization is directly proportional to $\boldsymbol{B}^{(3)}$ in the field theory. This is a simple result obtained after a long and complicated calculation based on the use of elementary action in the Hamilton-Jacobi equation. The same calculation produces the well known non - relativistic limit, which has been confirmed experimentally [1—5]. The elementary action is introduced in such a way as to spin the electron in the receiver, and in such a way as to reproduce the time it takes for the signal to reach the receiver from the transmitter. A combination of spinning and translating motions, combined with a time delay, means that there is a phase present, which is the electromagnetic phase. This appears only in the transverse components of the elementary action, which when put into the Hamilton-Jacobi equation of the electron in the receiver, produces the required circling motion. Because of the B Cyclic Theorem, this is identical with spinning the electron with a $\boldsymbol{B}^{(3)}$ field, and this is exactly what the result gives us in the ultra relativistic limit. The magnetization is directly proportional to $\boldsymbol{B}^{(3)}$. It is simply a magnetic field strength in the receiver produced by the magnetic flux density $\boldsymbol{B}^{(3)}$ of the vacuum, produced in turn by a magnetic field strength in the transmitter.

1.3 Calculation from the Hamilton-Jacobi Equation

The calculation given in Ref. 25 for the relativistic inverse Faraday effect is modified here for the ultra-relativistic limit by correcting the gyromagnetic ratio by the relativistic factor γ. In Gaussian units, the necessary expression is given by Talin *et al.* [26] in their Eq. (3),

$$M_Z = -\frac{|e|}{2mc\gamma} L_Z . \tag{2.1.1}$$

This corrects the usual gyromagnetic ratio [26], $e/2mc$ in Gaussian units. The origin of this correction is given by Talin *et al.* [26] in their Eq. (12),

$$M_Z = \frac{1}{V}\int \frac{dr}{2c}\left(\boldsymbol{r}\times \boldsymbol{j}(\boldsymbol{r},t)\right)_Z , \tag{2.1.2}$$

where M_Z is magnetization, V the sample volume, $\boldsymbol{r}$ the relativistic radius of gyration and $\boldsymbol{j}(\boldsymbol{r},t)$ is a symmetrized current density operator. From this equation they develop their gauge invariant expression (38), which is proportional to the conjugate product of vector potentials, $\boldsymbol{A} \times \boldsymbol{A}^*$. This is the B Cyclic theorem in plasma,

$$\boldsymbol{M}^{(3)*} = -ig'\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{2.1.3}$$

where g' is relativistic and depends on the plasma properties.

In Ref. 25, the relativistic Hamilton-Jacobi equation is used to calculate the angular momentum set up in one electron by a circularly polarized, classical, electromagnetic field. The result is the same as that of Talin *et al.*, except that *SI* units are used in Ref. 25. However, in Ref. 25, the non-relativistic gyromagnetic ratio was used to relate this angular momentum to the magnetization. In *SI* units this is $-e/2m$ as given in Eq. (9.3.1) of Atkins [27], and as used to define the usual Bohr magneton $e\hbar/2m$. From Eq. (1.3) of Talin *et al.* [26] it can be seen that the gyromagnetic ratio itself needs to be corrected relativistically under some conditions. Therefore the Bohr magneton is not a constant, it depends on the relativistic factor γ as defined by Talin *et al.* [26].

When the necessary correction is made to the Bohr magneton, the SI magnetization becomes

$$\boldsymbol{M}^{(3)} = -\frac{e^3c^2}{2m^2\omega^3 V}\left(\frac{1}{1+\left(\frac{eB^{(0)}}{m\omega}\right)^2}\right)B^{(0)}\boldsymbol{B}^{(3)}. \tag{2.1.4}$$

This is the magnetization in amps per meter (coulombs per second per meter) caused in one electron by a circularly polarized electromagnetic field. The magnitude of the magnetic flux density of the field is $B^{(0)}$, and its angular frequency is ω. The charge to mass ratio of the electron is e/m. The sample volume is V in cubic meters. Finally $\boldsymbol{B}^{(3)} := B^{(0)}\boldsymbol{k}$ where $\boldsymbol{k} = \boldsymbol{e}^{(3)}$. Equation (2.1.4) is valid over the whole range of existence of the

inverse Faraday effect, from non-relativistic to ultra-relativistic. In the non-relativistic limit we obtain the same result as Ref. 25,

$$\boldsymbol{M}^{(3)} \xrightarrow{eB^{(0)} \ll m\omega} -\frac{1}{V}\left(\frac{e^3c^2}{2m^2\omega^3}\right)B^{(0)}\boldsymbol{B}^{(3)}, \tag{2.1.5}$$

and the magnetization is proportional to intensity. This is the original inverse Faraday effect, first observed in 1965 [1] and repeated several times [4—5].

In the ultra-relativistic limit, $eB^{(0)} \gg m\omega$, Eq. (2.1.4) becomes,

$$\boldsymbol{M}^{(3)} \xrightarrow{eB^{(0)} \gg m\omega} -\frac{1}{V}\left(\frac{ec^2}{2\omega}\right)\boldsymbol{e}^{(3)}. \tag{2.1.6}$$

This seems to be independent of $\boldsymbol{B}^{(3)}$, but recall [27] that the magnetic dipole moment of the field is,

$$|\boldsymbol{m}^{(3)}| = \frac{ec^2}{\omega} = \frac{V_0}{\mu_0}|\boldsymbol{B}^{(3)}|. \tag{2.1.7}$$

Therefore in the ultra-relativistic limit,

$$\boldsymbol{M}^{(3)} = -\frac{1}{2}\frac{1}{\mu_0}\left(\frac{V_0}{V}\right)\boldsymbol{B}^{(3)}, \tag{2.1.8}$$

where V_0 is the local volume element used to describe electromagnetic energy in vacuo, and μ_0 is the vacuum permeability.

Equation (2.1.8) is clear proof that in the ultra-relativistic limit, the magnetization is directly proportional to the $\boldsymbol{B}^{(3)}$ field of the radiation. If $\boldsymbol{B}^{(3)}$ were zero there would be no magnetization, and this is inconceivable, because the structure of Eq. (2.1.4) is valid over the entire range from non-relativistic to ultra-relativistic. This means that there must be an ultra-relativistic effect because there is an observable non-relativistic effect. Equation (2.1.4), except for units, is identical with the second part of Eq. (3) of Talin *et al.* [26].

Under all conditions,

$$\nabla \times \boldsymbol{M}^{(3)} = \nabla \times \boldsymbol{B}^{(3)} = \boldsymbol{0} . \tag{2.1.9}$$

Equation (2.1.6) or (2.1.8) is potentially very useful for applications as discussed below.

1.4 Discussion

The ultra-relativistic limit (2.1.6) can be reached experimentally [22] in the laboratory with standard apparatus. It is expected that the effect can be observed in all materials with power line apparatus as standard in the industry. From Eq. (2.1.6), which is more suitable for electrical engineering applications than Eq. (2.1.8), it is clear that the expected effect is inversely proportional to the angular frequency and the sample volume. Therefore in order to maximize the effect it is necessary to maximize the power density of the radiation field, minimize its frequency and minimize the sample volume. Although the power density does not seem to be present in Eq. (2.1.6), recall that it is the limit of Eq. (2.1.4). The power density reveals itself through the product $V^{(0)}\boldsymbol{B}^{(3)}$ in Eq. (2.1.8). It is easily checked that both Eqs. (2.1.6) and (2.1.8) have the required units of coulombs per second per meter.

These are the *SI* units both of magnetization and of magnetic field strength. So in the ultra relativistic limit the entire magnetic dipole moment of the field is transferred to the electron. As shown by Eq. (2.1.6), there is no intensity dependence in this limit. Therefore a constant magnetization is set up in a sample, such as a ferrite core of a power line design [22] and this phenomenon is entirely new and unexplored, probably with many applications. Although $\boldsymbol{B}^{(3)}$ appears in Eq. (2.1.8), it is multiplied by $V^{(0)}$, and the product $V^{(0)}\boldsymbol{B}^{(3)}$ has no power density dependence. The latter enters indirectly however because we are considering a extreme high power density-low frequency limit of Eq. (2.1.4). Remarkably, this limit is easily accessible in the laboratory and in applications [22].

The conclusion is that if $\boldsymbol{B}^{(3)}$ were zero, there would be no observable magnetization and this contradicts experience in the non-relativistic limit of the main equation (2.1.4). Therefore there can be no further doubt that $\boldsymbol{B}^{(3)}$ is non-zero empirically. It is a non- Maxwellian field.

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