DERIVATION OF THE GAUSS LAW OF MAGNETISM, THE FARADAY LAW OF INDUCTION, AND O(3) ELECTRODYNAMICS FROM THE EVANS FIELD THEORY.

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ABSTRACT

The Gauss law of magnetism and the Faraday law of induction are derived from the Evans unified field theory. The geometrical constraints imposed on the general field theory by these well known laws lead self consistently to O(3) electrodynamics.

Keywords: Evans field theory, Gauss law applied to magnetism; Faraday law of induction; O(3) electrodynamics.

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1. INTRODUCTION.

In this paper the Gauss law applied to magnetism {1} and the Faraday Law of induction {1, 2} are derived from the Evans field theory {3-33} by the imposition of well defined constraints in differential geometry. Therefore the origin of these well known laws is traced to differential geometry and the properties of the general four dimensional manifold known as Evans spacetime. Such inferences are not possible in the Maxwell Heaviside (MH) theory of the standard model {1, 2} because MH is not an objective theory of physics, rather it is a theory of special relativity covariant only under the Lorentz transformation. An objective theory of physics must be covariant under any coordinate transformation{1} and this is a fundamental philosophical requirement for all physics, as first realized by Einstein. This fundamental requirement is known as general relativity and the general coordinate transformation leads to general covariance in contrast to the Lorentz covariance of special relativity. The fundamental lack of objectivity in the Lorentz covariant MH theory means that it is not able to describe the important mutual effects of gravitation and electromagnetism. In contrast the Evans unified field theory is generally covariant and is a direct logical consequence of Einstein's general relativity, which is essentially the geometrization of physics. The unified field theory is able by definition to analyse the effects of gravitation on electromagnetism and vice-versa.

In Section 2 a fundamental geometrical constraint on the general field theory is derived from a consideration of the first Bianchi identity of differential geometry. It is then shown that this constraint leads to O(3) electrodynamics {3-33} directly from the Bianchi identity. These inferences trace the origin of the Gauss law applied to magnetism and the Faraday law of induction to differential geometry and general relativity, as required by Einsteinian natural philosophy. Section 3 is a discussion of the numerical methods needed to solve the general and restricted Evans field equations.

2. GEOMETRICAL CONDITION NEEDED FOR THE GAUSS LAW OF MAGNETISM AND THE FARADAY LAW OF INDUCTION AND DERIVATION OF O(3) ELECTRODYNAMICS.

The geometrical origin of these laws in the Evans field theory is the first Bianchi identity of differential geometry {1}:

$$D \wedge T^a = R^a{}_b \wedge q^b \qquad -(1)$$

which can be rewritten as:

$$d \wedge T^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b. \qquad -(2)$$

Here T^a is the vector valued torsion two-form, $R^a{}_b$ is the tensor valued curvature or Riemann two-form; q^a is the vector valued tetrad one-form; $\omega^a{}_b$ is the spin connection, which can be regarded as a one-form {1}. The symbol D ^ denotes the covariant exterior derivative and d ^ denotes the ordinary exterior derivative.

The Bianchi identity becomes the homogeneous Evans field equation (HE) using:

$$A^a = A^{(0)} q^a, \qquad -(3)$$

$$F^a = A^{(0)} T^a. \qquad -(4)$$

Here $A^{(0)}$ is a scalar valued electromagnetic potential magnitude (whose S.I. unit is volt s / m). Thus A^a is the vector valued electromagnetic potential one-form and F^a is the vector valued electromagnetic field two-form. The HE is therefore:

$$d \wedge F^a = R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b = \mu_0 j^a \qquad -(5)$$

where:

$$j^a = \frac{1}{\mu_0}\left(R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b\right) \qquad -(6)$$

is the homogeneous current, a vector valued three-form. Here μ_0 is the S. I. vacuum permeability.

The homogeneous current is theoretically non-zero. However it is known experimentally to great precision that:

$$d \wedge F^a \sim 0. \qquad -(7)$$

Eq. (7) encapsulates the two laws which are to be derived here from Evans field theory. These are usually written in vector notation as follows. The Gauss law applied to magnetism is:

$$\underline{\nabla} \cdot \underline{B}^a \sim 0, \qquad -(8)$$

where $\underline{B}^a$ is magnetic flux density. The Faraday law of induction is:

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} \sim \underline{0} \qquad -(9)$$

where $\underline{E}^a$ is electric field strength. The index a appearing in Eqs. (8) and (9) comes from equation (5), i.e. from general relativity as required by Einstein. The physical meaning of a is that it indicates a basis set of the tangent bundle spacetime, a Minkowski or flat spacetime. Any basis elements (e.g. unit vectors or Pauli matrices) can be used {1}in the tangent spacetime of differential geometry, and the basis elements can be used to describe states of polarization {3-33}, for example circular polarization first discovered experimentally by Arago in 1811. Arago was the first to observe what is now known as the two transverse states of circular polarization. It is convenient {3-33} to describe these states of circular polarization by the well known {34} complex circular basis:

$$a = (1), (2) \text{ and } (3), \quad -(10)$$

whose unit vectors are:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) = \underline{e}^{(2)*}, \quad -(11)$$

$$\underline{e}^{(3)} = \underline{k}, \quad -(12)$$

where * denotes complex conjugation. Each state of circular polarization can be described by two complex conjugates. One sense of circularly polarized radiation is described by the complex conjugates:

$$\underline{A}_1^{(1)} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\phi}, \quad -(13)$$

$$\underline{A}_1^{(2)} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right)e^{-i\phi}. \quad -(14)$$

The other sense of circularly polarized radiation is described by the complex conjugates:

$$\underline{A}_2^{(1)} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right)e^{i\phi}, \quad -(15)$$

$$\underline{A}_2^{(2)} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{-i\phi} \quad -(16)$$

Here ϕ is the electromagnetic phase and Eqs. (13) ~~and~~ to (16) are solutions of Eq. (7). The experimentally observable Evans spin field $\underline{B}^{(3)}$ {3-33} is defined by the vector cross product of one conjugate with the other. In non-linear optics {3-33} this is known as the conjugate product, and is observed experimentally in the inverse Faraday effect, (IFE), the magnetization of any material matter by circularly polarized electromagnetic radiation. In the sense of circular polarization defined by Eqs. (13) and (14):

$$\underline{B}_1^{(3)} = -ig\,\underline{A}_1^{(1)} \times \underline{A}_1^{(2)} = B^{(0)}\underline{k} \quad -(17)$$

where

$$\gamma = \frac{\kappa}{A^{(0)}} \qquad -(18)$$

and where κ is the wavenumber. In the sense of circular polarization defined by Eqs. (15) and (16) $\underline{B}^{(3)}$ reverses sign:

$$\underline{B}_2^{(3)} = -i\gamma \underline{A}_2^{(1)} \times \underline{A}_2^{(2)} = -B^{(0)}\underline{k} \qquad -(19)$$

and this is observed experimentally {3-33} because the observable magnetization changes sign when the handedness or sense of circular polarization is reversed. Linear polarization is the sum of 50% left and 50% right circular polarization and in this state the IFE is observed to disappear. Thus $\underline{B}^{(3)}$ in a linearly polarized beam vanishes because half the beam has positive $\underline{B}^{(3)}$ and the other half negative $\underline{B}^{(3)}$. The $\underline{B}^{(3)}$ field was first inferred by Evans in 1992 {35} and it was recognised for the first time that the phase free magnetization of the IFE is due to a third (spin) state of polarization now recognized as a = (3) in the unified field theory. The Gauss law and the Faraday law of induction hold for a = (3), but it is observed experimentally that $\underline{E}^{(3)} = \underline{0}$ {3-33}. Therefore:

$$\underline{\nabla} \cdot \underline{B}^{(3)} \sim 0, \qquad -(20)$$

$$\frac{\partial \underline{B}^{(3)}}{\partial t} \sim \underline{0}. \qquad -(21)$$

The fundamental reason for this is that the spin of the electromagnetic field produces an angular momentum which is observed experimentally in the Beth effect {3-33}. The electromagnetic field is negative under charge conjugation symmetry (C), so the Beth angular momentum produces $\underline{B}^{(3)}$ directly, angular momentum and magnetic field being both axial vectors. The putative radiated $\underline{E}^{(3)}$ would be a polar vector if it existed, and would not be produced by spin. There is however no electric analogue of the inverse Faraday effect, a circularly polarized electromagnetic field does not produce an electric polarization experimentally, only a magnetization. Similarly, in the original Faraday effect, a static magnetic field rotates the plane of linearly polarized radiation, but a static electric field does not. The Faraday effect and the IFE are explained using the same hyperpolarizability tensor in the standard model, and in the Evans field theory by a term in the well defined Maclaurin expansion of the spin connection in terms of the tetrad, producing the IFE magnetization:

$$\underline{M}^{(3)} = \frac{-i}{\mu_0} \gamma' \underline{A}^{(1)} \times \underline{A}^{(2)} \qquad -(22)$$

where $\underline{A}^{(1)}$ and $\underline{A}^{(2)}$ are complex conjugate tetrad elements combined into vectors {3-33}. Similarly all non-linear optical effects in the Evans field theory are, self consistently, properties of the Evans spacetime or general four-dimensional base manifold. The unified field theory therefore allows non-linear optics to be built up from spacetime, as required in general relativity. In the MH theory the spacetime is flat and cannot be changed, so non-linear optics must be described using constitutive relations extraneous to the original linear theory.

In the unified field theory the Evans spin field and the conjugate product are deduced self consistently from the experimental observation:

$$j^a \sim 0 . \qquad -(23)$$

Eqs. (23) and (6) imply:

$$R^a{}_b \wedge A^b = \omega^a{}_b \wedge F^b \qquad -(24)$$

to high precision. In other words the Gauss law and Faraday law of induction appear to be true within contemporary experimental precision. The reason for this in general relativity (i.e. objective physics) is Eq. (24), a constraint of differential geometry. Using the Maurer Cartan structure equations of differential geometry {1}:

$$T^a = D \wedge q^a , \qquad -(25)$$

$$R^a{}_b = D \wedge \omega^a{}_b , \qquad -(26)$$

Eq. (24) becomes the following experimentally implied constraint on the general unified field theory:

$$(D \wedge \omega^a{}_b) \wedge q^b = \omega^a{}_b \wedge (D \wedge q^b) . \qquad -(27)$$

A particular solution of Eq. (27) is:

$$\omega^a{}_b = \kappa \, \epsilon^a{}_{bc} \, q^c \qquad -(28)$$

where $\epsilon^a{}_{bc}$ is the Levi Civita tensor in the flat tangent bundle spacetime. Being a flat spacetime, Latin indices can be raised and lowered in contravariant covariant notation and so we may rewrite Eq. (28) as:

$$\omega_{ab} = \kappa \, \epsilon_{abc} \, q^c . \qquad -(29)$$

Eq. (29) states that the spin connection is an antisymmetric tensor dual to the axial vector q^c within a scalar valued factor κ with the dimensions of inverse metres. Thus Eq. (29) defines the wave-number magnitude, κ , in the unified field theory. It follows from Eq. (29) that the covariant derivative defining the torsion form in the first Maurer Cartan structure equation (25) can be written as:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \qquad -(30)$$

$$= d \wedge q^a + \kappa q^b \wedge q^c \qquad -(31)$$

form which it follows, using Eqs. (3) and (4), that:

$$F^a = d \wedge A^a + g A^b \wedge A^c \qquad -(32)$$

In the complex circular basis, Eq. (32) can be expanded as the cyclically symmetric set of three equations:

$$F^{(1)*} = d \wedge A^{(1)*} - ig A^{(2)} \wedge A^{(3)} \quad -(33)$$

$$F^{(2)*} = d \wedge A^{(2)*} - ig A^{(3)} \wedge A^{(1)} \quad -(34)$$

$$F^{(3)*} = d \wedge A^{(3)*} - ig A^{(1)} \wedge A^{(2)} \quad -(35)$$

with O(3) symmetry {3-33}. These are the defining relations of O(3) electrodynamics, developed by Evans from 1992 to 2003 {3-33}.

It has been shown that O(3) electrodynamics is a direct result of the unified field theory given the experimental constraints imposed by the Gauss law and the Faraday law of induction. It follows that O(3) electrodynamics automatically produces these laws, i.e.:

$$d \wedge F^{(a)} = 0, \quad -(36)$$

$$a = 1, 2, 3,$$

as observed experimentally. The existence of the conjugate product has been DEDUCED in Eq. (32) from differential geometry, and it follows that the spin field and the inverse Faraday effect have also been deduced from differential geometry and the Evans unified field theory of general relativity or objective physics. This is a major advance from the standard model and the MH theory of special relativity.