# MANIFESTLY COVARIANT THEORY OF THE ELECTROMAGNETIC FIELD IN FREE SPACETIME, PART 2: THE LORENTZ FORCE EQUATION

## I. INTRODUCTION

In Part 1 of this series a manifestly covariant theory was developed for the electromagnetic field in free spacetime, in which[1] the electric and magnetic fields are treated as four vectors $E_\mu$ and $B_\mu$, all of whose components are physically meaningful. This is a departure from the conventional approach suggested by the recent discovery[2–5] that the longitudinal ((3)) and transverse ((1) and (2)) components of the electromagnetic field are linked by[2]

$$\mathbf{B}^{(3)} = \frac{\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}}{\mathrm{i}cE_0} \tag{1}$$

Here $\mathbf{B}^{(3)}$ is the longitudinal magnetic field of the electromagnetic plane wave, and $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$ define the transverse electric fields through

$$\mathbf{E}^{(1)} = E_0 \hat{\mathbf{e}}^{(1)} \mathrm{e}^{\mathrm{i}\phi} \tag{2a}$$

$$\mathbf{E}^{(2)} = E_0 \hat{\mathbf{e}}^{(2)} \mathrm{e}^{-\mathrm{i}\phi} \tag{2b}$$

where $E_0$ (proportional to the timelike polarization $E^{(0)}$ is the scalar amplitude of the plane wave, and $c$ is the speed of light in free spacetime, the universal constant of special relativity. The unit vectors $\hat{\mathbf{e}}^{(1)}$ and $\hat{\mathbf{e}}^{(2)}$ are defined in the circular basis[1] by

$$\hat{\mathbf{e}}^{(1)} = \frac{\mathbf{i} - \mathrm{i}\mathbf{j}}{\sqrt{2}} \tag{3a}$$

$$\hat{\mathbf{e}}^{(2)} = \frac{\mathbf{i} + \mathrm{i}\mathbf{j}}{\sqrt{2}} \tag{3b}$$

$$\hat{\mathbf{e}}^{(1)} \times \hat{\mathbf{e}}^{(2)} = \mathrm{i}\hat{\mathbf{e}}^{(3)} \tag{3c}$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in $X$ and $Y$, mutually orthogonal to the propagation axis $Z$ of the electromagnetic plane wave. The phase $\phi$ is defined as usual[6–15] by

$$\phi = \omega t - \boldsymbol{\kappa} \cdot \mathbf{r} \tag{4}$$



where $\boldsymbol{\kappa}$ is the wave vector at position $\mathbf{r}$, and $\omega$ is the angular frequency at instant $t$ in free spacetime.

Equation (1) is the key to the theory of covariant electrodynamics, essentially because the novel longitudinal field $\mathbf{B}^{(3)}$ is independent of the phase of the plane wave, and thus satisfies the conventionally defined Gauss theorem in differential form. Equation (1) is invariant to the fundamental symmetries, charge conjugation $\hat{C}$, parity inversion $\hat{P}$, and motion reversal $\hat{T}$, i.e., the right and left sides have the same $\hat{C}$, $\hat{P}$, and $\hat{T}$ symmetries.[1–5] Furthermore, the numerator on its right side is proportional[16–18] to the antisymmetric part of the light intensity tensor $I_{ij}$. The latter is known to be proportional to the third Stokes parameter $S_3$ and to mediate experimentally observable phenomena, such as antisymmetric light scattering[16–18] and the inverse Faraday effect,[19] and can therefore be considered a nonzero property of a circularly polarized electromagnetic wave in free space. Inter alia, $\mathbf{B}^{(3)}$ from Eq. (1) is similarly nonzero in free space, because the denominator on the right side of Eq. (1) is nonzero for finite $E^{(0)}$. It is more logical to state that the right side of Eq. (1) is nonzero because $\mathbf{B}^{(3)}$ is nonzero rather than the other way around, because $\mathbf{B}^{(3)}$ is a fundamental solution of Maxwell's equations in free spacetime. The quantity $\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}$ is, on the other hand, built up from a cross product of fundamental transverse electric fields. It is clear, however, that if the antisymmetric part of the light intensity is nonzero, then $\mathbf{B}^{(3)}$ is nonzero. In other words, $\mathbf{B}^{(3)}$ is the source of the antisymmetric part of light intensity and all concommitant experimental phenomena. It is worth noting in the context of $\hat{C}$ symmetry[19] that

$$\hat{C}(A_\mu) = -A_\mu \tag{5}$$

where $A_\mu$ is the well-known potential four vector in free spacetime. The $\hat{C}$ symmetries of $A_\mu$, $E_\mu$, and $B_\mu$ are all negative, so that the concomitant fields of the photon change sign under $\hat{C}$. Although the photon is stated[19] to be its own antiparticle, the antiphoton, generated by $\hat{C}$ from the photon, is associated with electric and magnetic fields of the opposite sign. For this reason, the antiphoton is a distinct entity from the photon. Furthermore, all four components of $A_\mu$, $E_\mu$, and $B_\mu$ must change sign under $\hat{C}$; i.e., all four polarizations (0), (1), (2), and (3) change sign. On the other hand, spacelike quantities, such as the propagation vector $\boldsymbol{\kappa}$, by definition are unaffected by $\hat{C}$, so that the $\hat{C}$ operator produces an electromagnetic wave propagating in the same direction, but with all four polarizations reversed. The electromagnetic wave produced in vacuo by $\hat{C}$ defines the antiphoton in the quantum field, a distinct entity from the photon. The fact that the concommitant fields are reversed in sign does not mean that $\mathbf{B}^{(3)}$ of Eq. (1) violates $\hat{C}$ symmetry. In the same way, Eq. (5) does not mean that $A_\mu$ violates $\hat{C}$ symmetry in vacuo. We conclude that Eq. (1) satisfies $\hat{C}$, $\hat{P}$, and $\hat{T}$ invariance in vacuo, and is a legitimate equation of electrodynamics.

In Part 1 of this series the vectors $E_\mu$ and $B_\mu$ were defined in terms of the electromagnetic field four tensor[6–15] $F_{\mu\nu}$, and its dual, $\tilde{F}_{\mu\nu}$. It was shown[1] that both $E_\mu$ and $B_\mu$ are Pauli-Lubanski types within the Poincaré group (the inhomogeneous Lorentz group), and that the products $E_\mu E_\mu$ and $B_\mu B_\mu$ form Casimir invariants of the Poincaré group. The Maxwell equations, Poynting theorem, and Stokes parameters were derived in manifestly covariant form, and it was shown that phenomena such as natural optical activity, ellipticity, and the electric Kerr effect can be expressed in terms of changes in $B_\mu$ (or its electric counterpart $E_\mu$). It was shown that optical absorption can be defined in terms of $B_\mu$ and $E_\mu$, and suggestions were made for experiments to detect the magnetizing effect of $B_\mu$ and the polarizing effect of $E_\mu$ as an electromagnetic wave interacts with matter. In this paper (part 2), the Lorentz force equation is investigated in manifestly covariant form; i.e., a manifestly covariant theory is given of the interaction of an electromagnetic wave with the electron.

In Section II, the Lorentz force equation is derived from the covariant definitions of $E_\mu$ and $B_\mu$, and expressed in terms of its magnetic and electric components. Section III examines the individual terms in the equation and shows that in manifestly covariant form, the Lorentz equation contains extra terms that, in principle, produce experimentally observable effects on the electron. There follows a discussion that suggests possible experiments for the detection of the extra manifestly covariant forces on the electron.

## II. DERIVATION OF THE MANIFESTLY COVARIANT LORENTZ FORCE EQUATION

Our aim is to derive the equation describing the interaction of $E_\mu$ and $B_\mu$ with an electron, this being a manifestly covariant description of the interaction of an electromagnetic wave with particulate matter. In Part 1, the four vectors $E_\mu$ and $B_\mu$ were defined as[1]

$$E_\mu = \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\tilde{F}_{\nu\rho}\delta_\sigma \tag{6a}$$

$$B_\mu = -\frac{\mathrm{i}}{2c}\varepsilon_{\mu\nu\rho\sigma}F_{\nu\rho}\delta_\sigma \tag{6b}$$



where $F_{\nu\rho}$ is the four curl of $A_\nu$ in free spacetime, and $\tilde{F}_{\nu\rho}$ is its dual. The unit tensors $\varepsilon_{\mu\nu\rho\sigma}$ and $\delta_\sigma$ are respectively the totally antisymmetric unit tensor in four dimensions and the unit generator of spacetime translations.[1, 20] These quantities are written out for reference as follows:

$$E_\mu \equiv (E^{(1)}, E^{(2)}, E^{(3)}, -\mathrm{i}E^{(0)}) \tag{7a}$$

$$B_\mu \equiv (B^{(1)}, B^{(2)}, B^{(3)}, -\mathrm{i}B^{(0)}) \tag{7b}$$

$$\delta_\mu \equiv (0, 0, 1, -\mathrm{i}) \tag{7c}$$

$$F_{\mu\nu} \equiv \begin{bmatrix} 0 & cB^{(3)} & -cB^{(2)} & -\mathrm{i}E^{(1)} \\ -cB^{(3)} & 0 & cB^{(1)} & -\mathrm{i}E^{(2)} \\ cB^{(2)} & -cB^{(1)} & 0 & -\mathrm{i}E^{(3)} \\ \mathrm{i}E^{(1)} & \mathrm{i}E^{(2)} & \mathrm{i}E^{(3)} & 0 \end{bmatrix} \tag{7d}$$

$$\tilde{F}_{\mu\nu} \equiv \begin{bmatrix} 0 & -\mathrm{i}E^{(3)} & \mathrm{i}E^{(2)} & cB^{(1)} \\ \mathrm{i}E^{(3)} & 0 & -\mathrm{i}E^{(1)} & cB^{(2)} \\ -\mathrm{i}E^{(2)} & \mathrm{i}E^{(1)} & 0 & cB^{(3)} \\ -cB^{(1)} & -cB^{(2)} & -cB^{(3)} & 0 \end{bmatrix} \tag{7e}$$

The need to define $E_\mu$ and $B_\mu$ as four vectors in spacetime is a direct consequence of Eq. (1), because the latter implies that there is a relation between the transverse and longitudinal spacelike components of the electromagnetic wave in vacuo. The conventional assertion that longitudinal components be "unphysical"[6–15] is no longer tenable in view of Eq. (1), because if $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$ be physically meaningful, then so must $\mathbf{B}^{(3)}$. It has been demonstrated[1–5] that the existence of $\mathbf{B}^{(3)}$ implies the existence of $\mathbf{E}^{(3)}$, and quantum field theory[20] leads to

$$\begin{gathered} |\mathbf{B}^{(3)}| - B^{(0)} = 0 \qquad |\mathbf{E}^{(3)}| - E^{(0)} = 0, \\ \mathbf{B}^{(3)} \cdot \mathbf{B}^{(3)} = B^{(0)2} \qquad \mathbf{E}^{(3)} \cdot \mathbf{E}^{(3)} = E^{(0)2} \end{gathered} \tag{8}$$

i.e., that physical states are admixtures of polarizations (3) and (0). Therefore, all four polarizations are physically meaningful. This is consistent with the fact that $A_\mu$ has four components.

The Lorentz force equation can be expressed covariantly by

$$f_\mu = F_{\mu\nu}J_\nu \tag{9}$$

where $f_\mu$ is a force four vector[6] and $J_\mu$ is the charge current four vector. In the conventional theory this is taken to be an adequate description of the interaction of the electric and magnetic components of the electromagnetic field with an electron. The conventional approach, however, assumes that the longitudinal and timelike components of these fields are unphysical, which means essentially that the longitudinal component is set to zero. In view of Eqs. (8) this is an illogical procedure, because if $E^{(3)}$ or $B^{(3)}$ be zero, then so must $E^{(0)}$ and $B^{(0)}$, but the latter are also proportional to the amplitudes of transverse components such as $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$, and in defining these, $E^{(0)}$ is obviously not zero. The conventional approach is therefore logically inconsistent. In the manifestly covariant theory,[1] on the other hand, this inconsistency is remedied. The inconsistency of the conventional approach is "hidden" by the mathematical nature of the four curl, which defines $F_{\mu\nu}$ as

$$F_{\mu\nu} \equiv \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu} \tag{10}$$

since this four curl leaves the timelike components of $E_\mu$ and $B_\mu$ undefined, i.e., the matrix $F_{\mu\nu}$ contains only the spacelike components on its off diagonals. The conventional antisymmetric tensor $F_{\mu\nu}$ contains no reference to the timelike polarizations $E^{(0)}$ and $B^{(0)}$. It follows, therefore, that the Lorentz force equation in covariant form (9) cannot be manifestly covariant, because $F_{\mu\nu}$ is used to define the Lorentz force vector $f_\mu$. Manifest covariance means that the physically meaningful polarizations (0) and (3) must be taken into consideration when calculating the force on the electron.

This is achieved by solving Eqs. (6a), (6b), and (9) simultaneously as follows. We note firstly the definitions of $J_\mu$ and $f_\mu$:

$$J_\mu \equiv \left( \rho\frac{v^{(1)}}{c}, \rho\frac{v^{(2)}}{c}, \rho\frac{v^{(3)}}{c}, \mathrm{i}\rho \right) \tag{11a}$$

$$f_\mu \equiv \left( f^{(1)}, f^{(2)}, f^{(3)}, f^{(0)} \right) \tag{11b}$$

where $\rho$ is the charge density, and $v^{(1)}$, $v^{(2)}$, and $v^{(3)}$ are the spacelike velocity components of the electron. The inverse of $J_\mu$ is defined so that

$$J_\mu J_\mu^{-1} = 1 \tag{12}$$

Multiplying both sides of Eq. (9) from the right by $J_\nu^{-1}$ we obtain

$$F_{\mu\nu} = f_\mu J_\nu^{-1} \tag{13}$$



so that in Eq. (6b)

$$cB_\mu = -\frac{\mathrm{i}}{2}\varepsilon_{\mu\nu\rho\sigma} f_\nu J_\rho^{-1} \delta_\sigma \tag{14}$$

Multiplying both sides from the right by $\delta_\sigma^{-1}$ yields

$$cB_\mu \delta_\sigma^{-1} = -\frac{\mathrm{i}}{2}\varepsilon_{\mu\nu\rho\sigma} f_\nu J_\rho^{-1} \tag{15}$$

and multiplying both sides of this equation from the right by $J_\rho$ gives

$$cB_\mu J_\rho \delta_\sigma^{-1} = -\frac{\mathrm{i}}{2}\varepsilon_{\mu\nu\rho\sigma} f_\nu \tag{16}$$

Here we have used the fact that

$$J_\rho \delta_\sigma^{-1} = \delta_\sigma^{-1} J_\rho \tag{17}$$

Finally, multiplying both sides of Eq. (16) from the right by $\delta_\sigma$ gives th magnetic part of the Lorentz force equation in manifestly covariant form

$$cB_\mu J_\rho = -\frac{\mathrm{i}}{2}\varepsilon_{\mu\nu\rho\sigma} f_\nu \delta_\sigma \tag{18}$$

Similarly, the electric part of the Lorentz force equation is

$$E_\mu J_\rho = \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma} g_\nu \delta_\sigma \tag{19}$$

where $g_\mu$ is defined through the dual $\tilde{F}_{\mu\nu}$ of the electromagnetic field tensor $F_{\mu\nu}$:

$$g_\mu \equiv \tilde{F}_{\mu\nu} J_\nu \tag{20}$$

## III. COMPONENTS OF THE MANIFESTLY COVARIANT LORENTZ EQUATION

In this section the structure of the tensor Eqs. (18) and (19) is investigated for individual terms, and the result is compared with the conventiona

TABLE I
Summary: Generalized Lorentz Equation

| Force | Components |
|---|---|
| $f_0$ | $E_1J_1i,\ E_2J_2i$ |
| | New $E_3J_3i$ |
| $f_1$ | $-2cB_2J_3$ |
| | $2cB_2\rho = 2E_1\rho$ |
| | New $2cB^{(0)}J_2 = 2cB^{(3)}J_2$ |
| $f_2$ | $2cB_1J_3$ |
| | $-2cB_1\rho = 2E_2\rho$ |
| | New $-2cB^{(3)}J_1 = -2cB^{(0)}J_1$ |
| $f_3$ | $c(B_2J_1 - B_1J_2)$ |
| | New $-E_3\rho$ |

an electron. Note that in the conventional approach, only the two transverse components (1) and (2) exist, the components (0) and (3) are discarded as unphysical. In our manifestly covariant approach, components (0), (1), (2), and (3) are physically meaningful. Explicit calculations are given, and the individual results from Eq. (18) are presented in Table I.

### A. Components $f_1$ and $f_2$

The Lorentz force on the electron due to $B_0$ is given as follows:

$$\begin{aligned} cB_0J_2 &= -\frac{i}{2}\varepsilon_{0123}f_1\delta_3 \\ cB_0J_1 &= -\frac{i}{2}\varepsilon_{0213}f_2\delta_3 \end{aligned} \tag{21}$$

and with the definitions $B_0 = -iB^{(0)}$ and $\delta_3 \equiv 1$ we obtain the 1 and 2 components of $f_\mu$

$$\begin{aligned} f_1 &= 2cB^{(0)}J_2 = 2\rho v_2 B^{(0)} \\ f_2 &= -2cB^{(0)}J_1 = -2\rho v_1 B^{(0)} \end{aligned} \tag{22}$$

Similarly, it may be shown that



so that

$$\begin{aligned} f_1 &= 2cB^{(0)}J_2 = 2cB^{(3)}J_2 \\ f_2 &= -2cB^{(0)}J_2 = -2cB^{(3)}J_2 \end{aligned} \tag{24}$$

which is consistent with Eq. (8), i.e., with the quantum theoretical result that physical photon states are admixtures of (0) and (3) polarizations such that Eq. (8) holds.

Clearly, the forces in Eq. (24) are absent from the conventional Lorentz equation. It is seen by inspection of Eqs. (22) and (23) that they have precisely the same form as the equations of motion[8] of a charge in a static magnetic field $\mathbf{B}^{(3)}$, whose magnitude is equal to $B^{(0)}$. This is consistent with the phase-independent definition of $\mathbf{B}^{(3)}$, Eq. (1), although $\mathbf{B}^{(3)}$ is generated by a photon traveling at the speed of light and cannot be regarded as a conventional magnetostatic field. It is a longitudinal magnetic field which travels with the photon at the speed of light. In principle, an experiment can be devised for measuring these extra forces on the electron in the manifestly covariant theory. This possibility is discussed further below.

Additional contributions to $f_1$ and $f_2$ arise from the timelike component of the charge current four vector $J_\mu$, but unlike the contributions from Eq. (24), these are also present in the conventional theory. They arise as follows:

$$\begin{aligned} cB_1J_0 &= -\frac{i}{2}\varepsilon_{1203}f_2 \\ f_2 &= -2cB_1\rho \end{aligned} \tag{25}$$

and using the relation in circular polarization,

$$cB_1 = -E_2 \tag{26}$$

we obtain

$$f_2 = 2E_2\rho \tag{27}$$

Similarly,

$$f_1 = 2cB_2\rho \tag{28}$$

we obtain

$$f_1 = 2E_1\rho \tag{30}$$

### B. The $f_0$ and $f_3$ Forces

The timelike $f_0$ components from Eq. (18) are obtained by

$$\begin{aligned} f_0 &= -2cB_1J_2i = E_2J_2i \\ f_0 &= 2cB_2J_1i = E_1J_1i \end{aligned} \tag{31}$$

and correspond to the well-known[8] $\mathbf{E} \cdot \mathbf{J}$ force from the conventional Lorentz force equation in covariant from. Therefore, the manifestly covariant Eq. (18) provides no new terms in $\mathbf{E} \cdot \mathbf{J}$.

The $f_3$ component is obtained from

$$f_0 + \mathrm{i}f_3 = \frac{2}{\mathrm{i}}cB_1J_2 \tag{32}$$

together with

$$f_0 + \mathrm{i}f_3 = -\frac{2}{\mathrm{i}}cB_2J_1 \tag{33}$$

Solving Eqs. (32) and (33) simultaneously gives

$$\begin{aligned} f_0 &= 0 \\ f_3 &= c(B_2J_1 - B_1J_2) \end{aligned} \tag{34}$$

i.e., there is no contribution to $f_0$ from the $B_2$ and $B_1$ components interacting with $J_1$ and $J_2$ respectively, and the overall $f_3$ component is the same as that in the conventional Lorentz force equation. Finally, there are manifestly covariant forces:

$$\begin{aligned} f_0 &= F_{03}J_3 = \mathrm{i}E_3J_3 \\ f_3 &= F_{30}J_0 = -E_3\rho \end{aligned} \tag{35}$$

direct from Eq. (9), the equation from which (18) is derived using (6a) and (6b).

These results are summarized in Table I, which shows that there are additions to the conventional $f_0$, $f_1$, $f_2$, and $f_3$ in the manifestly covariant ...tion. From Table I, Eq. (18) reduces to the conventional Lorentz



equation if $\mathbf{B}^{(3)} = \mathbf{0}$. However, this assumption of the conventional approach is illogical, because it conflicts with Eq. (1).

## IV. DISCUSSION

The question arises immediately as to whether the extra terms in Table I marked "new" are observable experimentally. This type of observation might be able to distinguish between the conventional theory and the manifestly covariant approach based on Eq. (1). It is probably difficult to isolate a single electron in a vacuum in order to test the theory directly, but the use of electron beams may be feasible. The resultant force on a single electron due to an electromagnetic plane wave is given in the manifestly covariant approach by a combination of Eqs. (9) (the conventional equation) and (18), taking into account thereby the existence of Eqs. (1) and (8). In the conventional approach, the force is described by the spacelike components of Eq. (9) alone, and Eqs. (1), (8), and (18) are not considered.

The conventional calculation of the trajectory of an electron in a monochromatic, circularly polarized plane wave is a standard problem (e.g., Landau and Lifshitz,[10] p. 118), in which there are no linear forces such as $-E_3\rho$ of the manifestly covariant theory. Presumably, such a force would cause the linear deflection of an electron beam when the latter is acted upon by a circularly polarized electromagnetic beam, such as an X-ray beam. However, this assumption does not consider statistical effects in either the electromagnetic or electron beam. Extra Lorentz precession terms due to $f_1$ and $f_2$ are expected in the manifestly covariant theory. If extra forces can be observed unequivocally, this would add to the considerable experimental evidence for the manifestly covariant theory reviewed in Part 1 of this series, evidence from sources such as absorption of circularly polarized light, circular dichroism, ellipticity, and the electric Kerr effect

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# *MANIFESTLY COVARIANT THEORY OF THE ELECTROMAGNETIC FIELD IN FREE SPACETIME, PART 3: $\hat{C}$, $\hat{P}$, AND $\hat{T}$ SYMMETRIES*

## I. INTRODUCTION

It has recently been observed[1–5] that there exists an equation of electrodynamics in vacuo that defines a longitudinal magnetic field, $\mathbf{B}^{(3)}$, which is independent of the phase of the electromagnetic plane wave, thus showing for the first time that there exist physically meaningful longitudinal solutions to Maxwell's equations in vacuo. Parts 1 and 2 of this series[1, 2] developed the theory of manifestly covariant electrodynamics from this basic observation, and recent work by Farahi and Evans[4] has shown that the existence of $\mathbf{B}^{(3)}$ implies the existence of its longitudinal electric counterpart $\mathrm{i}\mathbf{E}^{(3)}$. In Part 1[1] it was shown that $\mathrm{i}\mathbf{E}^{(3)}$ and $\mathbf{B}^{(3)}$ do not contribute to the electromagnetic energy density, and that Poynting's theorem can be expressed in terms of four, rather than two, polarizations. The existence of four photon polarizations, (0), (1), (2), and (3), was reconciled with two photon helicities, +1 and −1, by noting[1] that the helicities can be defined in terms either of (0) and (3) or of (1) and (2). Here (0) denotes the timelike photon polarization, (1) and (2) the transverse spacelike, and (3) the longitudinal spacelike. In Part (2), the Lorentz force equation was expressed in manifestly covariant form.



In this paper (Part 3), the fundamental symmetries of physics are applied to the basic equation

$$\mathbf{B}^{(3)} = \frac{\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}}{\mathrm{i} E_0 c} \tag{1}$$

of manifestly covariant electrodynamics (MCE). Here $\mathbf{B}^{(3)}$ is linked[1–5] to the transverse, oscillating, electric fields $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$ of the plane wave in vacuo, where $c$ is the speed of light. Here $\mathbf{E}^{(1)}$ is the complex conjugate of $\mathbf{E}^{(2)}$:

$$\mathbf{E}^{(1)} \equiv E_0 \hat{\mathbf{e}}^{(1)} \mathrm{e}^{\mathrm{i}\phi} \tag{2a}$$

$$\mathbf{E}^{(2)} \equiv E_0 \hat{\mathbf{e}}^{(2)} \mathrm{e}^{-\mathrm{i}\phi} \tag{2b}$$

where

$$\phi = \omega t - \boldsymbol{\kappa} \cdot \mathbf{r} \tag{3}$$

is the phase of the plane wave, with, as usual, $\omega$ as the angular frequency at instant $t$, $\boldsymbol{\kappa}$ the wave vector at position $\mathbf{r}$. The circular basis[6, 7] is used to define the unit vectors $\hat{\mathbf{e}}^{(1)}$ and $\hat{\mathbf{e}}^{(2)}$:

$$\hat{\mathbf{e}}^{(1)} = \frac{1}{\sqrt{2}}(\mathbf{i} - \mathrm{i}\mathbf{j}) \tag{4a}$$

$$\hat{\mathbf{e}}^{(2)} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathrm{i}\mathbf{j}) \tag{4b}$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in axes $X$ and $Y$ of the Cartesian frame $(X, Y, Z)$.

In Section II, it is shown that Eq. (1) is invariant under the following conditions:

1. The charge conjugation operator $\hat{C}$, which changes the sign of charge in classical electrodynamics, and in particle physics produces the antiparticle from the original particle
2. The parity inversion operator $\hat{P}$
3. The motion reversal operator $\hat{T}$

In other words, the left and right sides of Eq. (1) remain balanced after application of $\hat{C}$, $\hat{P}$, and $\hat{T}$ to each variable on both sides. Equation (1) is therefore a legitimate equation of electrodynamics, and $\mathbf{B}^{(3)}$ has the $\hat{C}$, $\hat{P}$, and $\hat{T}$ symmetries, and units, of magnetic flux density. $\mathbf{B}^{(3)}$ is also a