Group and phase velocities in the *R* theory of matter waves: conditions for superluminal signalling

M.W. Evans[1*] and H. Eckardt[2]**

**Alpha Institute for Advanced Studies (AIAS) (www.aias.us); **Unified Physics Institute of Technology (UPITEC) (www.upitec.org)*

The de Broglie relation between group and phase velocities of matter waves is a product of special relativity, which is now known to be valid only in narrow contexts, indicating the need for general relativity as corrected by the Einstein–Cartan–Evans (ECE theory. The de Broglie relation is corrected with *R* theory, and an expression obtained for group velocity of a matter wave in terms of *R*. The conditions and sample solutions for superluminal signalling are given.

Keywords: ECE theory, R theory, group and phase velocity in R theory, superluminal signalling.

1. Introduction

The concept of the group velocity of waves was introduced by Hamilton in the late eighteen thirties and published in abstracts in 1841. Little notice was taken of these abstracts until the concept was inferred independently by Stokes and by Rayleigh. The group velocity differs from the phase velocity. For a matter wave, de Broglie [1, 2] derived a relation between the two using the basic concepts of special relativity and quantum theory put together in the de Broglie Einstein equation of 1922 to 1924. In UFT 158 and following papers in this series [3-12] it has been shown that the de Broglie Einstein theory becomes unworkable and grossly self inconsistent within the context of special relativity. This disaster for the old twentieth century physics has been averted through the development of the concept of mass into covariant mass defined by the *R* parameter of ECE theory [312].

In Section 2 the de Broglie theory of the group velocity of a matter wave is developed in general relativity using *R* theory. The group velocity is defined

[1]e-mail: EMyrone@aol.com
[2]e-mail: horsteck@aol.com

as the partial derivative of total relativistic energy with respect to relativistic momentum. In special relativity this procedure produces a group velocity equal to the particle velocity v, and the group velocity cannot exceed c, the maximum velocity of special relativity. In R theory however the group velocity is defined by R and may exceed the velocity c. Under well defined conditions, superluminal signalling becomes possible using the group velocity. As observed experimentally the group velocity may exceed c, may be zero, or may be opposite in direction to the phase velocity. The results are developed in Section 2 for use with refraction theory. In Section 3 numerical solutions for dispersion are found using a differential equation obtained in Section 2.

2. Group velocity of a matter wave in R theory

The group velocity is defined by de Broglie for any wave / particle by:

$$v_g = \frac{\partial \omega}{\partial \mathrm{K}} = \frac{\partial (E/\hbar)}{\partial (p/\hbar)} = \frac{\partial E}{\partial p} \tag{1}$$

where the total relativistic energy E and relativistic momentum p are defined by the Einstein energy equation:

$$E^2 = c^2 p^2 + m_0^2 c^4 \tag{2}$$

where the mass m_0 is the conventional mass of special relativity, i.e. is a constant. In UFT 158 and following papers of this series it was found that the particle mass varies in experiments such as inelastic scattering. This indicates the necessity of generalizing particle mass in special relativity to the covariant mass, defined by:

$$R = \left(\frac{mc}{\hbar}\right)^2 \tag{3}$$

where c and $\hbar$ are the speed of light and reduced Planck constant respectively. The de Broglie group velocity is therefore: m_0

$$v_g = \frac{\partial}{\partial p}\left(c^2 p^2 + m_0^2 c^4\right)^{1/2} = \frac{p}{m_0}\left(\left(\frac{p}{m_0}\right)^2 + 1\right)^{-1/2} = \frac{1}{\gamma}\frac{p}{m} = \frac{\gamma m v}{\gamma m} = v \tag{4}$$

where the Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{5}$$

and where $\boldsymbol{v}$ is the particle velocity. Therefore, the group velocity is the same as the particle velocity:

$$\boldsymbol{v}_{\mathrm{g}} = \boldsymbol{v} \ . \tag{6}$$

The phase velocity of the wave is:

$$\boldsymbol{v}_{\mathrm{p}} = \frac{\omega}{\mathrm{K}} \tag{7}$$

where the de Broglie relations [1, 2] are given by:

$$E = \hbar\,\omega = \gamma\, mc^2, \tag{8}$$

$$p = \hbar\boldsymbol{\kappa} = \gamma\, m\boldsymbol{v}. \tag{9}$$

Therefore, the group velocity can be defined by:

$$v_g = \frac{\gamma m v}{\gamma m} = c^2 \frac{\kappa}{\omega} \tag{10}$$

and the de Broglie relation between group and phase velocities follows:

$$v_g \, v_p = c^2. \tag{11}$$

The generalization of these equations is based on the ECE wave equation [3–12]:

$$\left(\square + R\right) q^a_\mu = 0 \tag{12}$$

whose classical equivalent is:

$$E^2 = c^2\left(p^2 + \hbar^2 R\right) \tag{13}$$

Here q^a_μ is the Cartan tetrad. Therefore, the R factor is defined by:

$$R = \frac{1}{\hbar^2} p^\mu p_\mu = \frac{\omega^2}{c^2} - \kappa^2 \tag{14}$$

and the group velocity is defined by:

$$v_g = \frac{\partial\omega}{\partial\kappa}. \tag{15}$$

The relation between the angular frequency and wave vector is a dispersion relation, and is given from Eq. (14) as:

$$\omega = c\left(R + \kappa^2\right)^{1/2}. \tag{16}$$

Therefore:

$$v_g = \frac{c^2}{\omega}\left(\kappa + \frac{1}{2}\frac{\partial R}{\partial\kappa}\right). \tag{17}$$

Another equation for the particle velocity can be obtained directly from the equation:

$$\gamma\, m\, c^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \hbar\,\omega \tag{18}$$

giving:

$$v = c\left(1 - R\left(\frac{c}{\omega}\right)^2\right)^{1/2}. \tag{19}$$

It is observed experimentally [13–15] that the group velocity can be zero, greater than c or negative. In special relativity the photon is often assumed to be massless, despite the fact that the photon mass was assumed by de Broglie to be identically non-zero. This assumption means that the group velocity in special relativity is always:

$$v_g = \frac{c^2\kappa}{\omega} \tag{20}$$

and also for any type of matter wave as considered by de Broglie. In other words, in special relativity, the derivative of R with respect to κ vanishes in de Broglie theory because:

$$R = \left(\frac{m_0 c}{\hbar}\right)^2 \tag{21}$$

and m_0 is a constant. In R theory this is no longer the case because R is defined by geometry as follows:

$$R = \left(\frac{mc}{\hbar}\right)^2 = q^a_\mu \partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right) \tag{22}$$

and the concept of covariant mass m is the result of this geometry. It is possible to define the superluminal condition by:

$$v_g > c \tag{23}$$

where c is the maximum speed of special relativity. For the hypothetical massless photon c is the vacuum speed of light. The superluminal condition (23) means from Eq. (17) that

$$\kappa + \frac{1}{2}\frac{\partial R}{\partial \kappa} > \frac{\omega}{c} \tag{24}$$

In special relativity and de Broglie theory the condition (24) is not allowed because:

$$\omega^2 = c^2 \kappa + \left(\frac{m_0 c}{\hbar}\right)^2 \tag{25}$$

and

$$\kappa^2 = \frac{\omega^2}{c^2} - \left(\frac{m_0 c}{\hbar}\right)^2 \tag{26}$$

The fixed mass m_0 of special relativity is always greater than zero, so Eq. (24) can never apply. In R theory, m_2 in Eq. (22) is a positive valued proportionality factor between R and the fundamental constant $(c/\hbar)^2$. This makes a profound difference because R can be a variable that gives rise to new information in absorption and scattering theory as explained in UFT 158 to UFT 165 on www.aias.us. The fact that R is a variable makes superluminal signalling possible because $\partial R / \partial \kappa$ is non zero.

Therefore, from Eq. (24) the superluminal condition is:

$$\frac{\partial R}{\partial \kappa} > 2\left(\frac{\omega}{c} - \kappa\right) \tag{27}$$

i.e.:

$$\frac{\partial R}{\partial \kappa} > 2\left(\frac{v_p}{c} - 1\right)\kappa. \tag{28}$$

It is also observed experimentally [13–15] that the group velocity can be zero while the phase velocity remains finite. For the group velocity to vanish, Eq. (17) means:

$$\omega \to \infty. \tag{29}$$

Therefore, from Eq. (17):

$$\kappa + \frac{1}{2}\frac{\partial R}{\partial \kappa} \ll \omega \tag{30}$$

and so:

$$\frac{\partial R}{\partial \kappa} \ll 2 v_p \tag{31}$$

and from Eq. (17):

$$\kappa + \frac{1}{2}\frac{\partial R}{\partial \kappa} > 0. \tag{32}$$

In this case the phase velocity goes to infinity:

$$v_p = \frac{\omega}{\kappa} \to \infty \tag{33}$$

Finally, it is also observed experimentally [13–15] that the group velocity can become negative, i.e. opposite in direction to the phase velocity. From Eq. (17) the condition for this is:

$$\frac{\partial R}{\partial \kappa} < 0 \tag{34}$$

and from Eq. (17) it is required that:

$$\left|\frac{\partial R}{\partial K}\right| > 2\kappa \tag{35}$$

i.e. $\partial R / \partial \kappa$ must be negative.

It is possible to obtain the following relation between R, group velocity and phase velocity:

$$v_g = \frac{c^2}{v_p} + \frac{1}{2}\frac{c^2}{\omega}\frac{\partial}{\partial \kappa}\left(q_v^a \partial^\mu \left(\omega_{\mu v}^a - \Gamma_{\mu v}^a \right) \right) \tag{36}$$

This gives a relation between the geometrical definition of R in Eq. (22) and the partial derivative $\partial\omega / \partial\kappa$ that defines the group velocity:

$$\frac{\omega}{c^2}\frac{\partial \omega}{\partial \kappa} - \frac{1}{2}\frac{\partial R}{\partial \kappa} = \kappa. \tag{37}$$

The contravariant and covariant four momenta are well known to be:

$$p^\mu = \left(\frac{E}{c}, \boldsymbol{p} \right), \; p_\mu = \left(\frac{E}{c}, -\boldsymbol{p} \right) \tag{38}$$

so the R factor is:

$$R = \frac{1}{\hbar^2} p^\mu p_\mu = \frac{\omega^2}{c^2} - \kappa^2. \tag{39}$$

From Eqs. (17) and (39):

$$\frac{\partial E}{\partial p} = \frac{\partial \omega}{\partial \kappa} = \frac{1}{2\omega}\frac{\partial \omega^2}{\partial \kappa} \tag{40}$$

which relates the derivative $\partial\omega / \partial\kappa$ to E and p. For the hypothetical massless particle travelling at c:

$$\frac{\partial E}{\partial p} = \frac{\partial \omega}{\partial \kappa} = c = \frac{\omega}{\kappa} \tag{41}$$

so

$$\frac{\partial \omega^2}{\partial \kappa} = 2c^2\kappa \tag{42}$$

and from Eq. (40) the following self-consistent result is obtained:

$$\frac{\partial \omega}{\partial \kappa} = \frac{1}{2\omega}\frac{\partial \omega^2}{\partial \kappa} = c \tag{43}$$

with

$$\boxed{\frac{\partial\omega^2}{\partial\kappa} = 2\omega\frac{\partial\omega}{\partial\kappa}} \tag{44}$$

Q.E.D.

In this case, Eq. (39) gives:

$$R = \frac{\omega^2}{c^2} - \kappa^2 = 0 \tag{45}$$

i.e.

$$\omega^a_{\mu\nu} = \Gamma^a_{\mu\nu} \tag{46}$$

and

$$\Box q^a_\mu = 0. \tag{47}$$

Using the ECE hypothesis:

$$A^a_\mu = A^{(0)} q^a_\mu \tag{48}$$

we obtain the d'Alembert wave equation for a massless photon self-consistently for each polarization sense labelled *a*:

$$\Box A_\mu = 0 \tag{49}$$

Q.E.D.

This in turn means that the Proca equation for the massive photon is:

$$\left(\Box + R\right) A^a_\mu = 0 \tag{50}$$

and when the covariant mass vanishes *R* vanishes self consistently:

$$\Box A_\mu = 0, \;\; R = 0, \;\; m_0 = 0 \tag{51}$$

Q.E.D.

For a monochromatic plane wave traversing a dielectric [16] with no charge current density present the Helmholtz wave equation gives:

$$v_p = \frac{\omega}{\kappa} = \frac{c}{n} = \frac{1}{(\mu\varepsilon)^{1/2}} \tag{52}$$

where n is the refractive index, μ is the permeability and ε the permittivity of the dielectric. So, in this case R can be expressed simply in terms of the refractive index:

$$v_{\mathrm{g}} = nc + \frac{c^2}{2\omega}\frac{\partial R}{\partial \kappa} \tag{53}$$

and the refractive index is given by:

$$n = \frac{v_{\mathrm{g}}}{c} - \frac{c}{2\omega}\frac{\partial R}{\partial \kappa}. \tag{54}$$

At the interface of two dielectrics Snell's law is given by [16]:

$$\frac{\sin i}{\sin r} = \frac{\kappa'}{\kappa} = \frac{n'}{n} = \frac{v'_p}{v_p} \tag{55}$$

Therefore:

$$\frac{n'}{n} = \frac{A}{B} \tag{56}$$

where

$$A = \frac{v'_g}{c} - \frac{c}{2\omega}\frac{\partial R'}{\partial \kappa} \tag{57}$$

and where:

$$B = \frac{v_g}{c} - \frac{c}{2\omega}\frac{\partial R}{\partial \kappa} \tag{58}$$

Therefore Snell's law is:

$$\frac{\sin i}{\sin r} = f\left(\partial R' / \partial \kappa'\right) / f\left(\partial R / \partial \kappa\right) \tag{59}$$

and the R spectrum of any refraction process is:

$$\frac{n'}{n} = A' \tag{60}$$

with A' defined from Snell's law as:

$$A' = \left(\frac{v'_g}{c} - \frac{c}{2\omega} \frac{\partial R'}{\partial \kappa'} \right) / \left(\frac{v_g}{c} - \frac{c}{2\omega} \frac{\partial R}{\partial \kappa} \right). \tag{61}$$

The R theory of refraction can be expressed in terms of the R theory of the scattering process defined by:

$$\omega + \omega_0 = \omega' + \omega'' \tag{62}$$

and

$$\kappa''^2 = \kappa^2 + \kappa'^2 - 2\kappa\kappa' \cos\theta. \tag{63}$$

This is the scattering of a particle with initial angular frequency ω from a particle at rest with rest angular frequency ω_0. The scattered angular frequency of the incoming particle is ω' and the scattered angular frequency of the particle initially at rest is ω''. Eq. (63) is the accompanying conservation of momentum where the scattering angle is:

$$\theta = r - i \tag{64}$$

as defined in terms of r and i of Snell's law (see accompanying note 165(4) for more details). As in UFT 158 ff., solution of Eqs. (62) and (63) gives:

$$\omega_0 = c R_2^{1/2} = \frac{1}{(\omega - \omega')} \left(\omega\omega' - \left(c^2 R_1 + \left(\omega^2 - c^2 R_1 \right)^{1/2} \left(\omega'^2 - c^2 R_1 \right)^{1/2} \cos\theta \right) \right) \tag{65}$$

where R_2 is the R factor of the initially static particle, and R_1 is the R factor of the scattered particle. Solving Eq. (65) gives:

$$R_1 = \frac{1}{2ac^2} \left(-b \pm \left(b^2 - 4ac' \right)^{1/2} \right) \tag{66}$$

where:

$$\left.\begin{aligned} a &= 1-\cos^2\theta, \\ b &= \left(\omega'^2 + \omega^2\right)\cos^2\theta - 2\,\mathrm{A}, \\ c' &= \mathrm{A}^2 - 2\,\omega^2\,\omega'^2\,\cos\theta, \\ \mathrm{A} &= \omega\,\omega' - \left(\omega-\omega'\right)\omega_0. \end{aligned}\right\} \tag{67}$$

Therefore, from Eqs. (61) and (67):

$$\frac{n'}{n} = f\left(R_1\right) \tag{68}$$

which unifies the macroscopic theory of refraction with the particle theory of refraction considered as a scattering from an effective R_2.

3. Numerical solution for the dispersive dependence

The general dispersive relation between frequency go and wave number κ is given by Eq. (16):

$$\omega = c\left(R+\kappa^2\right)^{1/2} \tag{69}$$

where the curvature parameter R, which has a κ dependence too, enters the calculation. The group velocity is determined by R, ω and κ via Eq. (17):

$$v_g = \frac{c^2}{\omega}\left(\kappa + \frac{1}{2}\frac{\partial R}{\partial \kappa}\right). \tag{70}$$

To obtain an equation for R, Eq. (69) can be inserted into (70), leading to

$$\frac{\partial R}{\partial \kappa} = 2\frac{v_g}{c}\left(\sqrt{R+\kappa^2} - \kappa\right). \tag{71}$$

This is a first order differential equation for R. The ratio of v_g/c has to be predefined to obtain a solution.

In particular, superluminal cases of $v_g > c$ can be considered. Equation (71) must be solved numerically because the square root term prohibits an analytical solution. Having obtained $R(\kappa)$, the dispersion relation (69) can be calculated, and from that the optical refraction index $n(\kappa)$ using Eq. (54):

$$n = \frac{v_g}{c} - \frac{1}{2}\frac{c}{\omega}\frac{\partial R}{\partial \kappa}. \tag{72}$$

The numerical solution requires an initial value for $R(0)$. It results that only for positive initial values a solution exists. We did not consider complex-valued solutions. All numerical calculations were performed for three values of v_g/c:

$$\frac{v_g}{c} = \begin{cases} 0.5 \\ 1 \\ 1.5 \end{cases}$$

The solution $R(\kappa)$ for these cases is shown in Fig. 1. For $v_g = c$ the solution is linear, in the other cases it is decreasing or increasing, respectively. From Fig. 2 where the derivative of R is shown, it becomes clear that the derivatives are straight lines, i.e. R is a quadratic function of κ in all cases.

Interestingly, also the dispersive relation $\omega(\kappa)$ (Fig. 3) is linear as can also be concluded from Eq. (69) because R is quadratic. Finally, the optical refractive index $n(\kappa)$ can be seen in Fig. 4. This is asymptotically constant for large wave numbers. The behaviour is exactly as in classical optics, that means for $v_g = c$ it is exactly unity, for $v_g < c$ it is $n > 1$, and for $v_g > c$ (superluminal case) we have $n < 1$ as is also known from optics.

We conclude that the R theory of general relativity reproduces the results of classical optics. These were obtained from assuming a group velocity not being dependent on the frequency. The numerical method developed can easily be extended to arbitrarily predefined dependencies of $v_g(\kappa)$. Thus, special materials can be designed with certain properties. The method described is also applicable to quantum optics and de Broglie frequencies of particles so that these 'design principles' are even valid in the microscopic regime.

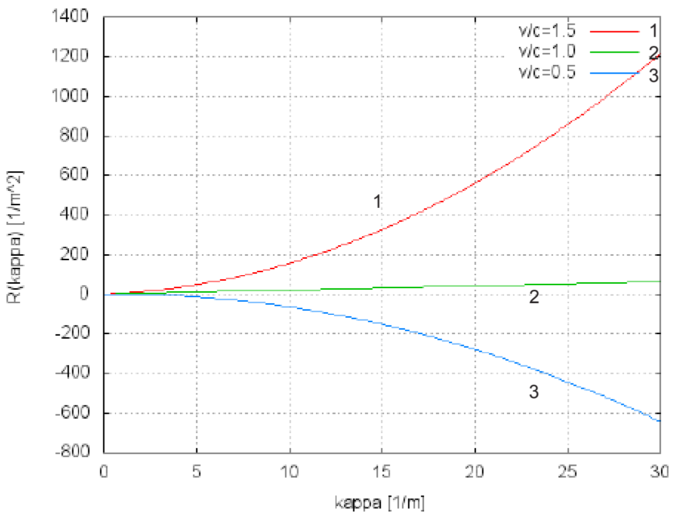

Fig. 1. Numerical solution of $R(\kappa)$ with $R(0)=1$.

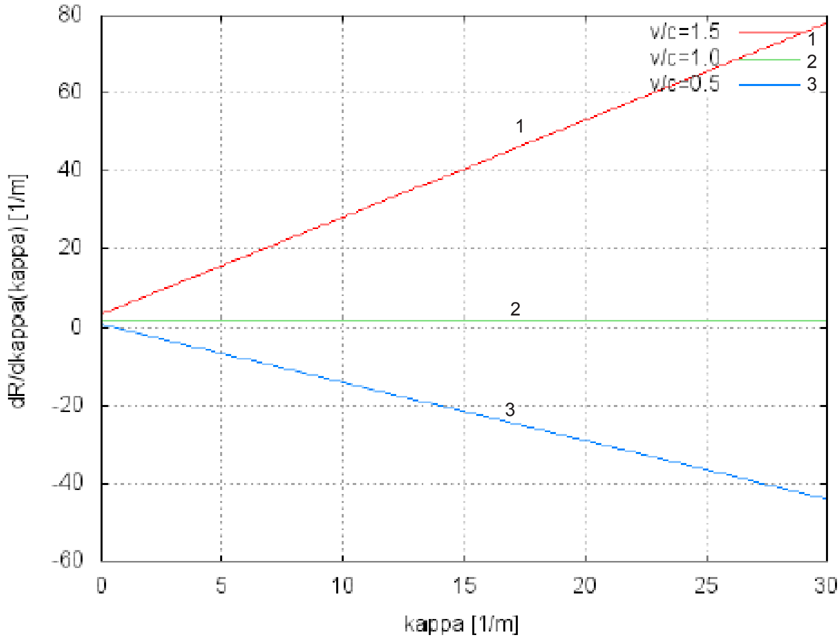

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Fig. 2. Derivative of $R(\kappa)$.

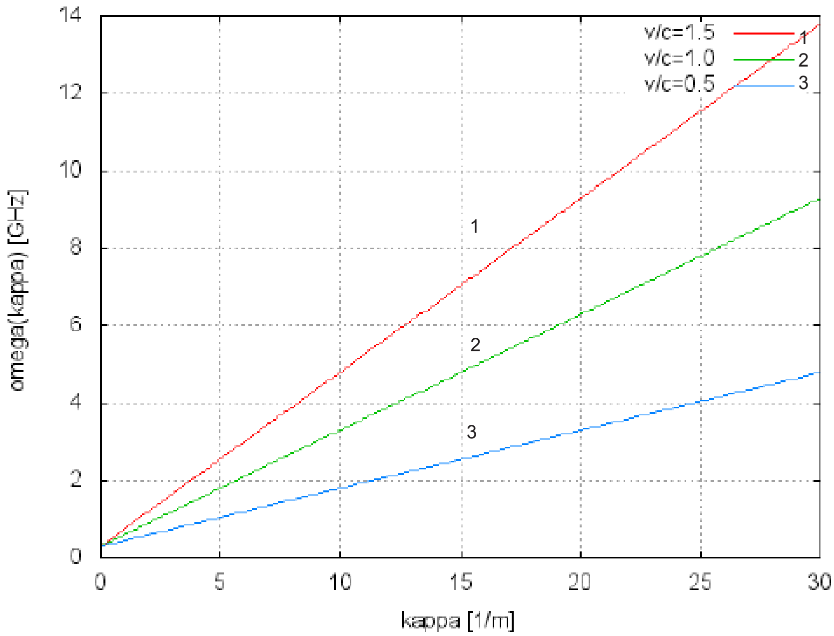

Fig. 3. Dispersive frequency relation $\omega(\kappa)$.

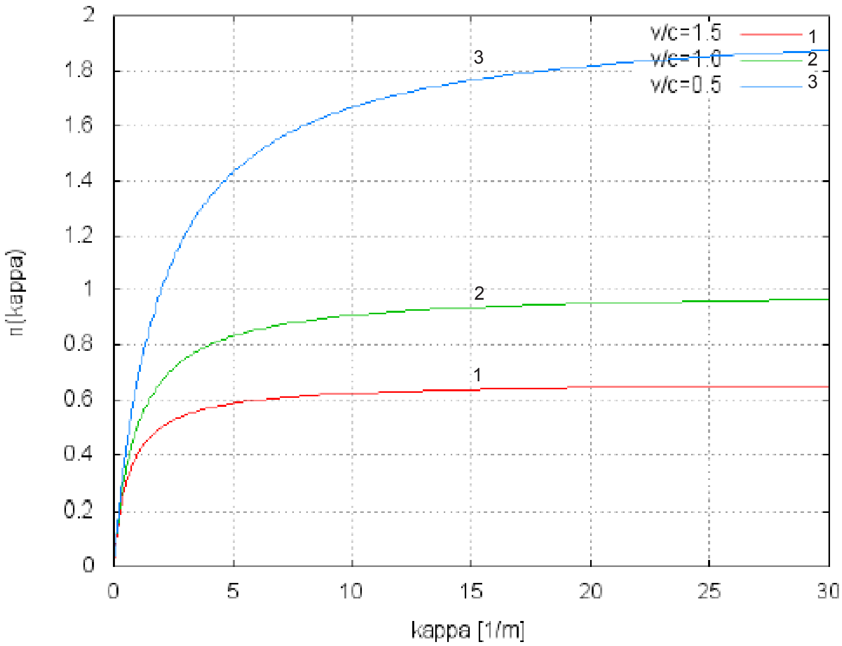

Fig. 4. Optical refractive index $n(\kappa)$.

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