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Electrostatic and magnetostatic fields generated by light in free space

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The classical theory of fields is used to demonstrate that the Maxwell equations support solutions of the type $\mathbf{E}^G=\mathbf{E}(\mathbf{r},t)+\mathbf{E}_\Pi$ and $\mathbf{B}^G=\mathbf{B}(\mathbf{r},t)+\mathbf{B}_\Pi$ where $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ are the usual oscillating electric and magnetic components of the plane wave, and where the real $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ are uniform electrostatic and magnetostatic fields directed in the propagation axis z of the wave. The fields $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ are nonzero in general and their presence does not affect the Poynting vector and the law of conservation of electromagnetic energy of the plane wave in free space. Thus, $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ are photon properties that take meaning when the photon beam interacts with matter, for example, an electron in the Lorentz equation. They are physically meaningful fields and their source is the same as that of $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$.

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INTRODUCTION

The phenomenological equations of Maxwell form the basis of the classical understanding of light. They were formulated in the mid-nineteenth century, before relativity was fully developed, and before the quantum theory came into existence. They were later put on a microscopic basis by Lorentz in his theory of the electron, and have become the starting point of a vast number of contemporary papers on the nature of light in free space and in materials. In this paper we show that there exist electrostatic and magnetostatic fields in the propagation axis of the classical electromagnetic plane wave, fields which propagate in free space, and which conserve the structure of the well-defined Poynting vector, and therefore do not affect the law of conservation of electromagnetic energy in free space. It is usually assumed that the following are solutions to the free-space Maxwell equations for a completely circularly polarized plane wave:

$$\mathbf{E}(\mathbf{r},t)=\frac{1}{\sqrt{2}}E_0(\mathbf{i}+i\mathbf{j})e^{i\phi}\,, \tag{1}$$

$$\mathbf{B}(\mathbf{r},t)=\frac{1}{\sqrt{2}}B_0(\mathbf{j}-i\mathbf{i})e^{i\phi}\,. \tag{2}$$

Here E_0 is the scalar electric field strength amplitude, and B_0 the scalar magnetic flux density amplitude, $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in X and Y of the laboratory frame, and ϕ is the phase of the plane wave. These solutions are oscillatory and time and space dependent through the phase

$$\phi=\omega t-\boldsymbol{\kappa}\cdot\mathbf{r}\,. \tag{3}$$

Here ω is the angular frequency of the wave, t the time, $\boldsymbol{\kappa}$ the wave vector, and $\mathbf{r}$ a position vector as usual. A whole literature is available concerning their properties.

However, the equations

$$\mathbf{E}^G=\mathbf{E}(\mathbf{r},t)+\mathbf{E}_\Pi\,, \tag{4}$$

$$\mathbf{B}^G=\mathbf{B}(\mathbf{r},t)+\mathbf{B}_\Pi \tag{5}$$

are also valid solutions to the free-space Maxwell equations. Here $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ are uniform, time-independent, electric and magnetic fields directed in the propagation axis z of the plane wave. It appears always to have been implicitly assumed that $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ are both zero in free space, and that there is no component in z of the plane wave *in vacuo*. There is no mathematical reason for this supposition, however, and as we shall see, the vectors $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ can be related to the well-known $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$. The source of $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ is therefore the same as the source of the well-accepted $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$. If the latter are nonzero, then so are *both* $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ in general.

Section I introduces $\mathbf{B}_\Pi$ using the well-known imaginary conjugate product:

$$\boldsymbol{\Pi}^{(A)}\equiv E_0 c\,\mathrm{Im}(\mathbf{B}_\Pi)=\mathbf{E}(\mathbf{r},t)\times\mathbf{E}^*(\mathbf{r},t)=-iE_0^2\mathbf{k} \tag{6}$$

of the electromagnetic plane wave [1–8], where $\mathbf{E}^*(\mathbf{r},t)$ is the complex conjugate of $\mathbf{E}(\mathbf{r},t)$, i.e.,

$$\mathbf{E}^*(\mathbf{r},t)=\frac{1}{\sqrt{2}}E_0(\mathbf{i}-i\mathbf{j})e^{-i\phi}\,. \tag{7}$$

We see in Appendix A that the real and imaginary parts of $\mathbf{B}_\Pi$ are the same. $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ are antiparallel in the propagation axis.

The law of conservation of energy for a plane wave in free space can be expressed through the continuity equation:

$$\nabla\cdot\mathbf{N}=-\frac{\partial U}{\partial t}\,, \tag{8}$$

where $\mathbf{N}$ is the Poynting vector,

$$\mathbf{N}=\frac{1}{\mu_0}\mathbf{E}(\mathbf{r},t)\times\mathbf{B}(\mathbf{r},t)\,, \tag{9}$$

and U a scalar field. Here μ_0 is the magnetic permeability of free space. The vector $\mathbf{N}$ is the flux of electromagnetic energy of the plane wave, and the scalar U is the electromagnetic fields's energy density. Therefore $\mathbf{N}$ is electromagnetic power per unit area, and U is power per unit volume. The scalar amplitude of the Poynting vector is the light intensity I_0. Therefore Eq. (8) expresses, in classical electrodynamics, the law of conservation of electromagnetic energy in free space. This idea of field energy has no meaning [9] unless the wave interacts with matter (e.g., an electron). In Sec. III it is shown that the continuity equation (8) is unchanged for nonzero $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ provided

$$\mathbf{B}_\Pi \times \mathbf{E}(\mathbf{r},t) = \mathbf{E}_\Pi \times \mathbf{B}(\mathbf{r},t) . \tag{10}$$

In other words Eq. (10) is the condition for conservation of free-space electromagnetic energy given the general solutions (4) and (5) of the Maxwell equations. Equation (10) shows that if $\mathbf{B}_\Pi$ is nonzero, and defined through the conjugate product (6), then so is $\mathbf{E}_\Pi$. Finally, a discussion is given of the physical meaning of the vectors $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$, with order-of-magnitude estimates, and experimental consequences.

I. THE DEFINITION OF $\mathbf{B}_\Pi$ THROUGH THE CONJUGATE PRODUCT

The conjugate product $\mathbf{E}\times\mathbf{E}^*$ appears in the antisymmetric part of Maxwell's stress tensor [10] and is a well-defined property of light. It is an axial vector with magnetic symmetry [11,12], i.e., that of angular momentum: positive to parity inversion $\hat{P}$, and negative to motion reversal $\hat{T}$. The vector notation $\mathbf{E}\times\mathbf{E}^*$ is equivalent to the tensor notation:

$$\Pi_i^{(A)} = \tfrac{1}{2}\epsilon_{ijk}(E_j E_k^* - E_k E_j^*) , \tag{11}$$

where ϵ_{ijk} is the Levi-Cività symbol. This shows that the axial vector $\mathbf{E}\times\mathbf{E}^*$ is equivalent to a polar rank-two tensor:

$$\Pi_{jk}^{(A)} = \tfrac{1}{2}(E_j E_k^* - E_k E_j^*) , \tag{12}$$

which is the antisymmetric part of the tensor $E_j E_k^*$. Therefore $\mathbf{E}\times\mathbf{E}^*$ is the vector part of light intensity.

The quantity

$$\mathrm{Im}(\mathbf{B}_\Pi) = \frac{i\mathbf{E}\times\mathbf{E}^*}{(E_0 c)} \tag{13}$$

is a uniform, divergentless, time-independent, magnetic flux density vector with the required symmetry and units. The magnetic field $\mathbf{B}_\Pi$ exists in free space because $\mathbf{E}\times\mathbf{E}^*$ exists in free space, and is defined in the z axis:

$$\mathrm{Im}(\mathbf{B}_\Pi) = +\frac{E_0}{c}\mathbf{k} = +B_0\mathbf{k} , \tag{14}$$

where $\mathbf{k}$ is an axial unit vector. The magnitude of $\mathbf{B}_\Pi$, i.e., $|\mathbf{B}_\Pi|$, is the scalar amplitude B_0 defined in the Introduction. A real interaction Hamiltonian is produced from $\mathbf{E}\times\mathbf{E}^*/(E_0 c)$ when it forms a scalar product with the usual imaginary magnetic dipole moment operator, $i\hat{m}''$, in quantum mechanics [13–15]. Similarly, the imaginary $\mathbf{E}\times\mathbf{E}^*$ produces a well-defined [1–8] real interaction Hamiltonian when it multiplies the imaginary part of the molecular electric polarizability operator, $i\hat{\alpha}''$. The latter is the well-known vectorial polarizability [16,17], which vanishes at zero frequency from time-dependent perturbation theory. Both $\hat{m}''$ and $\hat{\alpha}''$ are directly proportional (using the Wigner-Eckart Theorem, for example [16,17]) to the net molecular electronic angular-momentum operator $\hat{J}$:

$$\hat{m}'' = \gamma_e \hat{J} , \tag{15}$$

$$\hat{\alpha}'' = \gamma_\Pi \hat{J} , \tag{16}$$

where γ_e is the gyromagnetic ratio [13–15] and γ_Π is the gyroptic ratio [18–20]. Consequently

$$\hat{\alpha}'' = \frac{\gamma_\Pi}{\gamma_e}\hat{m}'' , \tag{17}$$

showing that $\hat{m}''$ and $\hat{\alpha}''$ have the same $\hat{T}$ negative, $\hat{P}$ positive symmetry, and are both axial vector operators. The conjugate product $\mathbf{E}\times\mathbf{E}^*$ forms a real Hamiltonian operator when it multiplies $i\hat{\alpha}''$, and because $\hat{\alpha}''$ is directly proportional to $\hat{J}$ and thus to $\hat{m}''$, it follows that $\mathbf{E}\times\mathbf{E}^*$ must be proportional to a magnetic field, which we have identified as $\mathbf{B}_\Pi$ in Eq. (13). Clearly, $\hat{m}''$ can form a real Hamiltonian operator only when multiplied by a magnetic field. The root of Eq. (13) is therefore found in the fact that the well-known molecular property tensors $\hat{\alpha}''$ and $\hat{m}''$ are both axial vectors with magnetic symmetry. This point can be emphasized by assuming that the real part of $\hat{m}''\cdot\mathbf{B}_\Pi$ is an interaction Hamiltonian and investigating the logical consequences. To do this, it is convenient to write the well-accepted [16,17] interaction Hamiltonian between $i\hat{\alpha}''$ and $\mathbf{E}\times\mathbf{E}^*$ as

$$\Delta\hat{H} = -i\hat{\alpha}''\cdot(\mathbf{E}\times\mathbf{E}^*)$$
$$= -iE_0 c\hat{\alpha}''\cdot\frac{(\mathbf{E}\times\mathbf{E}^*)}{E_0 c} \equiv -E_0 c\hat{\alpha}''\cdot\mathrm{Im}(\mathbf{B}_\Pi) , \tag{18}$$

where we have defined $\mathbf{B}_\Pi$ in terms of $\mathbf{E}\times\mathbf{E}^*$ as in Eq. (13). Using the proportionality (17) between the magnetic dipole moment and the vectorial polarizability, Eq. (18) becomes

$$\Delta\hat{H} = -E_0 c\frac{\gamma_\Pi}{\gamma_e}\hat{m}''\cdot\mathrm{Im}(\mathbf{B}_\Pi) = -i\hat{\alpha}''\cdot(\mathbf{E}\times\mathbf{E}^*) \tag{19}$$

showing that the product $\hat{m}\cdot\mathbf{B}_\Pi$ is directly proportional to the product $i\hat{\alpha}''\cdot(\mathbf{E}\times\mathbf{E}^*)$ through a nonzero proportionality constant. Therefore, if the energy $i\hat{\alpha}''\cdot(\mathbf{E}\times\mathbf{E}^*)$ is nonzero, then so must the energy $\hat{m}''\cdot\mathbf{B}_\Pi$ be nonzero.

Finally in this section, using the Wigner-Eckart theorem, the gyromagnetic and gyroptic ratios can be defined as follows in an atom with net electronic angular momentum, $\hat{J}$, showing that in this case γ_e and γ_Π are nonzero in general:

$$m_0^{1''} = \frac{\langle J\|\hat{m}^{1''}\|J\rangle}{\langle J\|\hat{J}^1\|J\rangle}J_0^1 = \gamma_e J_0^1 , \tag{20}$$

$$\alpha_0^{1''} = \frac{\langle J\|\hat{\alpha}^{1''}\|J\rangle}{\langle J\|\hat{J}^1\|J\rangle}J_0^1 = \gamma_\Pi J_0^1 , \tag{21}$$

$$\langle J\|\hat{J}^1\|J\rangle = [J(J+1)(2J+1)]^{1/2} . \tag{22}$$

II. THE CONSERVATION OF ELECTROMAGNETIC ENERGY

We have assumed that Eqs. (4) and (5) are solutions of the free-space Maxwell equations:

$$\nabla\times\mathbf{E}^G=-\frac{\partial\mathbf{B}^G}{\partial t}, \tag{23}$$

$$\nabla\times\mathbf{B}^G=\frac{1}{c^2}\frac{\partial\mathbf{E}^G}{\partial t} \tag{24}$$

in Système International (SI) units. It is clear that if $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ are defined in the z axis of the plane wave

$$\nabla\times\mathbf{E}_\Pi=\frac{\partial\mathbf{B}_\Pi}{\partial t}=0, \tag{25}$$

$$\nabla\times\mathbf{B}_\Pi=\frac{\partial\mathbf{E}_\Pi}{\partial t}=0 \tag{26}$$

because these fields are time independent and have no X and Y components. Consider the divergence of $\mathbf{E}^G(\mathbf{r},t)$ and $\mathbf{B}^G(\mathbf{r},t)$. Using the vector identity

$$\nabla\cdot(\mathbf{A}\times\mathbf{B})=\mathbf{B}\cdot(\nabla\times\mathbf{A})-\mathbf{A}\cdot(\nabla\times\mathbf{B}) \tag{27}$$

it follows that we may expand

$$\nabla\cdot(\mathbf{E}^G\times\mathbf{B}^G)=\nabla\cdot(\mathbf{E}\times\mathbf{B})+\nabla\cdot(\mathbf{E}\times\mathbf{B}_\Pi)+\nabla\cdot(\mathbf{E}_\Pi\times\mathbf{B})+\nabla\cdot(\mathbf{E}_\Pi\times\mathbf{B}_\Pi), \tag{28}$$

where $\mathbf{E}\times\mathbf{B}$ is proportional to the Poynting vector of the law of conservation of energy, Eq. (8). From the relations

$$\nabla\cdot(\mathbf{E}\times\mathbf{B}_\Pi)=\mathbf{B}_\Pi\cdot(\nabla\times\mathbf{E})-\mathbf{E}\cdot(\nabla\times\mathbf{B}_\Pi),$$
$$\nabla\times\mathbf{B}_\Pi=0 \tag{29}$$

and

$$\mathbf{B}_\Pi\cdot(\nabla\times\mathbf{E})=-\mathbf{B}_\Pi\cdot\frac{\partial\mathbf{B}}{\partial t}=0 \tag{30}$$

it follows that

$$\nabla\cdot(\mathbf{E}\times\mathbf{B}_\Pi)=0 \tag{31}$$

and similarly

$$\nabla\cdot(\mathbf{E}_\Pi\times\mathbf{B})=0. \tag{32}$$

Also, the last term in Eq. (28) vanishes because $\mathbf{E}_\Pi$ is parallel to $\mathbf{B}_\Pi$ in z. It follows therefore that

$$\nabla\cdot(\mathbf{E}^G\times\mathbf{B}^G)=\nabla\cdot(\mathbf{E}\times\mathbf{B}), \tag{33}$$

i.e., the continuity equation (8) is unaffected by the presence of $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$, and the relation [Eq. (8)] of the field energy-flux density (N) to the electromagnetic field energy density (U) is unchanged in the free-space electromagnetic plane wave. In other words the electromagnetic powers per unit area generated by $\nabla\cdot(\mathbf{E}\times\mathbf{B}_\Pi)$ and by $\nabla\cdot(\mathbf{E}_\Pi\times\mathbf{B})$ are both zero, and therefore so are the associated electromagnetic powers per unit volume.

This result is true only if $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ are both in the propagation axis of the plane wave. The argument so far shows that $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ may be separately nonzero, or that $\mathbf{E}_\Pi$ may be zero and $\mathbf{B}_\Pi$ nonzero, as defined in Eq. (13).

In order to obtain a relation between $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ we use the result, from Eq. (8),

$$\nabla\cdot(\mathbf{E}^G\times\mathbf{B}^G)=\nabla\cdot(\mathbf{E}\times\mathbf{B})=-\frac{\partial U}{\partial t}, \tag{34}$$

which implies that the divergence of the product $\mathbf{E}^G\times\mathbf{B}^G$ is nonzero and identical with the divergence of the product $\mathbf{E}\times\mathbf{B}$. This implies that

$$\mathbf{E}^G\times\mathbf{B}^G=\mathbf{E}\times\mathbf{B}+\text{const}. \tag{35}$$

However, we know that

$$\mathbf{E}^G\times\mathbf{B}^G=\mathbf{E}\times\mathbf{B}+\mathbf{E}_\Pi\times\mathbf{B}+\mathbf{E}\times\mathbf{B}_\Pi \tag{36}$$

and from Eqs. (35) and (36) we derive the key result:

$$\mathbf{E}_\Pi\times\mathbf{B}=\mathbf{B}_\Pi\times\mathbf{E} \tag{37}$$

assuming that the constant of integration in Eq. (35) is zero (see Appendix D) and demonstrating that *if* $\mathbf{B}_\Pi$ *is nonzero, then* E_Π *must also be nonzero.*

In precise analogy with $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$, the fields $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ take meaning only when there is wave-particle or wave-matter interaction, but these fields propagate through free space (i.e., vacuum). Clearly, electromagnetic waves can be detected only when there is particulate matter with which the waves can interact, otherwise there would be no experimental evidence at all for the existence of electromagnetic fields. The source of $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ is the same as the source of the oscillating fields $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$, because both the static and oscillating components are needed for the complete solution of the free-space Maxwell equations and the presence of oscillating components implies through Eqs. (13) and (37) the presence of static components. The static and oscillating components are both relativistic in nature, because the plane wave propagates at the speed of light. In quantum field theory, there are operator equivalents [18–21] of $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$. A fundamentally important difference between the oscillating and static components of the plane wave is that the former vanish upon time averaging and the latter do not. This is the source of several novel physical phenomena when there is wave-matter interaction. Equation (37) conserves $\hat{P}$ and $\hat{T}$ symmetry, and the static components of the solution are related through Eqs. (13) and (37) to the oscillating components with, as we have seen, conservation of electromagnetic energy. The components are therefore completely defined and the definition is self-consistent.

From the properties of the dual transform of special relativity (see Appendix A) the Maxwell equations are invariant to

$$c\mathbf{B}\to-i\mathbf{E},\qquad c\mathbf{B}_\Pi\to-i\mathbf{E}_\Pi. \tag{38}$$

The dual transform implies immediately that

$$\mathbf{B}_\Pi\times\mathbf{B}=-i\frac{\mathbf{E}_\Pi}{c}\times\left[-c\frac{\mathbf{B}}{i}\right]=\mathbf{E}_\Pi\times\mathbf{B}, \tag{39}$$

which confirms that the sum

$$\mathbf{E}_{\Pi}\times\mathbf{B}+\mathbf{E}\times\mathbf{B}_{\Pi}=0 \tag{40}$$

and that the general solution of Maxwell's equations must be of the form (see Appendix A)

$$\mathbf{E}^{G}=\mathbf{E}(\mathbf{r},t)\pm\frac{1}{\sqrt{2}}E_0(i-1)\mathbf{k}\ , \tag{41}$$

$$\mathbf{B}^{G}=\mathbf{B}(\mathbf{r},t)\pm\frac{B_0}{\sqrt{2}}(i+1)\mathbf{k} \tag{42}$$

to be consistent with the theory of special relativity applied to the Maxwell equations.

It is easily checked that Eq. (39) is consistent with Eqs. (1) and (2) with

$$\mathbf{B}_{\Pi}=\pm\frac{B_0}{\sqrt{2}}(i+1)\mathbf{k}\ , \tag{43}$$

$$\mathbf{E}_{\Pi}=\pm\frac{E_0}{\sqrt{2}}(i-1)\mathbf{k}\ . \tag{44}$$

Equation (39) is also consistent with the generalized continuity equation, and with the fact (see Appendix) that

$$\mathbf{F}_{\Pi}=\mathbf{E}_{\Pi}+ic\mathbf{B}_{\Pi} \tag{45}$$

and

$$F_{\Pi}^2=E_{\Pi}^2-c^2B_{\Pi}^2+2ic\mathbf{E}_{\Pi}\cdot\mathbf{B}_{\Pi} \tag{46}$$

are invariants of the Lorentz transform. From Eq. (40) the net contribution of $\mathbf{E}_{\Pi}$ and $\mathbf{B}_{\Pi}$ to the free-space electromagnetic energy is zero.

III. DISCUSSION

The orders of magnitude of $\mathbf{E}_{\Pi}$ and $\mathbf{B}_{\Pi}$ can be estimated directly from the intensity, I_0, of the light beam in $\mathrm{W\,m^{-2}}$, through the free-space relations:

$$|\mathbf{B}_{\Pi}|=B_0=\left[\frac{I_0}{\epsilon_0 c^3}\right]^{1/2},\quad |\mathbf{E}_{\Pi}|=E_0=\left[\frac{I_0}{\epsilon_0 c}\right]^{1/2}, \tag{47}$$

where ϵ_0 is the electric permittivity *in vacuo* ($8.854\times10^{-12}\ \mathrm{J^{-1}\,C^2\,m^{-1}}$ in SI units). Thus, for a beam of 10 000 $\mathrm{W\,m^{-2}}$ (1.0 $\mathrm{W\,cm^{-2}}$), B_0 is about 10^{-8} T, and E_0 about 20 $\mathrm{V\,m^{-1}}$. These are also the scalar amplitudes B_0 and E_0 of the oscillating part of the solution to Maxwell's equations, and the scalar intensity I_0 of the beam is unaffected by the presence of $\mathbf{E}_{\Pi}$ and $\mathbf{B}_{\Pi}$ because I_0 is the magnitude of the Poynting vector. However, $\mathbf{E}_{\Pi}$ and $\mathbf{B}_{\Pi}$ are nonzero after time averaging because they are independent of time, and form real, nonzero, interaction Hamiltonians with particulate matter. These Hamiltonians lead, therefore, to the prediction of novel physical phenomena which can be measured as a function of I_0 and of the polarization state of the light beam. If the latter is linearly or incoherently polarized, $\mathbf{E}\times\mathbf{E}^*$ is zero and in consequence, so are $\mathbf{B}_{\Pi}$ and $\mathbf{E}_{\Pi}$; otherwise $\mathbf{E}_{\Pi}$ and $\mathbf{B}_{\Pi}$ are proportional to the *square root* of I_0. Because $\mathbf{E}_{\Pi}$ and $\mathbf{B}_{\Pi}$ are electrostatic and magnetostatic fields which form part of the general solution of Maxwell's equations in free space, they have the properties of such fields when light interacts with matter. This is the main conclusion of this paper.

On the basis of this conclusion it is easy to see that the various theories of the interaction of conventionally generated electric and magnetic fields can be applied directly to the fields $\mathbf{E}_{\Pi}$ and $\mathbf{B}_{\Pi}$, and examples of these applications have been given elsewhere for $\mathbf{B}_{\Pi}$ [22–26]. These include the inverse Faraday effect; the optical Faraday and Zeeman effects; optically induced shifts in NMR resonances ("optical NMR," recently observed experimentally [27]); the optical Cotton-Mouton effect; optical electron-spin resonance (ESR); optical forward-backward birefringence; and a reinterpretation of antisymmetric light scattering and related phenomena in terms of $\mathbf{B}_{\Pi}$. It has also been deduced [18–21] that the quantum field equivalent of $\mathbf{B}_{\Pi}$ is the operator

$$\hat{B}_{\Pi}=B_0\frac{\hat{J}}{\hbar}\ , \tag{48}$$

where $\hat{J}$ is the quantized photon angular momentum, and $\hbar$ the reduced Planck constant. It has also been shown [25], using the properties of the classical Lorentz transformation, that there can be no Faraday induction in free space due to a time derivation of the type $d\mathbf{B}_{\Pi}/dt$, produced, for example, by modulating a laser beam. (Note, however, that Faraday induction occurs via the inverse Faraday effect [28] when a circularly polarized laser *interacts with matter* inside an induction coil.) The reason for this is that the Lorentz transformations do not allow free-space X and Y components either of $\mathbf{B}_{\Pi}$ or of $\mathbf{E}_{\Pi}$, and also show that the z components $\mathbf{E}_{\Pi}$ and $\mathbf{B}_{\Pi}$ must be relativistically invariant [25].

One of the simplest consequences of the presence of $\mathbf{B}_{\Pi}$ is an optical Zeeman effect, whose semiclassical theory regards $\mathbf{B}_{\Pi}$ as a classical vector [24]. In this approximation the theory of the optical Zeeman effect is the same as that of the conventional Zeeman effect [29], with the conventional magnetostatic $\mathbf{B}_z$ replaced by $\mathbf{B}_{\Pi}$. In the simplest case, the Zeeman shift is proportional to

$$\Delta f=\hat{\mathrm{m}}\cdot\frac{\mathbf{B}_{\Pi}}{h} \tag{49}$$

and therefore to the square root of the laser intensity $I_0^{1/2}$. This occurs in addition to an optical Zeeman shift caused [30] by the interaction of $\mathbf{E}\times\mathbf{E}^*$ with $\hat{\alpha}''$, a mechanism which is proportional to intensity I_0. There appear to be no experimental investigations to date of the optical Zeeman effect, which requires only a minor modification of optical Stark effect apparatus to circularly polarize the pump laser.

Similarly, a nonzero interaction Hamiltonian is formed in general between an electric dipole moment $\hat{\mu}$ and the optical electrostatic field $\mathbf{E}_{\Pi}$, leading to a new type of optical Stark effect, proportional to the square root of the pump laser intensity, which occurs in addition to the conventionally understood optical Stark effect [29], but only when the pump laser has some degree, at least, of circular polarization. Otherwise $\mathbf{E}_{\Pi}$ and $\mathbf{B}_{\Pi}$ vanish. As in the ordinary linear Stark effect, caused by an ordinary electric

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this work that E_Π and B_Π do not affect the law of conservation of electromagnetic energy, the widely accepted continuity equation (8) of the classical theory of fields. The notions of $\mathbf{E}\times\mathbf{E}^*$, $\mathbf{B}_\Pi$, and $\mathbf{E}_\Pi$ are inextricably and ineluctably interrelated, therefore, and experimental evidence for the presence of any one is evidence for all. Conversely, if there is no apparent evidence for one, then all must not exist. The inverse Faraday effect has been interpreted through the notion of $\mathbf{E}\times\mathbf{E}^*$ [7,8], but this provides an explanation in terms only of one mechanism, proportional to intensity. It has been argued here that there must be another mechanism present, proportional to the square root of intensity (the $\mathbf{B}_\Pi$ mechanism), and also effects due to $\mathbf{E}_\Pi$, also proportional to the square root of intensity. If these are found experimentally, contemporary understanding would be strengthened, but evidence for one mechanism (e.g., $\mathbf{E}\times\mathbf{E}^*$) is found, and evidence for another (e.g., $\mathbf{B}_\Pi$) not found, the theory of electromagnetic fields would be challenged at the most fundamental level.

Clearly, the notion of $\mathbf{E}\times\mathbf{E}^*$ implies that this object is transmitted through free space in an electromagnetic plane wave, and when this wave meets particulate matter, an interaction Hamiltonian is formed between $\mathbf{E}\times\mathbf{E}^*$ and a material property. In atoms and molecules with net electronic angular momentum, this property is the vectorial polarizability vector $\hat{\alpha}''$, well defined and accepted in semiclassical time-dependent perturbation theory, based on the time-dependent Schrödinger equation [29]. Since $\mathbf{B}_\Pi$ is directly proportional to $\mathbf{E}\times\mathbf{E}^*$, it cannot be argued that $\mathbf{E}\times\mathbf{E}^*$ exists and that $\mathbf{B}_\Pi$ does not. The source of $\mathbf{E}_\Pi$ and of $\mathbf{B}_\Pi$ is clearly the same as that of $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$. Furthermore, it has been shown that $\mathbf{B}_\Pi$ is part of the general solution of the equations of Maxwell, and is therefore phenomenologically indistinguishable from uniform, magnetostatic flux density, whose symmetry and units its possesses. It cannot therefore be argued that $\mathbf{B}_\Pi$ cannot form an interaction Hamiltonian with the appropriate material property (a magnetic dipole moment), and it has been shown in Eq. (19) that if $i\hat{\alpha}''\cdot\mathbf{E}\times\mathbf{E}^*$ is accepted as an interaction energy, then $\hat{\mathbf{m}}\cdot\mathrm{Im}(\mathbf{B}_\Pi)$ must also be accepted as such. Finally, the classical presence of $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ must have a meaning also in quantum field theory, where these vector fields become operators (Appendix B).

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Appendixes A–D: Short description of 21 manuscript pages on microfiche [32].

APPENDIX A

The original appendix on microfiche proves that $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ are entirely consistent with the Lorentz covariance of the Maxwell equations, and form invariants of the Lorentz transformation. The Appendix provides general solutions for $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$.

APPENDIX B

This appendix defines the quantum-mechanical structure of $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ in terms of creation and annihilation operators.

APPENDIX C

This provides definition for $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ in terms of vector potentials in free space.

APPENDIX D

This proves that the constant of integration in Eq. (35) is zero.

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