Chapter 5

On The Fundamental Origin of Angular Momentum in Cartan Geometry

by

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Abstract

It is shown that the fundamental origin of spacetime angular momentum in general relativity is the Cartan torsion. The canonical angular energy/momentum density is proportional by hypothesis to the Cartan torsion through a proportionality factor c/k, and integration over a hypersurface gives the anti-symmetric angular momentum tensor of spacetime. The latter is therefore proportional to an integral over the connection, which in turn may be expanded using the tetrad postulate. The meaning of the internal index of the electromagnetic potential and field is illustrated with respect to plane waves of electromagnetic radiation.

Keywords: Einstein Cartan Evans (ECE) theory, spacetime angular momentum, plane wave polarizations

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5.1 Introduction

The Einstein Cartan Evans (ECE) field theory [1] - [10] is a generally covariant unified field theory that develops natural science in terms of geometry. The type of geometry used in the theory is well known, and was first developed by Cartan [11]. In a series of 126 papers to date, the ECE theory has been applied to several areas of physics, chemistry, cosmology and electrical engineering. In paper 126 of the series (www.aias.us) it was shown that the conservation of spacetime angular momentum leads to a satisfactory description of all planar orbits, including the orbits of whirlpool galaxies, without the use of dark matter and without the use of the Einstein field equation of 1915. It has been shown during the development of the ECE theory that all solutions of the Einstein field equation violate the Cartan Evans dual identity of geometry because of the arbitrary neglect of spacetime torsion. The commutator of covariant derivatives in any spacetime of any dimension acts on any tensor to produce the spacetime torsion tensor in Riemann torsion. In the standard model of physics the torsion has been arbitrarily neglected with disastrous consequences. In Section 2 it is shown that angular momentum originates in the integral of spacetime torsion over a hypersurface, so the angular momentum can be obtained from the connection. The latter may be developed within the context of Cartan geometry in terms of the spin connection using the tetrad postulate. In Section 3 the theory is illustrated using the complex circular basis [1] - [10] superimposed on the Cartesian basis. It is shown that this superimposition produces a rigorously non-Minkowski spacetime. Thus electromagnetism is based on the Cartan torsion. The field equations of dynamics and electrodynamics in ECE theory are based on the Cartan Bianchi identity and the Cartan Evans dual identity.

5.2 Relation between Angular Momentum and Spacetime Torsion

The spacetime torsion is very fundamental in Riemann geometry and always exists in any spacetime in any dimension. It is generated through the action of the commutator of covariant derivatives on any tensor. For example the commutator may act on a vector V^ρ :

$$[D_\mu, D_\nu]V^\rho = R^\rho{}_{\sigma\mu\nu}V^\sigma - T^\lambda{}_{\mu\nu}D_\lambda V^\rho \tag{5.1}$$

to produce the torsion tensor:

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \tag{5.2}$$

and the curvature tensor :

$$R^\lambda{}_{\sigma\mu\nu} = \partial_\mu\Gamma^\rho_{\nu\sigma} - \partial_\nu\Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}. \tag{5.3}$$

Eq. (5.1) shows that the connection is anti-symmetric:

$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu}. \tag{5.4}$$

There is a one to one correspondence between the anti-symmetric commutator and the anti-symmetric connection, and this determines the anti-symmetry of the connection. For over a hundred years, the standard theory of gravitational physics has incorrectly omitted the torsion term and has asserted:

$$[D_\mu, D_\nu]V^\rho =?\ R^\rho{}_{\sigma\mu\nu}V^\sigma. \tag{5.5}$$

In Eq. (5.5) there is no constraint on the symmetry of the connection, from Eq. (5.5) it is possible to know only that the curvature tensor is anti-symmetric in its last two indices. However, from Eq. (5.1) the torsion and connection must be anti-symmetric. If they had any symmetric part the commutator would vanish, and both the torsion and curvature would vanish as follows:

$$0V^\rho = 0V^\sigma - 0D_\lambda V^\rho. \tag{5.6}$$

The hypothesis is now made that the three index angular momentum energy density is proportional to the three index torsion tensor as follows:

$$J^\lambda{}_{\mu\nu} = \frac{c}{k}T^\lambda{}_{\mu\nu} = 2\frac{c}{k}\Gamma^\lambda_{\mu\nu} \tag{5.7}$$

where c is the speed of light and k is the Einstein constant. This hypothesis may be viewed as a correction of the Einstein hypothesis of 1915, which is now known to be incorrect due to omission of torsion. The speed of light in vacuo c and the Einstein constant k are retained by dimensionality, and in Eq. (5.7) the S.I. units of the canonical angular energy/momentum density are $kgm\ m^{-1}s^{-1}$. The classical angular momentum of spacetime is then defined as in field theory Eq. (5.12) by the integration of a hypersurface as follows:

$$J_{\mu\nu} = \int J^\lambda{}_{\mu\nu}\, dV_\lambda. \tag{5.8}$$

The hypersurface is defined as the four volume:

$$V_\lambda = (V, -\boldsymbol{V}). \tag{5.9}$$

The angular momentum of spacetime in general is therefore:

$$J_{\mu\nu} = 2\frac{c}{k}\int \Gamma^\lambda_{\mu\nu}\, dV_\lambda \tag{5.10}$$

where the antisymmetry of the connection has been used. Therefore the fundamental origin of spacetime angular momentum is the antisymmetric connection integrated over a hypersurface.

An expression for the angular momentum can be derived from the anti-symmetric connection, and in so doing the theory of angular momentum and that of torsion are inter-related. Starting from the tetrad postulate:

$$\partial_\mu q^a_\nu + \omega^a_{\mu b}q^b_\nu = q^a_\lambda \Gamma^\lambda_{\mu\nu} \tag{5.11}$$

the Cartan convention:

$$q^a_\lambda q^\kappa_a = \delta^\kappa_\lambda \tag{5.12}$$

is used to obtain:

$$\Gamma^\lambda_{\mu\nu} = q^\lambda_a(\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu). \tag{5.13}$$

The antisymmetry of the connection implies that the torsion tensor is:

$$T^\lambda{}_{\mu\nu} = 2\Gamma^\lambda_{\mu\nu} = q^\lambda_a T^a{}_{\mu\nu} \tag{5.14}$$

so the first Cartan structure equation simplifies to:

$$T^a{}_{\mu\nu} = q^a_\lambda T^\lambda{}_{\mu\nu} = 2(\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu). \tag{5.15}$$

Therefore the angular momentum may be defined by:

$$J_{\mu\nu} = 2\frac{c}{k}\int q^\lambda_a(\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu)\, dV_\lambda. \tag{5.16}$$

In these expressions the tetrad is a (1, 1) mixed index tensor of rank two and also a vector valued one-form. For each value of the lower index μ it is a vector ([11]). So if the basis elements of a four dimensional vector are denoted: $\boldsymbol{e}_{(0)}, \boldsymbol{e}_{(1)}, \boldsymbol{e}_{(2)}, \boldsymbol{e}_{(3)}$ then

$$\boldsymbol{q}_\mu = q^{(0)}_\mu \boldsymbol{e}_{(0)} + q^{(1)}_\mu \boldsymbol{e}_{(1)} + q^{(2)}_\mu \boldsymbol{e}_{(2)} + q^{(3)}_\mu \boldsymbol{e}_{(3)}. \tag{5.17}$$

The inverse of the tetrad matrix q^a_μ is denoted q^μ_a and is defined in the Cartan convention by Eq. (5.11):

$$q^a_\mu q^\nu_a = \delta^\nu_\mu \tag{5.18}$$

where

$$\delta^\nu_\mu = \begin{cases} 1 & \text{if } \mu = \nu, \\ 0 & \text{if } \mu \neq \nu. \end{cases} \tag{5.19}$$

In ECE theory, as in any theory of relativity, the fundamental quantities are densities. For example the canonical energy momentum density appears in the covariant Noether Theorem, and in field theory ([12]) appears the canonical angular momentum/angular energy density. Therefore the fundamental quantities in the electromagnetic sector of ECE theory must also be canonical densities, as for its dynamical sector. The fundamental electromagnetic potential density is therefore defined by the ECE hypothesis:

$$A^a_\mu = A^{(0)} q^a_\mu \tag{5.20}$$

and the electromagnetic potential is the integral of this over a four vector with the units of volume:

$$V_a = (V, -\boldsymbol{V}). \tag{5.21}$$

Thus:

$$A_\mu = \int A^a_\mu \, dV_a. \tag{5.22}$$

The index a is therefore a fundamental feature of general relativity and does not appear in special relativity's Maxwell Heaviside (MH) theory ([12]). Note carefully that the spacetime of A^a_μ is not the Minkowski spacetime.

The torsion form $T^a{}_{\mu\nu}$ of differential geometry is a vector-valued two-form ([11]). Therefore for each double index $\mu\nu$ it is a vector:

$$\boldsymbol{T}_{\mu\nu} = T^{(0)}_{\mu\nu}\boldsymbol{e}_{(0)} + T^{(1)}_{\mu\nu}\boldsymbol{e}_{(1)} + T^{(2)}_{\mu\nu}\boldsymbol{e}_{(2)} + T^{(3)}_{\mu\nu}\boldsymbol{e}_{(3)}. \tag{5.23}$$

The index a of the torsion form may be integrated out over a volume four vector (a four dimensional hypersurface [12]) as follows:

$$T_{\mu\nu} = \int T^a{}_{\mu\nu} \, dV_a. \tag{5.24}$$

The canonical electromagnetic field density of ECE theory is therefore [1] - [10]:

$$F^a{}_{\mu\nu} = A^{(0)} T^a{}_{\mu\nu}. \tag{5.25}$$

The electromagnetic field of ECE theory is the integral of the field density over the four-volume:

$$F_{\mu\nu} = \int F^a{}_{\mu\nu} \, dV_a. \tag{5.26}$$

Similarly the differential form defining the canonical angular energy/momentum density of ECE theory is a vector valued two-form $J^a{}_{\mu\nu}$. The angular energy/momentum is therefore:

$$J_{\mu\nu} = \int J^a{}_{\mu\nu} \, dV_a. \tag{5.27}$$

By hypothesis:

$$J_{\mu\nu} = \frac{c}{k} T_{\mu\nu}. \tag{5.28}$$

Eq. (5.28) relates physics to geometry in the manner of relativity theory and philosophy. Here c is the vacuum speed of light and k is the original Einstein constant, retained by hypothesis in ECE theory. In the Einstein equation of 1915 the torsion was omitted, and so the equation is fundamentally incorrect.

The first Cartan structure equation [1] - [11]

$$F^a{}_{\mu\nu} = 2(\partial_\mu A^a_\nu + \omega^a_{\mu b} A^b_\nu) \tag{5.29}$$

links the field and potential densities through use of the spin connection form, $\omega^a_{\mu b}$. The spin connection is a non-tensor valued one-form [1] - [11], and for each value of the index μ it is a matrix. In Eq. (5.29) summation is implied over repeated b indices. The electromagnetic field tensor $F_{\mu\nu}$ in ECE theory is therefore a property of a non-Minkowski spacetime, and is:

$$F_{\mu\nu} = \int (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \omega^a_{\mu b} A^b_\nu - \omega^a_{\nu b} A^b_\mu)\, dV_a. \tag{5.30}$$

This equation reduces to:

$$F_{\mu\nu} = 2 \int (\partial_\mu A^a_\nu + \omega^a_{\mu b} A^b_\nu)\, dV_a \tag{5.31}$$

because of the anti-symmetry of the connection:

$$\Gamma^\kappa_{\mu\nu} = -\Gamma^\kappa_{\nu\mu}. \tag{5.32}$$

The electromagnetic field tensor is directly proportional as follows to the angular energy/momentum of spacetime:

$$F_{\mu\nu} = A^{(0)} T_{\mu\nu} = \frac{k}{c} A^{(0)} J_{\mu\nu} = 2k \frac{A^{(0)}}{c} \int q^\lambda_a (\partial_\mu A^a_\nu + \omega^a_{\mu b} q^b_\nu)\, dV_\lambda. \tag{5.33}$$

This proportionality relates ECE electromagnetic theory with ECE angular momentum theory. In the now obsolete Einsteinian gravitational physics, the angular momentum of spacetime is not considered because the torsion is incorrectly omitted. This leads to multiple errors in the standard model of physics. In particular, big bang theory, dark matter theory, black hole theory, and similar, are erroneous theories that have been replaced by a simpler and more powerful ECE cosmology based on torsion.

In general, the a index in ECE theory introduces canonical densities and also a higher order type of geometry via the fundamental definition of the tetrad [1] - [11]:

$$V^a = q^a_\mu V^\mu. \tag{5.34}$$

In any dimension and in any spacetime, the vector components V^a are associated with basis elements $e_{(a)}$, and the vector components V^μ are associated with basis elements ∂_μ. The complete vector field V is therefore [1] - [11]:

$$V = V^a e_{(a)} = V^\mu e_{(\mu)} = V^\mu \delta_\mu \tag{5.35}$$

and this equation leads to the tetrad postulate:

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu + \omega^a_{\mu b} - \Gamma^\lambda_{\mu\nu} q^a_\lambda = 0. \tag{5.36}$$

In a flat spacetime:

$$\omega^a_{\mu b} = 0, \tag{5.37}$$

$$\Gamma^\lambda_{\mu\nu} = 0, \tag{5.38}$$

so the condition for a flat spacetime is:

$$\partial_\mu q^a_\nu = 0 \tag{5.39}$$

in which case both V^a and V^μ are vector components (nearly always denoted as "vectors" [1] - [11] of a flat spacetime. In this case the tetrad may for example be a rotation matrix or a Lorentz boost matrix of Minkowski spacetime (the flat spacetime in four dimensions). If the condition Eq. (5.39) is not fulfilled (as an example see the plane wave tetrads of Section 3), the spacetime is not a flat spacetime, in which case there is a transition from special to general relativity as required. In the philosophy of relativity, every equation of physics must be an equation of general relativity. The obsolete standard model of physics failed to fulfill this basic requirement because its electromagnetic, weak and strong sectors were theories of special relativity, only its gravitational sector was a theory of general relativity and that sector was fundamentally incorrect because of neglect of torsion. There was a fundamental and well known schism in the basics of the now obsolete standard model of physics. The ECE theory is a correctly and generally covariant unified field theory based on correct geometry, all its sectors are sectors of general relativity, and its sectors are inter-related by correct geometry. All the experimentally known fundamental force fields of nature are inter-related by geometry. This has been an aim of human thought since classical times, in which geometry was equated with beauty. We now think of "nature" rather than "beauty", but the thought process is the same.

The index a as introduced originally by Cartan was an index of a tangential Minkowski spacetime, tangential at point P to a Riemannian spacetime labelled μ. The main advantage of this is that spinors (also inferred originally by Cartan in 1913) can be developed as the basis elements $e_{(a)}$. During the course of development of ECE theory the a index has been developed in some contexts to represent a spacetime spinning with respect to another. This allows the Dirac equation for example [1] - [10] to be derived from the ECE wave equation in a well defined limit. The Dirac four-spinor was thus identified as a well defined limit of a tetrad field. This method used a $SU(2)$ representation space, and the method can be extended to an $SU(n)$ or any other representation space. This is how ECE theory incorporates strong field theory and the electroweak theory without having to use the Higgs mechanism. The latter is a mechanism of special relativity with several ad hoc assumptions, it is therefore inconsistent with general relativity and obsolete. The Higgs boson has not been observed experimentally despite years of investigation. Claims as to the future observation of the Higgs boson are already being severely criticised form several angles.

The tetrad may also be used to relate two vectors V^κ and V^λ both written in a Riemannian spacetime:

$$V^\kappa = q^\kappa_\lambda V^\lambda \tag{5.40}$$

and previous developments such as the ECE engineering model have used the tetrad and have related it to the metric tensor as follows:

$$q^\kappa_\lambda = g^\kappa_\lambda = g^{\kappa\alpha} g_{\alpha\lambda}. \tag{5.41}$$

The symmetric metric used in the obsolete standard model is defined as

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab} \tag{5.42}$$

where η_{ab} is the Minkowski metric. Earlier development in ECE theory generalizes this concept to:

$$g^{ab}_{\mu\nu} = q^a_\mu q^b_\nu \tag{5.43}$$

in which the metric becomes a rank four mixed index tensor in general. For given μ and ν such a tensor is a sum of terms which are symmetric in μ and ν and antisymmetric in a and b.

Any basis elements $e_{(a)}$ of a flat spacetime may be used in general, in which case the most general type of electromagnetic potential and field densities in four dimensions may be expressed in a Cartesian basis for as:

$$\boldsymbol{A}_\mu = A^0_\mu \boldsymbol{e}_0 + A^1_\mu \boldsymbol{i} + A^2_\mu \boldsymbol{j} + A^3_\mu \boldsymbol{k} \tag{5.44}$$

and

$$\boldsymbol{F}_{\mu\nu} = F^0_{\mu\nu} \boldsymbol{e}_0 + F^1_{\mu\nu} \boldsymbol{i} + F^2_{\mu\nu} \boldsymbol{j} + F^3_{\mu\nu} \boldsymbol{k}. \tag{5.45}$$

In a generally covariant unified field theory such as ECE, the densities are the most fundamental components of electromagnetism, and of each of the four known fundamental force fields of nature currently thought to exist experimentally. Every fundamental field is an aspect of spacetime angular momentum. This result is reminiscent of the philosophy and conjectures for example of Leonardo and Descartes but may now be given a basis in rigorously correct twenty first century natural philosophy.

5.3 The Plane wave Tetrads

The original concept of plane wave tetrad was introduced early in the development of ECE theory and the plane wave tetrad is defined using the complex circular basis [1] - [10] used extensively in precursor gauge theories developed in the decade before ECE theory. The transverse plane wave tetrads are defined by the fundamental equation of Cartan geometry:

$$V^a = q^a_\mu V^\mu \tag{5.46}$$

as follows:

$$\begin{bmatrix} V^{(1)} \\ V^{(2)} \end{bmatrix} = \begin{bmatrix} q^{(1)}_1 & q^{(1)}_2 \\ q^{(2)}_1 & q^{(2)}_2 \end{bmatrix} \begin{bmatrix} V^1 \\ V^2 \end{bmatrix}. \tag{5.47}$$

In vector format:

$$\boldsymbol{q}^{(1)} = q^{(1)}_1 \boldsymbol{i} + q^{(1)}_2 \boldsymbol{j}, \tag{5.48}$$

$$\boldsymbol{q}^{(2)} = q_1^{(2)}\boldsymbol{i} + q_2^{(2)}\boldsymbol{j}. \tag{5.49}$$

Here:

$$q_1^{(1)} = q_X^{(1)} = \frac{1}{\sqrt{2}}\exp(i(\omega t - \kappa Z)) \tag{5.50}$$

$$q_2^{(1)} = q_Y^{(1)} = -\frac{i}{\sqrt{2}}\exp(i(\omega t - \kappa Z)) \tag{5.51}$$

$$q_1^{(2)} = q_X^{(2)} = \frac{1}{\sqrt{2}}\exp(-i(\omega t - \kappa Z)) \tag{5.52}$$

$$q_2^{(2)} = q_Y^{(2)} = \frac{i}{\sqrt{2}}\exp(-i(\omega t - \kappa Z)). \tag{5.53}$$

Therefore in this case the upper a indices (1) and (2) are complex conjugates and examples of the complex circular basis [1] - [10] for $e_{(a)}$. The complex circular basis vectors are expressed as combinations of Cartesian basis (unit) vectors. Therefore in Eq. (5.47) the complex circular basis is used for V^a and the Cartesian basis for V^μ. The electromagnetic potential density is the tetrad within the proportionality $A^{(0)}$, where $cA^{(0)}$ is the background voltage of ECE theory observed in the radiative corrections [1] - [10]. In general the electromagnetic potential density vector is:

$$\boldsymbol{A}_\mu = A_\mu^{(0)}\boldsymbol{e}^{(0)} + A_\mu^{(1)}\boldsymbol{e}^{(1)} + A_\mu^{(2)}\boldsymbol{e}^{(2)} + A_\mu^{(3)}\boldsymbol{e}^{(3)} \tag{5.54}$$

and the electromagnetic field density vector is:

$$\boldsymbol{F}_{\mu\nu} = F_{\mu\nu}^{(0)}\boldsymbol{e}^{(0)} + F_{\mu\nu}^{(1)}\boldsymbol{e}^{(1)} + F_{\mu\nu}^{(2)}\boldsymbol{e}^{(2)} + F_{\mu\nu}^{(3)}\boldsymbol{e}^{(3)} \tag{5.55}$$

and there are four polarizations: timelike (0), transverse ((1) and (2)) and longitudinal (3). Similarly the canonical angular energy/momentum density vector is:

$$\boldsymbol{J}_{\mu\nu} = J_{\mu\nu}^{(0)}\boldsymbol{e}^{(0)} + J_{\mu\nu}^{(1)}\boldsymbol{e}^{(1)} + J_{\mu\nu}^{(2)}\boldsymbol{e}^{(2)} + J_{\mu\nu}^{(3)}\boldsymbol{e}^{(3)}. \tag{5.56}$$

In these expressions the potential, field and angular momentum are defined by integration over the volume four vector:

$$A_\mu = \int \boldsymbol{A}_\mu \cdot d\boldsymbol{V}, \tag{5.57}$$

$$F_{\mu\nu} = \int \boldsymbol{F}_{\mu\nu} \cdot d\boldsymbol{V}, \tag{5.58}$$

$$J_{\mu\nu} = \int \boldsymbol{J}_{\mu\nu} \cdot d\boldsymbol{V}. \tag{5.59}$$

Note carefully that the tetrad plane wave vector is a vector in a non-Minkowski spacetime. This can be seen from applying Eq. (5.39) systematically to each component of the tetrad, whereupon results such as the following are obtained:

$$\partial_0 q_X^{(1)} = i\frac{\omega}{c}q_X^{(1)}, \tag{5.60}$$

$$\partial_3 q_X^{(1)} = -i\kappa q_X^{(1)}. \tag{5.61}$$

Eq. (5.39) for a flat spacetime is not obeyed. For the radiated or free electromagnetic field it has also been shown that the spin connection is proportional to the tetrad:

$$\omega^a_{\mu b} = -\frac{\kappa}{2}\epsilon^a{}_{bc} q^c_\mu \tag{5.62}$$

so that the spacetime is one in which the spin connection and gamma connection are non-zero. This result is obtained by superimposing one basis $((0),(1),(2),(3))$ on another basis (ct, X, Y, Z), a procedure which produces a transition from special relativity to general relativity, from Minkowski spacetime to a spacetime in which the spin connection and gamma connection is non-zero. This means that the electromagnetic field is the tetrad, a concept defined from Eq. (5.47) using two frames of reference. Restricting consideration to the transverse elements (1) and (2), it is seen that the left hand column vector in Eq. (5.47) is spinning and moving forward along Z with respect to the right hand column vector. This is the fundamental meaning of the electromagnetic field in a generally covariant unified field theory. The same type of philosophy and procedure is used for all four fundamental force fields in ECE theory [1] - [10], so it is a unified field theory. Note finally that the most fundamental quantities are always the field densities, which are introduced through the index a used in the definition of the tetrad.

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