

# Origin of the Photon Mass and ECE Spin Field in the Spin Connection of Space-Time

by

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## Abstract

By deriving the Proca equation from the tetrad postulate and the ECE hypothesis it is shown that the photon mass and ECE spin field are derived from the spin connection, proving that they are inter-related. The ECE wave and field equations for the photon with non-zero mass are inter-related through this derivation of the Proca equation. Thus, evidence for ECE theory as preferable to the standard model is also evidence for a non-zero photon mass. A first approximate estimate of the photon mass is made using this method.

**Keywords:** ECE unified field theory, Proca equation, photon mass, ECE spin field.

## 12.1 Introduction

The mass of the photon is centrally important in physics, it is the basis of the corpuscular theory of light (for example the photo-electric effect and the Compton effect), wave particle dualism and of the quantum theory, and is the basis of general relativity as proven in the deflection of light by gravity. Recently [1–12] it has been shown that quantum mechanics and general relativity can be described self-consistently and in a unified manner from geometry [13] in the Einstein Cartan Evans (ECE) unified field theory. Quantum mechanics is shown by ECE theory to be a causal and objective theory

and is derived from the fundamental property of geometry known as the tetrad postulate [13] - a complete vector field must be independent of the way in which it is expressed through components and basis elements. By covariant differentiation of the tetrad postulate the ECE Lemma is obtained straightforwardly, thereby defining a scalar curvature  $R$  in terms of the two connections of Cartan geometry, the general gamma connection and the spin connection. The ECE wave equation is then obtained through the application of the Einstein postulate:

$$R = -kT \quad (12.1)$$

to all radiated and matter fields. Here  $R$  is the well defined scalar curvature of the ECE Lemma [1–12],  $k$  is Einstein's constant and  $T$  is the scalar canonical energy momentum density defined by  $R$  of the ECE Lemma. Eq. (12.1) relates geometry and physics in ECE theory and is a generalization of Einstein's original gravitational postulate [14] to all radiated and matter fields of physics. The ECE postulate is:

$$A_\mu^a = A^{(0)} q_\mu^a \quad (12.2)$$

where  $A_\mu^a$  is the electromagnetic potential and  $q_\mu^a$  is the Cartan tetrad [13]. The least value of the voltage  $cA^{(0)}$  is a universal constant:

$$cA^{(0)} = \frac{E_0}{e} \quad (12.3)$$

where

$$E_0 = mc^2 \quad (12.4)$$

is the rest energy of the photon and where  $e$  is the magnitude of the charge on the electron. Here  $m$  is the non-zero photon mass responsible for the deflection of light by gravity and  $c$  is the vacuum speed of light, a universal constant of relativity theory. Special relativity is known to be precise to one part in thirty orders of magnitude and general relativity without torsion (Einstein Hilbert (EH) field theory) to one part in one hundred thousand in the solar system. The effects of torsion in ECE theory pervade cosmology, for example [1–12] they are responsible for the formation of spiral galaxies. Quantum mechanics is also known to be precise to many orders of magnitude, but still far less so than special relativity, by far the most precise theory in physics by many orders of magnitude.

Despite the central and obvious importance of a non-zero photon mass, the standard model's U(1) gauge theory of electrodynamics asserts that photon mass is identically zero. This is incorrect, because light would not be deflected by gravity if this were the case. Unsurprisingly, the massless photon assumption leads to insurmountable problems for the standard model. For example the Proca equation is not U(1) gauge invariant [15] and canonical quantization is not self consistent in the standard model because of the loss of manifest covariance [15]. The manifestly covariant theory of light requires that the three space components and the time component of the electromagnetic potential be physical. This is what is correctly given in canonical quantization of the potential by a theory with finite photon mass [15]. A theory with identically zero photon mass cannot be correctly quantized, and loses manifest covariance. The Gupta Bleuler method is used to save the hypothesis of identically zero photon mass. The Gupta Bleuler method leads to the incorrect conclusion that only transverse modes of electromagnetic radiation can be physical.

In Section 12.2 the wave structure of the Proca equation 12.15, the fundamental wave equation for the free photon with mass in relativistic quantum mechanics, is derived from the ECE wave equation and hypothesis. The gauge principle is replaced in ECE theory by the invariance of the tetrad postulate, and is not used. This method is an advance over the standard model in several ways, notably the abstract internal gauge indices of Yang and Mills are identified as being geometrical in origin. The Proca equation is also shown to be geometrical in origin, and the problem of lack of gauge invariance does not exist in ECE theory because the gauge principle is not needed. The gauge principle is superfluous to general relativity and is discarded in ECE theory using the Ockham Razor of natural philosophy. In ECE theory all four photon states are correctly physical, and manifestly covariant, and canonical quantization of the electromagnetic potential becomes rigorously consistent with the existence of photon mass. The Lorentz condition in ECE theory becomes a consequence of the derivation of the wave equation of photon mass from geometry. The Lorentz condition is no longer an additional postulate, as in the standard model. In consequence, conservation of charge - current density is also derived in ECE theory from geometry. It is a direct consequence of the derived Lorentz condition. This derivation of the Proca equation shows that free photon mass originates in the spin connection, as does the fundamental ECE spin field observable in the inverse Faraday effect. In the standard model neither the photon mass nor the ECE spin field exist, despite the fact that both concepts are fundamental and experimentally observable. Therefore the standard model is fragmented, flawed and obsolete in many ways [1–12]. In the course of this derivation the ECE wave and field equations are unified in the unified theory of photon mass.

In Section 12.3 the Proca equation is derived in the presence of matter, and for electromagnetic field/matter interaction.

## 12.2 Derivation of the Proca Equation from Geometry for the Free Photon with Mass

In ECE theory the relativistic quantum mechanics of a massive spin one particle are derived straightforwardly from the ECE Lemma:

$$\square q_\mu^a = Rq_\mu^a \quad (12.5)$$

and the general Einstein hypothesis:

$$R = -kT \quad (12.6)$$

as applied to all radiated and matter fields of physics. Eqs. (12.5) and (12.6) give the ECE wave equation [1–12]:

$$(\square + kT)q_\mu^a = 0 \quad (12.7)$$

of generally covariant quantum mechanics. In the standard model such a concept is non-existent because quantum mechanics cannot be unified with general relativity. In the standard model quantum mechanics of the Copenhagen School is acausal and subjective, and is a theory of special relativity, while general relativity in the standard model is restricted to classical gravitation and is causal and objective. So the standard model is hopelessly flawed and in ECE theory all radiated and matter fields are causal and objective and all are generally covariant [1–12] as required by the fundamentals of relativity. The latter cannot be discarded because it is the most precise theory in physics.

Using the ECE hypothesis (12.2) gives the generally covariant equation of quantum electrodynamics:

$$(\square + kT)A_\mu^a = 0 \quad (12.8)$$

For a photon free of the influence of other fields, the correspondence principle shows that:

$$kT = \left(\frac{mc}{\hbar}\right)^2 \quad (12.9)$$

where  $m$  is the mass of the photon and  $\hbar$  is the reduced Planck constant. For each state of polarization:

$$a = (0), (1), (2), (3) \quad (12.10)$$

Eq. (12.8) is the Proca equation. Here (1) and (2) denote complex conjugate transverse states and (3) denotes the longitudinal state of polarization. The

(0) state is time-like and all four states are physical. The longitudinal state (3) is observed through the ECE spin field responsible for the magnetization of matter by circularly polarized radiation, the inverse Faraday effect [1–12].

In ECE theory the electromagnetic field is defined by the ECE hypothesis (12.2) and the first Cartan structure equation:

$$F^a = d \wedge A^a + \omega^a_b \wedge A^b. \quad (12.11)$$

Here  $\wedge$  is the wedge product, so  $d\wedge$  denotes the exterior derivative [13]. This notation is slightly different from that used conventionally in order to emphasize the fact that  $d\wedge$  is anti-symmetric. In Eq. (12.11),  $\omega^a_b$  denotes the spin connection [1–13]. The spin connection is not a tensor, but its wedge product with  $A^b$  is a tensor. In ECE theory, as in EH theory, connections always appear in such a way that their non-tensorial nature is cancelled out by a wedge product, as is well known [13]. The purpose of the connection is to define a space-time that is not the Minkowski space-time. In EH theory the connection is the Christoffel connection:

$$\Gamma^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\nu\mu}. \quad (12.12)$$

In ECE theory the connections are more general in nature, and define torsion as well as the curvature of EH theory. In ECE theory a frame may spin with respect to another as well as curve with respect to another. The two frames in gravitational Cartan geometry are defined by the base manifold space-time (indexed  $\mu$ ) and the tangent space-time at point  $P$ , indexed  $a$ . The tangent space-time is a Minkowski space-time, or “flat space-time”. For the gravitational sector of ECE  $\mu$  curves with respect to  $a$ . In other sectors of ECE theory (e.g. electromagnetic, weak, strong and fermion) the  $a$  and  $\mu$  frames spin with respect to each other in a well defined way. So all physics is defined and unified by these geometrical concepts, and two hypotheses, Eqs. (12.1) and (12.2). So all physics becomes consistent with relativity, all physics is causal and objective.

In tensor notation [1–12] the electromagnetic field defined by Eq. (XX) is:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \omega_{\mu b}^a A_\nu^b - \omega_{\nu b}^a A_\mu^b. \quad (12.13)$$

Therefore:

$$\partial^\mu F_{\mu\nu}^a = \square A_\nu^a + \partial^\mu (\omega_{\mu b}^a A_\nu^b - \omega_{\nu b}^a A_\mu^b). \quad (12.14)$$

Comparing Eqs. (12.9) and (12.14):

$$\left(\frac{mc}{\hbar}\right)^2 A_\nu^a = \partial^\mu (\omega_{\mu b}^a A_\nu^b - \omega_{\nu b}^a A_\mu^b) \quad (12.15)$$

for the free photon, or classically, for the electromagnetic field in free space, no material matter being present. Now use the normalization:

$$A_\nu^a A_a^\nu = 4A^{(0)2} \quad (12.16)$$

derived from the tetrad normalization [13]:

$$q_\nu^a q_a^\nu = 4 \quad (12.17)$$

to find that the photon mass is given by the spin connection

$$m^2 = \frac{\hbar^2}{4c^2 A^{(0)2}} A_a^\nu \partial^\mu (\omega_{\mu b}^a A_\nu^b - \omega_{\nu b}^a A_\mu^b). \quad (12.18)$$

In a rough first approximation:

$$m \lesssim \frac{\hbar}{c} = 10^{-42} \text{kg} \quad (12.19)$$

a result which is meant only to indicate that the photon mass is very small. Experiments [16] indicate that it is less than  $10^{-51}$  kilograms.

From Eq. (12.15):

$$\left(\frac{mc}{\hbar}\right)^2 \partial^\nu A_\nu^a = 0 \quad (12.20)$$

so the Lorenz condition is derived for each  $a$ :

$$\partial^\nu A_\nu^a = 0 \quad (12.21)$$

and is always true. In the standard model the Lorenz condition is still the nineteenth century assumption used to arrive at the d'Alembert equation, in which there is no photon mass. In ECE theory the Lorenz condition always applies, meaning that there can be no gauge freedom in ECE theory. The gauge principle therefore becomes obsolete and is replaced by the invariance of the tetrad postulate [1–12]. The Proca equation in ECE theory is also derived from geometrical first principles and assumes its rightful place as a fundamentally important equation of physics. In the standard model the Proca equation is rarely mentioned. When it is mentioned [15], its equations are as follows:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \quad (12.22)$$

$$\partial_\mu F^{\mu\nu} = -\left(\frac{mc}{\hbar}\right)^2 A^\nu, \quad (12.23)$$

$$\partial_\nu A^\nu = 0. \quad (12.24)$$

These imply the wave equation:

$$\left(\square + \left(\frac{mc}{\hbar}\right)^2\right) A_\nu = 0 \quad (12.25)$$

whose lagrangian density in reduced notation, ( $c = \hbar = 1$ ), is:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu. \quad (12.26)$$

This is not U(1) gauge invariant because of the mass term [15]. In the standard model therefore a spin one particle with non-zero mass is not gauge invariant. The natural consequence has been the development of ECE theory and the abandonment of the gauge principle [1–12]. This development simplifies physics as required by the Ockham Razor. The fact that Eq. (12.24) is needed by Proca theory means that the gauge freedom:

$$A^\nu \rightarrow A^\nu + \partial^\nu \chi \quad (12.27)$$

is lost. This is because if Eq. (12.27) were true, Eq. (12.24) would not always be zero. The generally covariant form of Eq. (12.24) is Eq. (12.21). In the standard model the potential  $A^\nu$  is asserted to be unphysical, but this can no longer be true by Eqs. (12.24) and (12.26). In ECE theory the potential is physical, manifestly and generally covariant. The physical potential is observed through the Aharonov Bohm effects, for example, the inverse Faraday effect, the minimal prescription leading to ESR, NMR and MRI, and so on.

### 12.3 Proca Equation in the Presence of Matter

In the presence of matter the inhomogeneous ECE field equation is [1–15]:

$$\partial^\mu F_{\mu\nu}^a = \mu_0 J_\nu^a \quad (12.28)$$

where  $J_\nu^a$  is the inhomogeneous charge-current density. Thus:

$$\partial^\mu F_{\mu\nu}^a \neq 0. \quad (12.29)$$

The Proca equation in the presence of matter becomes:

$$\left(\square + \left(\frac{mc}{\hbar}\right)^2\right) A_\mu^a = \frac{\mu_0}{c} J_\mu^a. \quad (12.30)$$

This equation is the ECE wave equation:

$$(\square + kT)A_\mu^a = 0 \quad (12.31)$$

provided that:

$$J_\mu^a = \frac{c}{\mu_0} \left( \left( \frac{mc}{\hbar} \right)^2 - kT \right) A_\mu^a. \quad (12.32)$$

As indicated experimentally [16] the photon mass is very small, so for all practical purposes in the laboratory, Eq. (12.32) becomes:

$$J_\mu^a = -\frac{ckT}{\mu_0} A_\mu^a. \quad (12.33)$$

It follows that:

$$\partial^\mu J_\mu^a = 0 \quad (12.34)$$

which is the conservation of charge-current density on ECE theory. The latter follows from the derived Lorenz condition, Eq. (12.21). Eq. (12.34) means that for each state of polarization, charge-current density is always conserved, and that this is the result of geometry as required by the principle of relativity.

There are several fundamentally important effects of photon mass [16] and these should be looked for experimentally because ECE theory now shows that photon mass is centrally important to relativity. For example, the equivalence principle may be applied straightforwardly to electrodynamics through the photon mass, as for any particle. Eq. (12.3) shows that electrodynamics is defined by a universal constant, the least voltage  $\left(\frac{E_0}{e}\right)$ , and electrodynamics is geometrical as is gravitation. In ECE theory there is only one field, the unified field. All physics is manifested in different limits of this one field. In contrast the standard model is hopelessly fragmented and self-inconsistent

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