1)

GENERALLY COVARIANT HEISENBERG EQUATION.

The starting point for the derivation of the generally covariant Heisenberg equation is the Cartan structure equation:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \quad -(1)$$

Here T^a is the torsion form of differential geometry, $D\wedge$ is the covariant exterior derivative, and q^a is the tetrad form. The covariant exterior derivative is expanded in terms of the ~~covar~~ ordinary exterior derivative $d\wedge$ and the spin connection $\omega^a{}_b \wedge$.

The torsion form is governed by the Bianchi identity:

$$D \wedge T^a = R^a{}_b \wedge q^b \quad -(2)$$

where $R^a{}_b$ is the Riemann or curvature form. Reinstating the indices of the base manifold:

$$T^a{}_{\mu\nu} = (D \wedge q^a)_{\mu\nu} \quad -(3)$$

$$= - T^a{}_{\nu\mu}.$$

Similarly:

$$R^a{}_{b\mu\nu} = (D \wedge \omega^a{}_b)_{\mu\nu} = - R^a{}_{b\nu\mu}. \quad -(4)$$

2) Equations (3) and (4) are the first and second Cartan structure equations of standard differential geometry. It is seen that both the torsion and curvature forms are antisymmetric in the indices μ and ν of the base manifold. The torsion form is a vector valued two-form and the curvature form is a tensor valued two-form.

In order to construct the Heisenberg equation in generally covariant form the <u>angular momentum two-form</u> is defined from the torsion two-form:

$$J^a_{\mu\nu} = \frac{\hbar}{\kappa} T^a_{\mu\nu} = - J^a_{\nu\mu} \quad -(5)$$

Here $\hbar$ is the least angular momentum in the universe, the reduced Planck constant with units of J s. In eqn (5) κ is the wavenumber with units of inverse metres. The units of the torsion form are also inverse metres, so $J^a_{\mu\nu}$ has the correct units of J s. Therefore

$$\boxed{En^a = c\kappa J^a = c\hbar T^a} \quad -(6)$$

Eq. (6) is the generally covariant form of the Planck quantization:

$$En = \hbar\omega \quad -(7)$$

3)

where:

$$\omega = \kappa c. \quad —(8)$$

Here c is the speed of light, ω is the angular frequency (2π f) in radians per second, and

$$En^a_{\mu\nu} = - En^a_{\nu\mu} \quad —(9)$$

is the vector valued energy two-form. More precisely, $En^a_{\mu\nu}$ is the angular energy / angular momentum two-form.

It is well known that the generally covariant quantity is the canonical energy / momentum density, $T_{\mu\nu}$, of the Einstein field equation:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \quad —(10)$$
$$= k T_{\mu\nu}.$$

where:

$$T_{\mu\nu} = T_{\nu\mu}. \quad —(11)$$

It is seen from eqns. (9) and (11) that $En^a_{\mu\nu}$ is antisymmetric whereas $T_{\mu\nu}$ is symmetric.

Therefore in order to construct the generally covariant form of the Heisenberg equation a cyclic commutator equation is needed made up of:

$$\epsilon^a_{\mu\nu} = -\epsilon^a_{\nu\mu}. \quad -(12)$$

These are the <u>densities</u> corresponding to eq. (9). These densities $\epsilon^a_{\mu\nu}$ are vector valued two-forms with the units of $J m^{-3}$, i.e energy divided by <u>volume</u>.

We therefore expect wedge products of the type:

$$\epsilon^a \wedge \epsilon^b = \epsilon_{min} \epsilon^c \quad -(13)$$

where ϵ_{min} is the least energy density of a given elementary particle.

The Heisenberg equation is an equation of special relativity and is a commutator relation between angular momenta. In cartesian coordinates

$$[J_x, J_y] = i\hbar J_z \quad -(14)$$

et cyclicum. The angular momentum commutator equations (14) can be derived without any choice of operator representation. Within a

5) factor $\hbar$ the Heisenberg equation (14) is the fundamental commutator relation between rotation generators of the Poincaré group of special relativity. It is seen that volume does not enter into eqn (14). This is because eqn. (14) is one of special relativity and Minkowski spacetime. Re-instating the wavefunction ψ:

$$[J_x, J_y]\psi = i\hbar J_z \psi \quad -(15)$$

and this is equivalent to the Schrödinger wave equation in the non-relativistic limit, and to the Dirac equation in special relativity for one particle.

The Heisenberg equation of motion (15) is therefore not a correctly objective equation of physics. This is because it is not generally covariant. The wavefunction ψ is not recognized to be the correctly covariant wavefunction of the Palatini variation of general relativity – the tetrad q^a_μ. As described by Atkins eqn (15) is the basis for quantum mechanics applied to atoms and molecules (P.W. Atkins, "Molecular Quantum Mech...

Oxford Univ. Press, 1983). However, recent experimental data from at least three independent sources have shown beyond doubt that the Heisenberg equation can fail qualitatively. These are:

1) the advanced microscopy experiments of the Croca group in Lisbon, summarized in: J. R. Croca, "Towards a Non-linear Quantum Physics" (World Scientific, 2003). In this book many examples of the qualitative failure of the Heisenberg uncertainty principle are given.

2) The Young interferometric experiments of Afshar, reported in {Physics Today or New Scientist} in 2004 and replicated at Harvard. These experiments show that a photon and an electromagnetic wave can be observed simultaneously. This is not possible in the Bohr Heisenberg idea of "complementarity".

3) Experiments very near to absolute zero on Anderson condensates have long indicated that the Heisenberg uncertainty principle fails qualitatively under these circumstances (New Scientist, 2004).

Major theoretical advances have also

7) been made recently in unified field theory. Notably the origin of the wave equations of physics has been discovered in differential geometry. The origin is the fundamental tetrad postulate:

$$D_\nu q^a_\mu = 0 . \quad -(16)$$

This is fundamental to differential geometry and can be proven in several ways. From eqn. (16):

$$D^\nu (D_\nu q^a_\mu) := 0 \quad -(17)$$

or

$$\boxed{\square q^a_\mu := R q^a_\mu} . \quad -(18)$$

Eqn (18) is the Evans Lemma. (Found. Phys. Lett., 2004). A lemma is a subsidiary proposition in mathematics, and eqn. (18) leads to the Evans wave equation (Found. Phys. Lett., 2003):

$$\boxed{(\square + kT) q^a_\mu = 0} . \quad -(19)$$

Here:

$$R = -kT \quad -(20)$$

where R is a scalar curvature, k is the

Einstein constant and T is the index contracted canonical energy-momentum density.

So in using the correctly covariant Evans wave equation the concept of volume is introduced through the use of T. The wavefunction is also correctly described as the tetrad. The latter is the fundamental field in the Palatini variation of general relativity. Finally the correspondence principle shows that eqn. (19) must reduce to the Dirac equation in the limit of special relativity. For one particle in this limit

$$\left(\Box + (mc/\hbar)^2\right) q^a_\mu = 0, \quad -(21)$$

the wave form of the Dirac equation. Therefore:

$$\boxed{kT = \frac{m^2c^2}{\hbar^2}} \quad -(22)$$

In the rest frame of one particle:

$$T = \frac{m}{V_0} = \frac{E_0}{c^2 V} \quad -(23)$$

where the rest energy is

$$E_0 = mc^2$$

9) Therefore:

$$\frac{\hbar m}{V_0} = \frac{m^2 c^2}{\hbar^2} \quad -(25)$$

and the ~~minimum~~ rest volume of an elementary particle is:

$$\boxed{V_0 = \frac{k\hbar^2}{mc^2} = \frac{k\hbar^2}{E_0}} \quad -(26)$$

For the electron:

$$V_0 = 2.53 \times 10^{-81} m^3. \quad -(27)$$

Every elementary particle, including the photon and neutrinos, is characterized by its rest volume.

using the de Broglie equation:

$$E_0 = mc^2 = \hbar\omega_0 \quad -(28)$$

we obtain:

$$\boxed{V_0 = \frac{\hbar k}{\omega_0}}. \quad -(29)$$

where ω_0 is the rest frequency of the elementary particle.

In the rest frame in the special relativistic limit therefore:

10)

$$\epsilon_{min} = \frac{\hbar\omega}{V_0} \quad -(30)$$

which is the quantum of energy density for any elementary particle, not just the photon.

Finally, the energy densities ϵ^a, ϵ^b and ϵ^c appearing in eqn. (13) must in general be defined with respect to a given sample volume V in the rest frame in the non-relativistic limit, so:

$$\epsilon^a = E^a / V \quad -(31)$$

and so on, where:

$$E_n^a = \omega J^a \quad -(32)$$

and so on. We therefore obtain:

$$\left[\frac{J^a}{V_0}, \frac{J^b}{V_0}\right] = i\frac{\hbar}{V_0}\frac{J^c}{V_0} \quad -(33)$$

i.e.

$$\left[J^a, J^b\right] = i\hbar\frac{V_0}{V} J^c \quad -(34)$$

or

$$\left[V\epsilon^a, V\epsilon^b\right] = i\left(V_0\,\epsilon_{min}\right)\left(V\epsilon^c\right) \quad -(35)$$

11)

DISCUSSION

Eq. (34) is the correctly covariant form of the Heisenberg equation. Having been derived from differential geometry it is causal in the same sense that rotation generator relations are causal. In general:

$$V \gg V_0 \quad -(36)$$

and so for a sample volume V it is possible that:

$$[J^a, J^b]\psi \sim 0 \quad -(37)$$

in direct contradiction of the Heisenberg uncertainty principle. Eqn. (37) means that particle and wave may be observed simultaneously in agreement with the Afshar experiments. The volume of the particle (photon) is V_0 in its rest frame while the volume of the wave occupying the whole of the apparatus is V.

Similarly the Croca experiments and Aspect experiments show that the wave can occupy the whole of the apparatus.

Therefore we are justified in introducing the sample volume V. The rest volume V_0 is rigorously defined by general relativity