SELF CONSISTENT DERIVATION OF THE EVANS LEMMA AND APPLICATION TO THE GENERALLY COVARIANT DIRAC EQUATION.

by

Myron W. Evans,

Alpha Foundation's Institute for Advanced Study (AIAS)

www.aias.us and www.atomicprecision.com

ABSTRACT

The self consistency of two derivations of the Evans Lemma is demonstrated rigorously and the Evans wave equation derived therefrom. The wave equation is reduced to the Dirac equation in the appropriate limit and the meaning discussed of the generally covariant Dirac and Pauli spinors. The effect of gravitation on particle physics may be investigated with the Evans equation.

Keywords : Evans unified field theory, lemma and wave equation; generally covariant Dirac equation; effect of gravitation on particle physics.

Thirty Sixth paper of the unified field theory; 1st paper of Series III.

1. INTRODUCTION

Recently a generally covariant unified field theory has been developed {1 – 14} which gives a plausible description of all radiated and matter fields in terms of the tetrad. The latter is the fundamental field of the Palatini variation of general relativity {15 – 17}. The Evans lemma {3, 4} is an identity of Cartan geometry which is the subsidiary proposition to the Evans wave equation. The latter unifies general relativity and quantum mechanics and is a wave equation of causal and objective physics {3, 4}. It has been demonstrated experimentally {11, 12} beyond reasonable doubt that the Heisenberg uncertainty principle is an intellectual aberration, so should be abandoned in favor of a causal and generally covariant interpretation {1 - 14} of quantum mechanics based on Cartan geometry. The lemma and wave equation are therefore fundamentally important to a consistent interpretation of natural philosophy as an objective subject in which every event has a cause. It is therefore necessary to demonstrate the lemma rigorously in more than one way, and to demonstrate its geometrical self consistency.

In Section 2 the self consistency of the Lemma is demonstrated with two independent methods. In Section 3 the Lemma is transformed into a wave equation using the index contracted form of the Einstein field equation, and the resulting equation reduced to the single particle Dirac equation in the appropriate limit, proving that the origin of the Dirac four spinor is Cartan geometry. The effect of gravitation on the single particle Dirac equation can therefore be calculated from the generally covariant single particle Dirac equation derived directly from the Evans wave equation.

2. GEOMETRICAL SELF CONSISTENCY OF THE EVANS LEMMA.

The lemma is an identity which is derived from the standard tetrad postulate of Cartan geometry { 18 }:

$$D_\mu q^a_{\ \nu} = 0. \qquad -(1)$$

Covariant differentiation of Eq. (1) gives the identity:

$$D^\mu \left(D_\mu q^a_{\ \nu} \right) := 0. \qquad -(2)$$

The covariant derivative is defined { 18 } such that:

$$D^\mu (\phi) = \partial^\mu (\phi) \qquad -(3)$$

where ϕ is a scalar. Thus for every scalar element defined in Eq. (1), Eq. (3) applies. It follows that:

$$\partial^\mu \left(D_\mu q^a_{\ \nu} \right) := 0. \qquad -(4)$$

The tetrad postulate is expanded out next as follows { 18 }:

$$D_\mu q^a_{\ \lambda} = \partial_\mu q^a_{\ \lambda} + \omega^a_{\ \mu b} q^b_{\ \lambda} - \Gamma^\nu_{\ \mu\lambda} q^a_{\ \nu} = 0 \qquad -(5)$$

where $\omega^a_{\ \mu b}$ is the spin connection and where $\Gamma^\nu_{\ \mu\lambda}$ is the gamma connection for a spacetime both with curvature and torsion. Using the inverse tetrad relation:

$$q^a_{\ \mu} q^\mu_{\ a} = 1 \qquad -(6)$$

it follows directly from Eq. (4) that:

$$\Box q^a_{\ \mu} = R q^a_{\ \mu} \qquad -(7)$$

where

$$R = q^{\lambda}_{a}\, \partial^{\mu} \left(\Gamma^{\nu}_{\mu\lambda}\, q^{a}_{\nu} - \omega^{a}_{\mu b}\, q^{b}_{\lambda} \right) \quad - (8)$$

and where the d'Alembertian operator is defined by:

$$\Box = \partial^{\mu} \partial_{\mu} . \quad - (9)$$

Eq. (7) is the lemma, or subsidiary geometrical proposition, that leads to the Evans wave equation. It is a simple identity of Cartan geometry and is the structure that leads directly to the generally covariant, causal and thus objective wave equations of physics. From the tetrad postulate (1):

$$\Gamma^{\nu}_{\mu\lambda}\, q^{a}_{\nu} - \omega^{a}_{\mu b}\, q^{b}_{\lambda} = \partial_{\mu} q^{a}_{\lambda} \quad - (10)$$

and using Eq. (10) in Eq. (8) it is found self-consistently that:

$$R = q^{\lambda}_{a}\, \partial^{\mu} \left(\partial_{\mu} q^{a}_{\lambda} \right) = q^{\lambda}_{a}\, \Box\, q^{a}_{\lambda} \quad - (11)$$

which leads back self consistently to Eq. (7) upon use of Eq. (6). It has therefore been shown that the most basic structure of Cartan geometry is the wave equation (7). This wave equation of geometry is the source of quantum mechanics in physics. The importance of the lemma is therefore clear, it indicates that all of physics is derived from Cartan geometry. Geometry is transformed into physics using:

$$R = -kT. \quad - (12)$$

Eq. (12) is the most fundamental equation of relativity, and is the simplest way in which geometry can be translated into physics via the scalar energy-momentum density T and the Einstein constant k. Here R is the scalar curvature in inverse square meters. Eq. (12) applies to all radiated and matter fields as intended originally by Einstein himself { 19 }.

Not only can we recover the Einstein Hilbert field equation from Eq. (12) but also a number of other field equations {1-14}. The Einstein Hilbert field equation is derived from the second Bianchi identity of Riemann geometry on the assumption { 18 } of the Christoffel connection which is symmetric in its lower two indices. This assumption implies that the torsion tensor is zero. Therefore the Einstein Hilbert field theory assumes that there is spacetime curvature but no spacetime torsion. In some circumstances this assumption is perfectly adequate, for example for the sun (Cassini experiments at NASA, 2002 to present), but in other circumstances it is well known that there are cosmological anomalies {20 }, some of them appear to be very large anomalies. So the Einstein Hilbert field equation appears to be only partially successful in a cosmological context when we take all the data into account.

In the Evans field theory on the other hand curvature and torsion are both present in general {1- 14} and the connection is the spin connection of the Palatini variation of general relativity in which the fundamental field is the tetrad and not the symmetric metric of the Einstein–Hilbert field theory. The symmetric metric is the dot product of two tetrads, as is well known { 18 }:

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab} \quad -(13)$$

where η_{ab} is the Minkowski metric of the tangent spacetime. It follows immediately that there always exists an antisymmetric metric - the wedge product of two tetrads:

$$g^c_{\mu\nu} = q^a_\mu \wedge q^b_\nu . \quad -(14)$$

The antisymmetric metric is a vector valued two-form of differential geometry. The most general metric is the outer product of two tetrads {1-14}. The outer product is a matrix, and therefore can always be written { 21 } as the sum of a symmetric and antisymmetric matrix.

The trace of the symmetric matrix is essentially the dot product and the antisymmetric traceless part is essentially the cross product. A simple example is vector analysis in three dimensional Euclidean space. If the dot product $\underline{A} \cdot \underline{B}$ is defined of two vectors, we can always define a cross product $\underline{A} \times \underline{B}$. This rule can be generalized to n dimensional non-Euclidean geometry through the use of tetrads. The dot product is generalized to Eq. (13) and the cross product is generalized to Eq. (14).

The antisymmetric metric is missing from the Einstein Hilbert field theory of gravitation, but is a special case of the Evans field theory when the spin connection is dual to the tetrad {3, 4}. In this special case the wedge product of the spin connection and the tetrad that appears in the first Cartan structure equation:

$$T^{a}_{\mu\nu} = (d \wedge q^{a})_{\mu\nu} + \omega^{a}_{\mu b} \wedge q^{b}_{\nu} \quad -(15)$$

reduces to the antisymmetric metric within a factor κ with the dimensions of wavenumber. This duality condition:

$$\omega^{a}_{\mu b} = \kappa \epsilon^{a}{}_{bc} q^{c}_{\mu} \quad -(16)$$

defines free space electromagnetic radiation {1 - 14} decoupled from gravitation - a special case of the general Evans unified field theory. $T^{a}_{\mu\nu}$ is the torsion form (a vector valued two-form) d ^ denotes the exterior derivative, and $\omega^{a}_{\mu b}$ denotes the spin connection of the well known Palatini variation {15-17} of relativity theory. Therefore the Einstein-Hilbert field theory of gravitation, although well known and well used, is severely constrained by its fundamental geometrical assumption of a Christoffel (symmetric) connection:

$$\Gamma^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\nu\mu}. \quad -(17)$$

This could well be the source of the well observed { 20 } anomalies of cosmology and the

Evans field theory should be used to address these anomalies. If relativity theory is abandoned, objective physics is abandoned, leaving essentially no physics at all. The use of the Christoffel connection means that:

$$R_{\rho\sigma\mu\nu} + R_{\rho\mu\nu\sigma} + R_{\rho\nu\sigma\mu} = 0 \qquad -(18)$$

whereas more generally {1 – 14} Eq. (18) is the first Bianchi identity of Cartan geometry:

$$\left(d \wedge T^{a}\right)_{\mu\nu\sigma} + \omega^{a}_{\mu b} \wedge T^{b}_{\nu\sigma} := R^{a}{}_{b\mu\nu} \wedge q^{b}_{\sigma} . \qquad -(19)$$

It has been shown {1 - 14} that Eq (19) is the same as the following identity of Riemann geometry:

$$\begin{aligned} &\partial_\mu \Gamma^{\lambda}_{\nu\rho} - \partial_\nu \Gamma^{\lambda}_{\mu\rho} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\rho} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\mu\rho} \\ &+ \partial_\nu \Gamma^{\lambda}_{\rho\mu} - \partial_\rho \Gamma^{\lambda}_{\nu\mu} + \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\rho\mu} - \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\nu\mu} \\ &+ \partial_\rho \Gamma^{\lambda}_{\mu\nu} - \partial_\mu \Gamma^{\lambda}_{\rho\nu} + \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\rho\nu} \\ &:= R^{\lambda}{}_{\rho\mu\nu} + R^{\lambda}{}_{\mu\nu\rho} + R^{\lambda}{}_{\nu\rho\mu} \qquad -(20) \end{aligned}$$

using both the Riemann and torsion tensors, both being non-zero in general. In general the Riemann ~~torsion~~ form of Cartan geometry is { 1 – 14 }:

$$R^{a}{}_{b\mu\nu} = q^{a}_{\sigma}\, q^{\tau}_{b}\, R^{\sigma}{}_{\tau\mu\nu} \qquad -(21)$$

where $R^{\sigma}{}_{\tau\mu\nu}$ is the Riemann tensor of Riemann geometry. The symmetries:

$$R^{a}{}_{b\mu\nu} = -R^{a}{}_{b\nu\mu} \qquad -(22)$$

$$R^{\sigma}{}_{\tau\mu\nu} = -R^{\sigma}{}_{\tau\nu\mu} \qquad -(23)$$

are always true, but in general the Riemann form and Riemann tensor are asymmetric in their first two indices. The Riemann tensor becomes antisymmetric in its first two indices if and only if Eq (18) is true { 18 }. This is another illustration of the rather severe geometrical constraints on the Einstein Hilbert field theory. In the Evans field theory these constraints are lifted and a lot of new physics awaits exploration.

A simple example of a new field equation from Eq. (12) is:

$$R q^a_\mu = -kT q^a_\mu \quad -(24)$$

which is a classical field equation closely similar to the Evans wave equation of generally covariant quantum mechanics:

$$(\square + kT) q^a_\mu = 0 \quad -(25)$$

obtained from Eqs. (7) and (12). Therefore the Evans lemma of geometry translates into physics using Eq. (12). To solve Eq. (25) it is possible for example to first define T and then derive the eigenfunctions q^a_μ if possible analytically or otherwise computationally. There are various model situations that may be used for T. One of the simplest is the single particle special relativistic limit where:

$$T \rightarrow \frac{m}{V_0} \quad -(26)$$

in the particle rest frame. Here m is the mass of an elementary particle and V is its rest volume, a new fundamental concept introduced by the Evans field theory {1-14}. The correspondence principle states essentially that general relativity reduces to special relativity under well defined conditions. The wave equation of special relativistic quantum mechanics is the experimentally well tested Dirac equation:

$$\left(\square + \frac{m^2c^2}{\hbar^2}\right) q^a_\mu = 0 \qquad (27)$$

where c is the speed of light in vacuo and $\hbar$ the reduced Planck constant. It is deduced therefore that the Dirac four spinor, the wavefunction of the Dirac equation, is a tetrad. The latter is the fundamental field of the Palatini variation of general relativity and as such must remain the fundamental field in special relativity. This important conclusion is a direct consequence of the correspondence principle.

Using this argument and comparing Eqs. (25) and (27) it follows that the fundamental rest volume is defined by:

$$V_0 = \frac{\hbar^2 k}{mc^2} := \frac{\hbar^2 k}{En_0} \qquad (28)$$

for all elementary particles, including the photon, neutrinos, gravitons and gravitinos. This is one of the major discoveries of the Evans field theory because it removes the necessity for Feynman calculus and renormalization in quantum electrodynamics and quantum chromodynamics. It also removes the unphysical infinities of classical electrodynamics, infinities which originate in the notion of point electron without volume. From Eq. (28) it is deduced from general relativity that there are no point particles in nature, and that every elementary particle must have mass. From this deduction it follows that the Higgs mechanism must be abandoned and that theories based on the Higgs mechanism, such as the GWS theory, must be modified. The first steps towards such a modification have been taken {1-14}.