

APPENDIX 1 : HODGE DUAL TRANSFORMATION

The general Hodge dual of a tensor is defined { || } as:

$$\tilde{\nabla}_{\mu_1 \dots \mu_{n-p}} = \frac{1}{p!} \epsilon_{\mu_1 \dots \mu_{n-p}}^{\nu_1 \dots \nu_p} \nabla_{\nu_1 \dots \nu_p} \quad - (A1)$$

where:

$$\epsilon_{\mu_1 \mu_2 \dots \mu_n} = |g|^{1/2} \bar{\epsilon}_{\mu_1 \mu_2 \dots \mu_n} \quad - (A2)$$

is the totally anti-symmetric tensor, defined as the square root of the modulus of the determinant of the metric multiplied by the Levi-Civita symbol:

$$\bar{\epsilon}_{\mu_1 \mu_2 \dots \mu_n} = \left\{ \begin{array}{l} 1 \text{ for even permutation} \\ -1 \text{ for odd permutation} \\ 0 \text{ otherwise} \end{array} \right\} \quad - (A3)$$

Using the metric compatibility condition { || }:

$$D_\mu g_{\nu\rho} = 0 \quad - (A4)$$

it is seen that:

$$D_\mu |g|^{1/2} = \partial_\mu |g|^{1/2} = 0 \quad - (A5)$$

because the determinant of the metric is made up of individual elements of the metric tensor.

The covariant derivative of each element vanishes by Eq. (A4), so we obtain Eq. (A5).

The pre-multiplier $|g|^{1/2}$ is a scalar, and we use the fact that the covariant derivative of a scalar is the same as its four-derivative { || }:

$$D_\mu \nabla = \partial_\mu \nabla \quad - (A6)$$

The homogeneous field equation (4) in tensor notation is:

other words if we write down the sum:

$$\partial_{\mu} \tilde{F}^a_{\nu\rho} + \partial_{\rho} \tilde{F}^a_{\mu\nu} + \partial_{\nu} \tilde{F}^a_{\rho\mu} := d \wedge \tilde{F}^a - (A14)$$

it is identically equal to the sum:

$$-A^{(6)} \left(\tilde{R}^a_{\mu\nu\rho} + \tilde{R}^a_{\rho\mu\nu} + \tilde{R}^a_{\nu\rho\mu} + \omega^a_{\mu b} \tilde{T}^b_{\nu\rho} + \omega^a_{\rho b} \tilde{T}^b_{\mu\nu} + \omega^a_{\nu b} \tilde{T}^b_{\rho\mu} \right) := -A^{(6)} \left(q^b \wedge R^a_b + \omega^a_b \wedge T^b \right) - (A15)$$

So the inhomogeneous field equation is:

$$d \wedge \tilde{F}^a = \mu_0 J^a = -A^{(6)} \left(q^b \wedge R^a_b + \omega^a_b \wedge T^b \right) - (A16)$$

which is equivalent to:

$$\partial_{\mu} F^{a\mu\nu} = \mu_0 J^{a\nu} - (A17)$$

as given in the text.