

# THE GENERALLY COVARIANT INVERSE SQUARE LAW OF ALL ORBITS.

by

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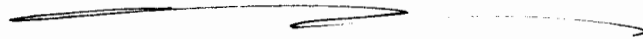
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## ABSTRACT

Using the theory of fluid gravitation it is shown that all observable orbits can be expressed as a generally covariant inverse square law of universal gravitation. The law can be expressed in several ways, notably that the acceleration due to gravity between a mass  $m$  orbiting a mass  $M$  is the derivative of the orbital velocity of  $m$  in a moving frame of reference. This is the Lagrange derivative, and in fluid gravitation becomes the convective derivative.

Keywords: ECE2, fluid gravitation, generally covariant inverse square law.

UFT 36



## 1. INTRODUCTION

In immediately preceding papers of this series {1-12} the subject of fluid gravitation has been developed by unifying the ECE2 field equations of gravitation and fluid dynamics. Similarly, in earlier papers, the subject of fluid electrodynamics was developed by unifying the ECE2 field equation of electrodynamics and fluid dynamics. In the preceding paper, fluid dynamics led to a new inverse square law of attraction between a mass  $m$  orbiting a mass  $M$ . In Section 2 this law is developed in various ways and applied to three examples of planar orbits: the conic section, the precessing ellipse and the hyperbolic spiral. The law can also be applied in three dimensions as shown in the preceding paper.

This paper is a brief synopsis of detailed calculations given in the notes accompanying UFT360 on [www.aias.us](http://www.aias.us). Note 360(1) gives a detailed self consistency check for the elliptical planar orbit. Notes 360(2) gives the generally covariant inverse square law of all orbits. Note 360(3) derives the new force law for a whirlpool galaxy. Note 360(4) gives the new force law for a precessing elliptical orbit, and Note 360(5) gives new general expressions for acceleration due to gravity.

Section 3 is a numerical and graphical analysis of the new law.

## 2. DEVELOPMENT OF THE NEW LAW

The law can be expressed as follows:

$$\underline{g} = \left( \underline{v} \cdot \underline{\nabla} \right) \underline{v} \quad - (1)$$

in two or three dimensions. Here  $\underline{g}$  is the acceleration due to gravity between a mass  $m$  orbiting a mass  $M$ . The orbital velocity of the mass  $m$  is  $\underline{v}$ . Therefore  $\underline{g}$  is the derivative of  $\underline{v}$  in a moving frame of reference - the Lagrange derivative. In fluid gravitation it is referred to

as the convective derivative using the traditional terminology. For planar orbits, the inverse square law is:

$$\underline{g} = \frac{-mG}{x^2 + y^2} \underline{e}_r \quad - (2)$$

From Eq. ( 1 ), the orbital velocity is:

$$\underline{v} = \frac{(mG)^{1/2} (-x \underline{i} + y \underline{j})}{(x^2 + y^2)^{3/4}} \quad - (3)$$

The law ( 1 ) is generally covariant because it is derived from a generally covariant unified field theory. For a circular orbit:

$$x^2 + y^2 = r^2 \quad - (4)$$

and the law ( 1 ) reduces to the form of the Hooke Newton inverse square law of universal gravitation. However, note carefully that for all orbits, including the circular orbit, the new law is generally covariant. The Hooke Newton law is empirical, non relativistic and galilean covariant.

In plane polar coordinates ( r,  $\theta$  ):

$$v^2 = \frac{mG}{(x^2 + y^2)^{1/2}} = \dot{r}^2 + r^2 \dot{\theta}^2 \quad - (5)$$

For the elliptical orbit it is well known that:

$$v^2 = mG \left( \frac{2}{r} - \frac{1}{a} \right) \quad - (6)$$

where a is the semi major axis, so for the elliptical orbit:

$$\frac{1}{(x^2 + y^2)^{1/2}} = \frac{2}{r} - \frac{1}{a} = \frac{1}{mG} (\dot{r}^2 + r^2 \dot{\theta}^2) \quad - (7)$$

and the acceleration due to gravity in fluid gravitation is:

$$\underline{g} = -\frac{mG}{x^2 + y^2} \underline{e}_r = -mG \left( \frac{2}{r} - \frac{1}{a} \right)^2 \underline{e}_r = -\frac{1}{mG} (\dot{r}^2 + r^2 \dot{\theta}^2)^2 \underline{e}_r \quad - (8)$$

being defined by a generally covariant inverse square law. In ECE fluid dynamics (an extension of Kambe fluid dynamics as described in previous papers):

$$\underline{g} = (\underline{v} \cdot \underline{\nabla}) \underline{v} = -\frac{\partial \underline{v}}{\partial t} - \underline{\nabla} \phi \quad - (9)$$

where  $\underline{v}$  plays the role of the vector potential and where

$$\phi = h \quad - (10)$$

is the gravitational scalar potential. Here h is the enthalpy per unit mass m.

Therefore Eq. (7) shows that the acceleration due to gravity is the Lagrange derivative of the orbital velocity and can be expressed as the following law of general relativity, the generally covariant law::

$$\underline{g} = -\frac{v^4}{mG} \underline{e}_r \quad - (11)$$

where  $\underline{e}_r$  is the radial unit vector.

In the galilean covariant Newtonian theory:

$$\phi = -\frac{mG}{r} \quad - (12)$$

and

$$H = \frac{1}{2} m v^2 - \frac{m M G}{r} \quad - (13)$$

where H is the hamiltonian.

The new law applies to all orbits, in the sense that all orbits can be described by the generally covariant inverse square law ( 2 ). It is well known that the Newtonian law applies only to conic sections. The hyperbolic orbit of a whirlpool galaxy is considered in Note 360(3) and the precessing elliptical orbit in Note 360(4).

For all planar orbits X and Y are defined as:

$$X = m G \frac{(\dot{r} \sin \theta + r \dot{\theta} \cos \theta)}{(\dot{r}^2 + r^2 \dot{\theta}^2)^{3/2}} \quad - (14)$$

and

$$Y = m G \frac{(r \dot{\theta} \sin \theta - \dot{r} \cos \theta)}{(\dot{r}^2 + r^2 \dot{\theta}^2)^{3/2}} \quad - (15)$$

and this is the general definition of the moving frame of reference. Astronomical observation of any planar orbit defines:

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta} \frac{d\theta}{dt} \quad - (16)$$

and

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{L}{m r^2} \quad - (17)$$

by establishing the functional dependence of r on  $\theta$ .

### 1) Whirlpool Galaxy

In this case:

$$r = \frac{r_0}{\theta} \quad (18)$$

where  $r_0$  is a constant. It follows that:

$$\dot{r}^2 + r^2 \dot{\theta}^2 = \frac{L^2}{m^2} \left( \frac{1}{r^2} + \frac{1}{r_0^2} \right) \quad (19)$$

where  $L$  is the angular momentum of the system defined by  $m$  orbiting  $M$ . It is a constant of motion worked out from the relevant Euler Lagrange equation (see Note 360(4)). It follows as in Note 360(3) that:

$$X = mG \frac{\left( \frac{L}{mr} \cos\left(\frac{r_0}{r}\right) - \frac{L}{mr_0} \sin\left(\frac{r_0}{r}\right) \right)}{\left( \frac{L^2}{m^2} \left( \frac{1}{r^2} + \frac{1}{r_0^2} \right) \right)^{3/2}} \quad (20)$$

and

$$Y = mG \frac{\left( \frac{L}{mr} \sin\left(\frac{r_0}{r}\right) - \frac{L}{mr_0} \cos\left(\frac{r_0}{r}\right) \right)}{\left( \frac{L^2}{m^2} \left( \frac{1}{r^2} + \frac{1}{r_0^2} \right) \right)^{3/2}} \quad (21)$$

define the relevant moving frame of Eq. (1) for the whirlpool galaxy and hyperbolic spiral orbit of  $m$  about. These coordinates  $X$  and  $Y$  are graphed in Section 3.

## 2) Precessing Elliptical Orbit in a Plane

It is observed astronomically to high precision that all solar system objects of mass  $m$  orbit the sun of mass  $M$  in a planar orbit defined by:

$$r = \frac{d}{1 + e \cos(\gamma\theta)} \quad (22)$$

where:

$$x = \frac{1}{1 + \frac{3MG}{c^2 d}} \quad (23)$$

in which  $d$  is the half right latitude. This experimental or empirical result is also true in binary objects outside the solar system, in which  $x$  is very close to unity. It follows as in Note 360(4) that:

$$\dot{r} = \frac{dr}{dt} = \frac{xEL}{md} \left( 1 + \frac{1}{\epsilon^2} \left( \frac{d}{r} - 1 \right)^2 \right)^{1/2} \quad (24)$$

and

$$\dot{\theta} = \frac{L}{mr^2} \quad (25)$$

So X and Y can be graphed using Eqs. ( 14 ), ( 15 ), ( 24 ) and ( 25 ). This procedure is carried out in Section 3 and defines the moving frame of Eq. ( 1 ), which is the generally covariant inverse square law of the precessing elliptical orbit observed astronomically.

As shown in Note 350(5):

$$\underline{g} = -\frac{v^4}{MG} \underline{e}_r \quad (26)$$

for the precessing elliptical orbit, so its generally covariant acceleration due to gravity can be expressed as:

$$\underline{g} = -\frac{L^4}{m^4 MG} \frac{x^2 \epsilon^2}{d^2} \left( 1 - \frac{1}{\epsilon^2} \left( \frac{d}{r} - 1 \right)^2 \right) + \frac{1}{r^2} \quad (27)$$

The same equation applies to the generally covariant acceleration due to gravity between a star of mass  $m$  orbiting a central mass  $M$  in a whirlpool galaxy. These results are also graphed in Section 3. The precessing ellipse reduces to the static ellipse when:

$$x = 1. \quad (28)$$

Therefore:

$$\underline{F} = -\frac{mM G}{x^2 + 1} \frac{e}{r} \quad (29)$$

is the generally covariant inverse square law for any planar orbit.

### 3. NUMERICAL RESULTS AND GRAPHICS.

Section by Dr. Horst Eckardt



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