DESCRIPTION OF THE PERIHELION PRECESSION AND LIGHT DEFLECTION DUE TO GRAVITATION WITH THE GRAVITOMAGNETIC AMPERE LAW.

by

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ABSTRACT

The ECE2 gravitomagnetic field is calculated for dynamics in general and for a three dimensional orbit. For the planar part of this orbit the gravitomagnetic Ampere law is used to calculate the light deflection due to gravitation and the precession of the perihelion, so that both phenomena are expressed in terms of the gravitomagnetic field of the relevant mass, for example the sun and the planet Mercury.

Keywords: ECE2 theory, gravitomagnetic Ampere law, light deflection due to gravitation and precession of the perihelion.

UFT 322

1. INTRODUCTION

In recent papers of this series {1 - 12} the Jacobi Cartan Evans (JCE) identity of UFT313 has been developed into vector field equations of electromagnetism and gravitation, and named the ECE2 theory. These equations have the same structure as the Maxwell Heaviside (MH) equations but the ECE2 equations are part of a generally covariant unified field theory, so ECE2 is a theory of general relativity. The nineteenth century MH theory is a theory of special relativity and is Lorentz covariant and not generally covariant. However, the vector and tensor structures of ECE2 and MH are the same, so the general covariance of ECE2 can be described by the Lorentz transformation. This property of ECE2 is referred to as Lorentz-like covariance. The covariance of the field tensors of ECE2 produces the Lorentz force and Biot Savart law as in classical electrodynamics. This was the subject of the immediately preceding paper. In this paper the Lorentz like covariance is applied to define the gravitomagnetic Biot Savart law and Ampère law. These laws of gravitomagnetism are applied to dynamics in general and to orbits in particular. A self consistent description is found of light deflection due to gravitation and perihelion precession in terms of the gravitomagnetic field. A similar procedure was used with ECE theory to describe the results of Gravity Probe B in UFT117 and equinoctial precession in UFT119 using the gravitomagnetic Ampère law.

As usual this paper should be read with its accompanying notes. Note 322(1) defines the gravitomagnetic field and the mass current of planar orbits. Note 322(2) is a calculation of the current of mass density, Note 322(3) is a summary of the gravitomagnetic description of orbits, Note 322(4) is the gravitomagnetic description of dynamics in general, Note 322(5) is the calculation of light deflection due to the sun's gravitation in terms of the gravitomagnetic field, Note 322(6) is the calculation of the perihelion precession of Mercury in terms of the gravitomagnetic field.

Section 2 is a summary of the main conclusions given in the notes.

2. DYNAMICS WITH THE GRAVITOMAGNETIC AMPERE LAW

The main results of Notes 322(1) and 322(2) have been used in Section 3 of UFT320, and so this Section begins with a summary. In planar orbit theory {1 - 12} it is well known that the angular velocity is defined by the angular momentum $\underline{L}$, a constant of motion:

$$\underline{\omega} = \frac{L}{mr^2}\underline{k} \quad -(1)$$

where r is the distance between a mass m orbiting a mass M and $\underline{k}$ is the unit vector perpendicular to the plane of the orbits. In this case the gravitomagnetic field is:

$$\underline{\Omega} = -\left(\frac{MG}{mc^2}\right)\frac{L}{r^3}\underline{k} \quad -(2)$$

and the current of mass density is:

$$\underline{J}_m = -\frac{3M}{4\pi m}\frac{L}{r^4}\underline{e}_\theta \quad -(3)$$

where the unit vectors of the cylindrical polar coordinate system are defined as:

$$\underline{e}_\theta = -\underline{i}\sin\theta + \underline{j}\cos\theta \quad -(4)$$

$$\underline{e}_r = \underline{i}\cos\theta + \underline{j}\sin\theta. \quad -(5)$$

If the force of attraction between m and M is the central inverse square law:

$$\underline{F} = -\frac{mMG}{r^2}\underline{e}_r \quad -(6)$$

then

$$L^2 = m^2 MG\alpha \quad -(7)$$

where α is the half right latitude of the conical section orbit:

$$r = \frac{\alpha}{1+\epsilon \cos\theta} \quad -(8)$$

and where ϵ is its eccentricity. For an ellipse:

$$\alpha = \left(1-\epsilon^2\right) a \quad -(9)$$

and for a hyperbola:

$$\alpha = \left(\epsilon^2 - 1\right) a \quad -(10)$$

where a is the semi major axis of the ellipse and is well defined for the hyperbola.

The ECE2 gravitomagnetic description of dynamics in general and orbital dynamics in particular is the gravitomagnetic Ampère law {UFT117, UFT119}:

$$\underline{\nabla} \times \underline{\Omega} = \frac{4\pi G}{c^2} \underline{J}_m . \quad -(11)$$

As in Note 322(4), dynamics in general and orbital theory in particular can be described with Eq. (11). In cylindrical polar coordinates the position vector is:

$$\underline{r} = r \underline{e}_r + Z \underline{k} \quad -(12)$$

the velocity vector is:

$$\underline{v} = \dot{r} \underline{e}_r + \omega r \underline{e}_\theta + \dot{Z} \underline{k} \quad -(13)$$

and the acceleration vector is:

$$\underline{a} = \left(\ddot{r} - r\dot{\theta}^2\right) \underline{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) \underline{e}_\theta + \ddot{Z} \underline{k} \quad -(14)$$

where by definition in orbital theory:

$$\underline{\omega} = \frac{d\theta}{dt} \underline{k} . \quad -(15)$$

In dynamics in general the gravitomagnetic field is:

$$\underline{\Omega} = -\frac{1}{c^2} \underline{v} \times \underline{a} \quad -(16)$$

and contains Newtonian and non Newtonian forces. The lagrangian for the inverse square law (6) is:

$$\mathcal{L} = T - U \quad -(17)$$

where the kinetic energy is:

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\dot{r}^2 + \dot{\theta}^2 r^2 + \dot{Z}^2 \right) \quad -(18)$$

and the potential energy is:

$$U = -\frac{mG}{r} . \quad -(19)$$

The three Euler Lagrange equations are:

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \quad -(20)$$

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(21)$$

$$\frac{\partial \mathcal{L}}{\partial Z} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{Z}} . \quad -(22)$$

Eq. (21) gives the Leibnitz equation of orbits:

$$F(r) = -\frac{\partial U}{\partial r} = m(\ddot{r} - r\dot{\theta}^2) \quad -(23)$$

and Eq. (22) gives:

$$\frac{d^2 Z}{dt^2} = 0. \quad -(24)$$

Eq. (20) defines the conserved angular momentum:

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\theta}}. \quad -(25)$$

In cylindrical polar coordinates:

$$\underline{L} = m\underline{r} \times \underline{v} = m(\omega r Z \underline{e}_r + \dot{r} Z \underline{e}_\theta + \omega r^2 \underline{k}) \quad -(26)$$

so $\underline{L}$ is not in general perpendicular to the orbital plane. This has been pointed out in previous work on three dimensional orbit theory using spherical polar coordinates. It follows from Eq. (26) that the Z component of angular momentum is:

$$L_Z = mr^2\omega \quad -(27)$$

and is a conserved constant of motion:

$$\frac{dL_Z}{dt} = 0 \quad -(28)$$

if the angular velocity is defined as in Eq. (15). Note that the total angular momentum defined by:

$$L^2 = L_r^2 + L_\theta^2 + L_Z^2 \quad -(29)$$

is not conserved, i.e. :

$$\frac{dL}{dt} \neq 0. \quad -(30)$$

The well known Binet equation of orbits is defined {1 - 12} by Eqs. (23) and (27):

$$F(r) = -\frac{L_Z^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right). \quad -(31)$$

This equation gives the force law for any orbit.

For planar orbits it can be shown as in Note 322(4) that:

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \quad -(32)$$

so the velocity is:

$$\underline{v} = \dot{r}\,\underline{e}_r + \omega r\,\underline{e}_\theta \quad -(33)$$

and the acceleration is:

$$\underline{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r \quad -(34)$$

with position vector:

$$\underline{r} = r\,\underline{e}_r + Z\,\underline{k} \quad -(35)$$

and angular momentum vector:

$$\underline{L} = m\left(Z\left(\omega r\,\underline{e}_r + \dot{r}\,\underline{e}_\theta\right) + \omega r^2\,\underline{k}\right). \quad -(36)$$

Note carefully that the planar orbit is embedded in three dimensions defined by r, θ and Z. In the usual planar theory of orbits if the textbooks it is assumed that:

$$Z = 0. \quad -(37)$$

For clarity, Eq. (35) can be defined as:

$$\underline{r}_{total} = r\underline{e}_r + Z\underline{k} \quad -(38)$$

so:

$$r^2_{total} = r^2 + Z^2 \quad -(39)$$

and a conical section orbit in cylindrical polar coordinates is defined in general by:

$$r^2_{total} = \left(\frac{\alpha}{1+\epsilon\cos\theta}\right)^2 + Z^2. \quad -(40)$$

In the most general dynamics the lagrangian is:

$$\mathcal{L} = \frac{1}{2}m\left(\dot{r}^2 + \dot{\theta}^2 r^2 + \dot{Z}^2\right) - U(r, \theta, Z) \quad -(41)$$

and the potential energy and force depend on orientation and Z as well as on r. Most generally the velocity in the observer frame is:

$$\underline{v} = \dot{r}\underline{e}_r + \underline{\omega} \times \underline{r} \quad -(42)$$

and the acceleration in the observer frame is

$$\underline{a} = \ddot{r}\underline{e}_r - \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \frac{d\underline{\omega}}{dt} \times \underline{r} + 2\underline{\omega} \times \frac{dr}{dt}\underline{e}_r. \quad -(43)$$

The gravitomagnetic field depends on the vector cross product of $\underline{v}$ from Eq. (42) with $\underline{a}$ form Eq. (43).

The phenomena of light deflection due to gravitation and perihelion precession are non Newtonian phenomena which are described straightforwardly in ECE2 theory as follows.

A precessing orbit is described {1 - 12} by:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \qquad -(44)$$

where the orbit advances by:

$$\Delta\theta = (x-1)\theta. \qquad -(45)$$

In the solar system x is very close to unity so to an excellent approximation:

$$L^2 = m^2 M G \alpha. \qquad -(46)$$

From Eqs. (31) and (44) the force responsible for a precessing orbit is:

$$\underline{F} = mMG\left(-\frac{x^2}{r^2} + \frac{(x^2-1)\alpha}{r^3}\right)\underline{e}_r. \qquad -(47)$$

For light grazing the sun, the orbit is a hyperbola with half right latitude:

$$\alpha = a(\epsilon^2 - 1) \qquad -(48)$$

where a is the distance of closest approach:

$$a = R_0. \qquad -(49)$$

The angle of deflection is:

$$\Delta\psi = \frac{2}{\epsilon} \qquad -(50)$$

$$-(51)$$

and the gravitomagnetic field is:

$$\underline{\Omega} = \frac{MGL}{mc^2}\left(-\frac{x^2}{r^3} + \frac{(x^2-1)\alpha}{r^4}\right)\underline{k}$$

If it is assumed that the hyperbolic orbit of the beam of light grazing the sun is not precessing, then:

$$x \sim 1 \quad -(52)$$

so to an excellent approximation:

$$\Omega_2^2 = \frac{(MG)^3}{c^4 r^6} a(\epsilon^2 - 1). \quad -(53)$$

At closest approach:

$$r = a = R_0 \quad -(54)$$

so:

$$\Omega_2^2 = \frac{(MG)^3}{c^4 R_0^5} (\epsilon^2 - 1). \quad -(55)$$

In light deflection by the sun the path of the beam of light is nearly a straight line with very large eccentricity {1 - 12}, so to an excellent approximation:

$$\Omega_2^2 = \frac{(MG)^3}{c^4 R_0^5} \epsilon^2. \quad -(56)$$

The angle of deflection is then:

$$\Delta\xi = \frac{2}{\epsilon} = \frac{2}{\Omega_2 c^2} \left(\frac{(MG)^3}{R_0^5} \right)^{1/2}. \quad -(57)$$

Therefore light deflection due to gravitation is expressed in terms of known quantities and is due to the gravitomagnetic field of general relativity. From Eq. (57) the gravitomagnetic field for a light deflection $\Delta\xi$ can be calculated from quantities that are all known experimentally and this method can be extended to all known precessional phenomena such as that measured by Gravity Probe B (UFT117).

For light deflection by the sun:

$$\Omega_Z = \frac{2}{\Delta\psi\, c^2}\left(\frac{(MG)^3}{R_0^5}\right)^{1/2} \quad -(58)$$

and as in UFT150:

$$\begin{aligned}
\Delta\psi &= 8.4848 \times 10^{-6} \text{ radians} \\
R_0 &= 6.955 \times 10^{8} \text{ m} \\
M &= 1.9891 \times 10^{30} \text{ kg} \\
G &= 6.67428 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \\
c &= 2.9979 \times 10^{8} \text{ m s}^{-1}
\end{aligned} \quad -(59)$$

So the gravitomagnetic field for light deflection by the sun is:

$$\Omega_Z = 0.000314 \text{ radians per second.} \quad -(60)$$

The precession of the perihelion of Mercury is defined by the Z component of the gravitomagnetic field as follows:

$$\Omega_Z = \frac{MGL}{mc^2}\left(-\frac{x^2}{r^3} + \frac{(x^2-1)\alpha}{r^4}\right) \quad -(61)$$

where:

$$\alpha = b(1-\epsilon^2)^{1/2} \quad -(62)$$

and where b is the perihelion. Therefore at the perihelion:

$$\Omega_Z = -\frac{(MG)^{3/2}(1-\epsilon^2)^{1/4}x^2}{c^2 b^{5/2}} \quad -(63)$$

and:

$$\Delta\theta = (x-1)\frac{\pi}{2}. \quad -(64)$$

The observed precession of the perihelion of Mercury is:

$$\Delta\theta = 43.11'' \text{ per century} \tag{65}$$
$$= 7.9673 \times 10^{-7} \text{ radians per year}$$

and at the perihelion:

$$\theta = \frac{\pi}{2}. \tag{66}$$

So the x factor is:

$$x = 1 + 1.268 \times 10^{-7}. \tag{67}$$

So to an excellent approximation:

$$x^2 - 1 = 0 \tag{68}$$

thus justifying Eq. (63). The required experimental data are:

$$\begin{aligned} \underline{M} &= 3.285 \times 10^{23} \text{ kg} \\ b &= 4.60012 \times 10^{10} \text{ m} \\ \epsilon &= 0.205630 \end{aligned} \tag{69}$$

so the gravitomagnetic field responsible for the precession of the perihelion of Mercury is

$$\Omega_2 = -2.489 \times 10^{-24} \text{ rad s}^{-1}. \tag{70}$$

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