The Present Status of the Quantum Theory of Light

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ORIGIN, OBSERVATION AND CONSEQUENCES OF THE $B^{(3)}$ FIELD

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Dedicated to Jean-Pierre Vigier

A summary overview is given of the origin, methods of observation, and consequences of the $\boldsymbol{B}^{(3)}$ field of free space electromagnetism, with suggestions for further work.

1. Introduction

The $\boldsymbol{B}^{(3)}$ field of vacuum electromagnetism introduces a new paradigm of field theory, summarized in the cyclically symmetric equations linking it [1—8] to the usual transverse magnetic plane wave components $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$. The $\boldsymbol{B}^{(3)}$ field was first (and obliquely) inferred in January, 1992 at Cornell University from a careful re-examination of known magneto-optic phenomena [9,10] which had previously been interpreted in orthodoxy through the conjugate product $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$ of electric plane wave components $\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)*}$. In the intervening three and a half years its understanding has developed substantially into monographs and papers [1—8] covering several fundamental aspects of field theory.. This paper is a summary overview of the origin, observation, and consequences of the fundamental $\boldsymbol{B}^{(3)}$ field of electromagnetism in the vacuum.

2. Origin

The $\boldsymbol{B}^{(3)}$ field originates [1—8] in an experimental observable of magneto-optics known to specialists as the conjugate product [11]. This observable is responsible, for

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example, for the phenomenon of magnetization by light first confirmed experimentally by van der Ziel *et al.* [9]. If 3-D space is represented by unit vectors [1,2] in the complex basis

$$\boldsymbol{e}^{(1)} \times \boldsymbol{e}^{(2)} = i\boldsymbol{e}^{(3)*}, \text{ et cyclicum,} \tag{1}$$

then $\boldsymbol{B}^{(3)}$ is defined in vacuo by [1,2]

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \text{ et cyclicum.} \tag{2}$$

On the left hand side the observable conjugate product appears in orthodox form [11], while on the right hand side it is expressed through the second order observable $iB^{(0)}\boldsymbol{B}^{(3)*}$. Here $B^{(0)}$ is the scalar magnitude of the magnetic plane wave components $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$, which, as usual, are transverse to the direction of light propagation. Equations (2) show that the standard vacuum Maxwell equations are self-inconsistent, because $\boldsymbol{B}^{(3)}$ is evidently *longitudinal* in vacuo and is not a transverse magnetic flux density. Equations (2) are relations between infinitesimal generators of the rotation group O(3), with the startling consequences that both the Wigner little group [12] and the sector group symmetry [13] of vacuum electromagnetism become O(3). This implies that the photon, if particulate, is also massive. The E cyclics,

$$\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} = -E^{(0)}\left(i\boldsymbol{E}^{(3)}\right)^{*}, \text{ et cyclicum,} \tag{3}$$

can be generated from the B cyclics (2) by fundamental tensorial duality [1,2] in vacuo. In contrast, to Eqs. (2), Eqs. (3) contain the unphysical $i\boldsymbol{E}^{(3)}$ field, formally a purely imaginary quantity [1,2]. The conjugate product thus generates the real and physical $\boldsymbol{B}^{(3)}$, but this is *not* accompanied by a real and physical $\boldsymbol{E}^{(3)}$. The physical equations (2) conserve the known discrete symmetries [14].

Since $iB^{(0)}\boldsymbol{B}^{(3)*}$ is a second order *observable*, then $\boldsymbol{B}^{(3)}$, a physical and real magnetic flux density, acts also at first order under the right observational conditions [3], discussed in the following section. The $\boldsymbol{B}^{(3)}$ field propagates in vacuo at the speed of light, F.A.P.P., and it is incorrect to think of it as a static magnetic field. The B cyclics, Eq. (2), are non-linear and non-Abelian [2], so that the self-generating source of $\boldsymbol{B}^{(3)}$ is the cross product $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$. Similarly, the source of one axis $\boldsymbol{e}^{(3)}$, in 3-D space is the cross product of the other two, $\boldsymbol{e}^{(1)}$ and $\boldsymbol{e}^{(2)}$, emphasizing that the B cyclics originate in spacetime itself, be this flat, as in special relativity, or curved as in general relativity. It would have been quite natural to accept this if it had been realized prior to Maxwell's great work in the nineteenth century. (Phenomena of magnetization by light were of course unknown to Maxwell.) About 130 years later, we have become used to thinking of electromagnetism in vacuo as a 2-D phenomenon, so a sudden shift into 3-D paradigm appears startlingly new. If we accept $\boldsymbol{B}^{(3)}$, and base electromagnetism as a phenomenon on Eqs. (2), then the task of unification with gravitation becomes much easier, because both electroweak and gravitational fields become properties of curved space-time. Several fundamental inferences of this nature have been developed and are summarized in the Table [3].

A first attempt has been made by Evans and Vigier [2] to write the Maxwell equations more consistently in O(3), non-Abelian form, using methods [15] borrowed directly from general relativity. This work shows that in vacuo

$$\boldsymbol{B}^{(3)*} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{4}$$

where $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$ is the vector potential plane wave, e is the elementary charge, and $\hbar$ is Dirac's constant. Equation (4), in turn, reveals that the interaction of $\boldsymbol{B}^{(3)}$ with matter (e.g. a fermion) is dictated by the usual transverse vector potentials $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$ through the minimal prescription [1—8]. Dirac's equation of motion should therefore be used to describe this interaction self-consistently [3]. If this is done correctly, the conditions of observation of $\boldsymbol{B}^{(3)}$ become properly defined, as described in the next section. The same thing can be done classically [1] with the *relativistic* Hamilton-Jacobi equation of motion. The relativistic factor reveals that the magnetization profile generated by the interaction of $\boldsymbol{B}^{(3)}$ with one fermion is in general a mixture of terms to first and second order in $B^{(0)}$. In the *weak field limit* (visible frequency radiation of any practically accessible intensity), the profile is second order, and proportional to beam power density, I (W m^{-2}). In the *strong field limit* (intense, radio frequency pulses) the profile is expected to be proportional to $\sqrt{I}$. These considerations are developed as follows.

3. Observation

Equations (2) show that $iB^{(0)}\boldsymbol{B}^{(3)*}$ is observable whenever $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ is observable [9,10], i.e., whenever circularly polarized electromagnetic radiation interacts with matter. If the latter can be represented for simplicity by an ensemble of N non-interacting electrons in a volume V, the relativistic Hamilton-Jacobi equation shows [1—8] that in general,

$$B^{(3)}_{\text{in sample}} = \frac{N}{V}\cdot\frac{\mu_0 e^3 c^2}{2m\omega^2}\left(\frac{B^{(0)}}{(m^2\omega^2 + e^2B^{(0)2})^{1/2}}\right)B^{(3)}_{\text{free space}}, \tag{5}$$

in which $B^{(3)}$ in free space is related to the beam power density I by

$$B^{(3)}_{\text{free space}} = \left(\frac{\mu_0}{c}I\right)^{1/2} e^{(3)} = \left(\frac{I}{\epsilon_0 c^3}\right)^{1/2} e^{(3)}. \tag{6}$$

In these equations ϵ_0 and μ_0 are respectively the free space permittivity and permeability, c the speed of light in vacuo, e/m the charge to mass ratio of the fermion, and ω is the beam angular frequency. In the weak field (visible frequency) limit, $m\omega \gg eB^{(0)}$, and Eq. (5) reduces to

$$B^{(3)}_{\text{in sample}} \to \frac{N}{V}\left(\frac{\mu_0 e^3 c^2 B^{(0)}}{2m^2\omega^3}\right)B^{(3)}_{\text{free space}}. \tag{7}$$

In the opposite strong field (radio frequency) limit,

$$B^{(3)}_{\text{in sample}} \to \frac{N}{V}\left(\frac{\mu_0 e^2 c^2}{2m\omega^2}\right)B^{(3)}_{\text{free space}}. \tag{8}$$

The observable quantity is $B^{(3)}$ in the sample, which is different from $B^{(3)}$ in free space. Typically, as shown by van der Ziel *et al.* [9], the sample $B^{(3)}$ in the weak field limit is about 10^{-9} T (10^{-5} gauss), and to observe it requires a skillful inverse Faraday effect experiment [9,10]. On the basis of arguments in Sec. 2, these magneto-optic effects become fundamentally important. The observation has not been attempted to date of $B^{(3)}$ acting at first order (to order $I^{1/2}$), but is the straightforward result of the relativistic factor in Eq. (5), obtainable both from the Hamilton-Jacobi equation and the classical limit of the Dirac equation [3]. Such an experiment, carried out with circularly polarized radio-frequency pulses of high intensity, is of great interest. Many other variations are possible, using well-developed contemporary techniques in non-linear optics and magneto-optics.

TABLE 1. Theory of Electrodynamics

	Fundamental Concept	Standard Theory	New Theory
1	Structure of Theory	linear, Abelian	non-linear, Non-Abelian
2	Maxwell's Equations in Vacuo	$\frac{\partial F_{\mu\nu}}{\partial x_\nu} = 0,\ \boldsymbol{B}^{(3)} = 0$	$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}$, et cyclicum
3	Electromagnetic Sector Symmetry	$U(1) = O(2)$	$O(3)$
4	Poincaré Group Symmetry	$\hat{J}^{(3)}$ generator missing, unphysical little group	magnetic fields are rotation generators
5	Wigner Little Group	$E(2)$, unphysical planar, Euclidean	$O(3)$, physical space rotation
6	Source of $\boldsymbol{B}^{(3)}$ at Observer Point R and Instant t	not considered	circling e at time $t - \frac{R}{c}$ earlier
7	Propagation of $\boldsymbol{B}^{(3)}$	not considered	through the Liénard-Wiechert potentials $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$
8	Gauge Group Definition of $\boldsymbol{B}^{(3)}$	not considered	$\boldsymbol{B}^{(3)*} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$
9	Free Space Four Tensor	$F_{\mu\nu}$, Abelian in space ((1), (2), (3))	$G_{\mu\nu}$, Non-Abelian in space ((1), (2), (3))
10	Photon Helicities	-1 and 1	-1, 0, 1
11	Translational Poynting Theorem	$\nabla \cdot \boldsymbol{N} = -\frac{\partial U}{\partial t}$	same
12	Rotational Poynting Theorem	not considered	$\nabla \cdot \boldsymbol{J}^{(3)} = -\frac{\partial U^{(3)}}{\partial t}$
13	Magnetic Fields	$\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$	$\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*},\ \boldsymbol{B}^{(3)}$
14	Electric Fields	$\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)*}$	$\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)*},\ i\boldsymbol{E}^{(3)}$

	Fundamental Concept	Standard Theory	New Theory
15	Vector Potentials	$\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$	$\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$, $i\boldsymbol{A}^{(3)}$, A_0
16	Planck-Einstein Relation	$En = \hbar\omega$	$En = \hbar\omega = h_1\lambda$, $h_1 = ec\lvert\boldsymbol{B}^{(3)}\rvert$
17	de Broglie Relation	$p = \hbar\kappa$	$p = \hbar\kappa = \frac{h_1\lambda}{c}$
18	Quantum of Energy	$\hbar\omega = \frac{1}{\mu_0}\int(\boldsymbol{B}^{(1)}\cdot\boldsymbol{B}^{(1)*}$ $+\boldsymbol{B}^{(2)}\cdot\boldsymbol{B}^{(2)*})dV$	$\hbar\omega = \frac{1}{\mu_0}\int(\boldsymbol{B}^{(1)}\cdot\boldsymbol{B}^{(1)*}$ $+\boldsymbol{B}^{(2)}\cdot\boldsymbol{B}^{(2)*}+\boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)*})dV_1$
19	Quantum of Angular Momentum	$\hbar$	$\hbar = h_1\frac{\lambda}{\omega}$
20	Quantum of Torque	$\hbar\omega$	$\hbar\omega = h_1\lambda$
21	Momentum Equivalence Condition	not considered	cyclic relations imply in vacuo $eA^{(0)} = \hbar\kappa$, the quantum of linear momentum
22	Mass of Photon	identically zero	$m = \frac{e\lambda_0\lvert\boldsymbol{B}^{(3)}\rvert}{c}$
23	Gauge Invariant Lagrangian Mass Term	not considered	$En = \frac{V}{\mu_0}\left(\frac{Mc^2}{\hbar\omega}\right)^2\boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)*}$ in $O(3)$ gauge group
24	Gauge Conditions on Four Potential	1) transverse 2) Coulomb	1) not allowed 2) scalar, non-zero 3) $A_\mu A_\mu - 0$
25	Field Quantization	canonical, beset with difficulties because A_μ is not covariant	direct quantization of cyclic field relations

	Fundamental Concept	Standard Theory	New Theory
26	de Broglie Theorem	not considered	$\hbar\omega_0 = h_1\lambda_0 = mc^2$
27	Interaction with Fermion	via $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$ in minimal prescription, $A_0 = \phi = 0$	same as Standard Theory but finite scalar potential $A_0 = \phi \neq 0$
28	Magneto-optics	I dependence from $A_0 = 0$	$I^{1/2}$ dependence observable under the right conditions, $A_0 \neq 0$
29	Q.E.D.	no mass term	finite mass term
30	Observation of $\boldsymbol{B}^{(3)}$ Field	not considered	routinely observable through $i\boldsymbol{B}^{(0)}\boldsymbol{B}^{(3)*}$ in magneto-optics

Notes $\boldsymbol{N}$:= Poynting vector; U = energy density; $\boldsymbol{J}^{(3)}$ = radiation angular momentum; $U^{(3)} = \frac{1}{\mu_0}\boldsymbol{B}^{(3)} \cdot \boldsymbol{B}^{(3)*}$; λ := wavelength; λ_0 = rest wavelength; V = volume of radiation; M = mass of radiation; $\phi = A_0$ = scalar potential.

4. Consequences

Since $i\boldsymbol{B}^{(0)}\boldsymbol{B}^{(3)*}$ is a routine observable it is possible to conclude with currently available data that the two dimensional picture of vacuum electromagnetism is incomplete. There are several major consequences of this deduction if accepted (Table 1). For example, the gauge group symmetry of the electromagnetic sector of unified field theory becomes O(3), not U(1), requiring appropriate theoretical development. The Wigner little group [12] becomes O(3), the physical space rotation group, which replaces the unphysical E(2) of orthodoxy [15]. In special relativity, a massless particle can have two degrees of polarization only, so the existence in the vacuum of $\boldsymbol{B}^{(3)}$ means that the photon if particulate is massive. Experimental efforts to measure limits on the photon mass are therefore shown to be well-founded in the B cyclic equations (2). The latter indicate the need to develop field equations of electromagnetism in an O(3) gauge symmetry [2], more completely, a Lorentz group gauge symmetry. When this is done, a self-consistent expression for $\boldsymbol{B}^{(3)}$ is obtained (Eq. (4)) in terms of transverse potentials $\boldsymbol{A}^{(1)}$ and $\boldsymbol{A}^{(2)}$. Radiation theory must be developed to explain the mode of propagation of $\boldsymbol{B}^{(3)}$ through the vacuum, and some inroads to this question have been made in the literature [3]. There are several interesting experimental consequences of a $\boldsymbol{B}^{(3)}$ field, among these are the optical Aharonov-Bohm effect [1—8] and the inference that $\boldsymbol{B}^{(3)}$ is the relict magnetic field in relativistic cosmology [3]. Satellite observations of the anisotropy in the cosmic background radiation would lead to a determination of $\boldsymbol{B}^{(3)}$, and inter alia, show that $\boldsymbol{B}^{(3)}$ is the seed field responsible for fundamental cosmic magnetic effects. If $\boldsymbol{B}^{(3)}$ is accepted as indicating a massive photon, new impetus is given to interpretations of quantum field theory which rely on finite photon mass, for example theories developed by the Vigier school [16]. Finally in a list of consequences mentioned at random, field theory in the vacuum reduces to the theory of angular momentum commutators in quantum mechanics, allowing a straightforward route to quantization of Eqs. (2). In general, magnetic fields become proportional to rotation generators, electric fields to boost generators, expressed either in pseudo-Euclidean or Riemannian geometry. This realization leads to new ways of unifying quantized electroweak theory and quantized gravitation.

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