

DERIVATION OF THE GAUSS LAW OF MAGNETISM, THE FARADAY LAW OF
INDUCTION, AND O(3) ELECTRODYNAMICS FROM THE EVANS FIELD THEORY.

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ABSTRACT

The Gauss law of magnetism and the Faraday law of induction are derived from the Evans unified field theory. The geometrical constraints imposed on the general field theory by these well known laws lead self consistently to O(3) electrodynamics.

Keywords: Evans field theory, Gauss law applied to magnetism; Faraday law of induction; O(3) electrodynamics.

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1. INTRODUCTION.

In this paper the Gauss law applied to magnetism {1} and the Faraday Law of induction {1, 2} are derived from the Evans field theory {3-33} by the imposition of well defined constraints in differential geometry. Therefore the origin of these well known laws is traced to differential geometry and the properties of the general four dimensional manifold known as Evans spacetime. Such inferences are not possible in the Maxwell Heaviside (MH) theory of the standard model {1, 2} because MH is not an objective theory of physics, rather it is a theory of special relativity covariant only under the Lorentz transformation. An objective theory of physics must be covariant under any coordinate transformation{1} and this is a fundamental philosophical requirement for all physics, as first realized by Einstein. This fundamental requirement is known as general relativity and the general coordinate transformation leads to general covariance in contrast to the Lorentz covariance of special relativity. The fundamental lack of objectivity in the Lorentz covariant MH theory means that it is not able to describe the important mutual effects of gravitation and electromagnetism. In contrast the Evans unified field theory is generally covariant and is a direct logical consequence of Einstein's general relativity, which is essentially the geometrization of physics. The unified field theory is able by definition to analyse the effects of gravitation on electromagnetism and vice-versa.

In Section 2 a fundamental geometrical constraint on the general field theory is derived from a consideration of the first Bianchi identity of differential geometry. It is then shown that this constraint leads to O(3) electrodynamics {3-33} directly from the Bianchi identity. These inferences trace the origin of the Gauss law applied to magnetism and the Faraday law of induction to differential geometry and general relativity, as required by Einsteinian natural philosophy. Section 3 is a discussion of the numerical methods needed to solve the general and restricted Evans field equations.

2. GEOMETRICAL CONDITION NEEDED FOR THE GAUSS LAW OF MAGNETISM AND THE FARADAY LAW OF INDUCTION AND DERIVATION OF O(3) ELECTRODYNAMICS.

The geometrical origin of these laws in the Evans field theory is the first Bianchi identity of differential geometry {1}:

$$D \wedge T^a = R^a_b \wedge q^b - (1)$$

which can be rewritten as:

$$d \wedge T^a = R^a_b \wedge q^b - \omega^a_b \wedge T^b - (2)$$

Here T^a is the vector valued torsion two-form, R^a_b is the tensor valued curvature or Riemann two-form; q^a is the vector valued tetrad one-form; ω^a_b is the spin connection, which can be regarded as a one-form {1}. The symbol D^{\wedge} denotes the covariant exterior derivative and d^{\wedge} denotes the ordinary exterior derivative.

The Bianchi identity becomes the homogeneous Evans field equation (HE) using:

$$A^a = A^{(0)} q^a, \quad - (3)$$

$$F^a = A^{(0)} T^a. \quad - (4)$$

Here $A^{(0)}$ is a scalar valued electromagnetic potential magnitude (whose S.I. unit is volt s / m). Thus A^a is the vector valued electromagnetic potential one-form and F^a is the vector valued electromagnetic field two-form. The HE is therefore:

$$d \wedge F^a = R^a_b \wedge A^b - \omega^a_b \wedge F^b = \mu_0 j^a - (5)$$

where:

$$j^a = \frac{1}{\mu_0} \left(R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b \right) \quad - (6)$$

is the homogeneous current, a vector valued three-form. Here μ_0 is the S. I. vacuum permeability.

The homogeneous current is theoretically non-zero. However it is known experimentally to great precision that:

$$d \wedge F^a \sim 0. \quad - (7)$$

Eq. (7) encapsulates the two laws which are to be derived here from Evans field theory.

These are usually written in vector notation as follows. The Gauss law applied to magnetism is:

$$\underline{\nabla} \cdot \underline{B}^a \sim 0, \quad - (8)$$

where \underline{B}^a is magnetic flux density. The Faraday law of induction is:

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} \sim \underline{0} \quad - (9)$$

where \underline{E}^a is electric field strength. The index a appearing in Eqs. (8) and (9) comes from equation (5), i.e. from general relativity as required by Einstein. The physical meaning of a is that it indicates a basis set of the tangent bundle spacetime, a Minkowski or flat spacetime.

Any basis elements (e.g. unit vectors or Pauli matrices) can be used $\{1\}$ in the tangent

spacetime of differential geometry, and the basis elements can be used to describe states of polarization $\{3-33\}$, for example circular polarization first discovered experimentally by

Arago in 1811. Arago was the first to observe what is now known as the two transverse states

of circular polarization. It is convenient {3-33} to describe these states of circular polarization by the well known {34} complex circular basis:

$$a = (1), (2) \text{ and } (3), \quad - (10)$$

whose unit vectors are:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}} (\underline{i} - i\underline{j}) = \underline{e}^{(2)*}, \quad - (11)$$

$$\underline{e}^{(3)} = \underline{k}, \quad - (12)$$

where * denotes complex conjugation. Each state of circular polarization can be described by two complex conjugates. One sense of circularly polarized radiation is described by the complex conjugates:

$$\underline{A}_1^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i\phi}, \quad - (13)$$

$$\underline{A}_1^{(2)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{-i\phi}. \quad - (14)$$

The other sense of circularly polarized radiation is described by the complex conjugates:

$$\underline{A}_2^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{i\phi}, \quad - (15)$$

$$\underline{A}_2^{(2)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{-i\phi}. \quad - (16)$$

Here ϕ is the electromagnetic phase and Eqs. (13) and (16) are solutions of Eq. (7).

The experimentally observable Evans spin field \underline{B} {3-33} is defined by the vector cross product of one conjugate with the other. In non-linear optics {3-33} this is known as the conjugate product, and is observed experimentally in the inverse Faraday effect, (IFE), the magnetization of any material matter by circularly polarized electromagnetic radiation. In the sense of circular polarization defined by Eqs. (13) and (14):

$$\underline{B}_1^{(3)} = -ig \underline{A}_1^{(1)} \times \underline{A}_1^{(2)} = B^{(0)} \underline{k} \quad - (17)$$

where

$$g = \frac{\kappa}{A^{(0)}} \quad - (18)$$

and where κ is the wavenumber. In the sense of circular polarization defined by Eqs. (15)

and (16) $\underline{B}^{(3)}$ reverses sign:

$$\underline{B}^{(3)} = -ig \underline{A}^{(1)} \times \underline{A}^{(2)} = -B^{(0)} \underline{k} \quad - (19)$$

and this is observed experimentally {3-33} because the observable magnetization changes

sign when the handedness or sense of circular polarization is reversed. Linear polarization is

the sum of 50% left and 50% right circular polarization and in this state the IFE is observed to

disappear. Thus $\underline{B}^{(3)}$ in a linearly polarized beam vanishes because half the beam has positive

$\underline{B}^{(3)}$ and the other half negative $\underline{B}^{(3)}$. The $\underline{B}^{(3)}$ field was first inferred by Evans in 1992 {35}

and it was recognised for the first time that the phase free magnetization of the IFE is due to a

third (spin) state of polarization now recognized as $a = (3)$ in the unified field theory. The

Gauss law and the Faraday law of induction hold for $a = (3)$, but it is observed experimentally

that $\underline{E}^{(3)} = 0$ {3-33}. Therefore:

$$\underline{\nabla} \cdot \underline{B}^{(3)} \sim 0, \quad - (20)$$

$$\frac{\partial \underline{B}^{(3)}}{\partial t} \sim 0. \quad - (21)$$

The fundamental reason for this is that the spin of the electromagnetic field produces an

angular momentum which is observed experimentally in the Beth effect {3-33}. The

electromagnetic field is negative under charge conjugation symmetry (C), so the Beth angular

momentum produces $\underline{B}^{(3)}$ directly, angular momentum and magnetic field being both axial

vectors. The putative radiated $\underline{E}^{(3)}$ would be a polar vector if it existed, and would not be

produced by spin. There is however no electric analogue of the inverse Faraday effect, a

circularly polarized electromagnetic field does not produce an electric polarization.

experimentally, only a magnetization. Similarly, in the original Faraday effect, a static magnetic field rotates the plane of linearly polarized radiation, but a static electric field does not. The Faraday effect and the IFE are explained using the same hyperpolarizability tensor in the standard model, and in the Evans field theory by a term in the well defined Maclaurin expansion of the spin connection in terms of the tetrad, producing the IFE magnetization:

$$\underline{M}^{(3)} = \frac{-i}{\mu_0} \gamma' \underline{A}^{(1)} \times \underline{A}^{(2)} \quad - (22)$$

where $\underline{A}^{(1)}$ and $\underline{A}^{(2)}$ are complex conjugate tetrad elements combined into vectors {3-33}.

Similarly all non-linear optical effects in the Evans field theory are, self consistently, properties of the Evans spacetime or general four-dimensional base manifold. The unified field theory therefore allows non-linear optics to be built up from spacetime, as required in general relativity. In the MH theory the spacetime is flat and cannot be changed, so non-linear optics must be described using constitutive relations extraneous to the original linear theory.

In the unified field theory the Evans spin field and the conjugate product are deduced self consistently from the experimental observation:

$$j^a \sim 0. \quad - (23)$$

Eqs. (23) and (6) imply:

$$R^a_b \wedge A^b = \omega^a_b \wedge F^b \quad - (24)$$

to high precision. In other words the Gauss law and Faraday law of induction appear to be true within contemporary experimental precision. The reason for this in general relativity (i.e. objective physics) is Eq. (24), a constraint of differential geometry. Using the Maurer Cartan structure equations of differential geometry {1}:

$$T^a = D \wedge \vartheta^a, \quad - (25)$$

$$R^a_b = D \wedge \omega^a_b, \quad - (26)$$

Eq. (24) becomes the following experimentally implied constraint on the general unified field theory:

$$(D \wedge \omega^a_b) \wedge \vartheta^b = \omega^a_b \wedge (D \wedge \vartheta^b). \quad - (27)$$

A particular solution of Eq. (27) is:

$$\omega^a_b = \kappa \epsilon^a_{bc} \vartheta^c \quad - (28)$$

where ϵ^a_{bc} is the Levi Civita tensor in the flat tangent bundle spacetime. Being a flat spacetime, Latin indices can be raised and lowered in contravariant covariant notation and so we may rewrite Eq. (28) as:

$$\omega_{ab} = \kappa \epsilon_{abc} \vartheta^c. \quad - (29)$$

Eq. (29) states that the spin connection is an antisymmetric tensor dual to the axial vector ϑ^c within a scalar valued factor κ with the dimensions of inverse metres. Thus Eq. (29) defines the wave-number magnitude, κ , in the unified field theory. It follows from Eq.

(29) that the covariant derivative defining the torsion form in the first Maurer Cartan

structure equation (25) can be written as:

$$T^a = d \wedge \vartheta^a + \omega^a_b \wedge \vartheta^b \quad - (30)$$

$$= d \wedge \vartheta^a + \kappa \vartheta^b \wedge \vartheta^c \quad - (31)$$

form which it follows, using Eqs. (3) and (4), that:

$$F^a = d \wedge A^a + g A^b \wedge A^c \quad - (32)$$

In the complex circular basis, Eq. (32) can be expanded as the cyclically symmetric set of three equations:

$$F^{(1)*} = d \wedge A^{(1)*} - ig A^{(2)} \wedge A^{(3)} \quad - (33)$$

$$F^{(2)*} = d \wedge A^{(2)*} - ig A^{(3)} \wedge A^{(1)} \quad - (34)$$

$$F^{(3)*} = d \wedge A^{(3)*} - ig A^{(1)} \wedge A^{(2)} \quad - (35)$$

with O(3) symmetry {3-33}. These are the defining relations of O(3) electrodynamics, developed by Evans from 1992 to 2003 {3-33}.

It has been shown that O(3) electrodynamics is a direct result of the unified field theory given the experimental constraints imposed by the Gauss law and the Faraday law of induction. It follows that O(3) electrodynamics automatically produces these laws, i.e.* :

$$d \wedge F^{(a)} = 0, \quad - (36)$$

$$a = 1, 2, 3,$$

as observed experimentally. The existence of the conjugate product has been DEDUCED in Eq. (32) from differential geometry, and it follows that the spin field and the inverse Faraday effect have also been deduced from differential geometry and the Evans unified field theory of general relativity or objective physics. This is a major advance from the standard model and the MH theory of special relativity.

3. NUMERICAL METHODS OF SOLUTION.

In general the homogeneous and inhomogeneous Evans field equations must be solved simultaneously for given initial and boundary conditions. In this section the two equations are written out in tensor notation and subsidiary information summarized. The homogeneous field equation in tensor notation is:

$$\begin{aligned} & \partial_{\mu} F_{\nu\rho}^a + \partial_{\nu} F_{\rho\mu}^a + \partial_{\rho} F_{\mu\nu}^a \\ &= R^a{}_{b\mu\nu} A_{\rho}^b + R^a{}_{b\nu\rho} A_{\mu}^b + R^a{}_{b\rho\mu} A_{\nu}^b \\ & - \omega_{\mu b}^a F_{\nu\rho}^b - \omega_{\nu b}^a F_{\rho\mu}^b - \omega_{\rho b}^a F_{\mu\nu}^b \end{aligned} \quad - (37)$$

and this is equivalent to the barebones or minimalist notation of differential geometry:

$$d \wedge F = R \wedge A - \omega \wedge F \quad - (38)$$

where all indices have been suppressed. For the purposes of electrical engineering Eq. (37)

is, to an excellent approximation:

$$\partial_{\mu} F_{\nu\rho}^a + \partial_{\nu} F_{\rho\mu}^a + \partial_{\rho} F_{\mu\nu}^a = 0. \quad - (39)$$

Eq. (39) is equivalent to the Gauss Law applied to magnetism:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad - (40)$$

and the Faraday Law of induction:

$$\underline{\nabla} \times \underline{B}^a + \frac{\partial \underline{E}^a}{\partial t} = \underline{0} \quad - (41)$$

for all polarization states a.

The exceedingly important influence of gravitation on electromagnetism and vice versa must however be computed in general when the right hand side of Eq. (37) is non-zero. This tiny but in general non-zero influence leads to a violation of the well known laws (40) and (41), and this must be searched for with high precision instrumentation. Eq. (39) may be rewritten as the Hodge dual equation:

$$d^\mu \tilde{F}_{\mu\nu}^a = 0 \quad - (42)$$

and for each index a this is a homogeneous Maxwell Heaviside field equation. In general $\tilde{F}_{\mu\nu}^a$ is the Hodge dual of $F_{\mu\nu}^a$ in Evans spacetime and \tilde{j}_ν^a is the Hodge dual of the charge-current density three-form defined by the right hand side of Eq. (37), i.e. by:

$$d_\mu F_{\nu\rho}^a + d_\nu F_{\rho\mu}^a + d_\rho F_{\mu\nu}^a = \mu_0 (j_{\mu\nu\rho}^a + j_{\nu\rho\mu}^a + j_{\rho\mu\nu}^a) \quad - (43)$$

where:

$$j_{\mu\nu\rho}^a = \frac{1}{\mu_0} (R^a{}_{b\mu\nu} A_\rho^b - \omega_{\mu b}^a F_{\nu\rho}^b) \quad - (44)$$

Thus:

$$\tilde{j}_\sigma^a = \frac{1}{6} \epsilon^{\mu\nu\rho\sigma} j_{\mu\nu\rho}^a \quad - (45)$$

and

$$\begin{aligned} \tilde{F}_{\mu\sigma}^a &= \frac{1}{6} \epsilon^{\nu\rho\mu\sigma} j_{\nu\rho\mu}^a \\ \tilde{F}_{\nu\sigma}^a &= \frac{1}{6} \epsilon^{\rho\mu\nu\sigma} j_{\rho\mu\nu}^a, \\ \tilde{F}_{\rho\sigma}^a &= \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a, \end{aligned} \quad - (46)$$

In computing these Hodge duals the correct general definition maps from a p-form of

differential geometry to an (n-p)-form of differential geometry in the general n dimensional manifold. The general four dimensional manifold is the Evans spacetime, so named to distinguish it from the Riemannian spacetime used in Einstein's field theory of gravitation.

The Hodge dual is in general $\{ \underline{1} \}$:

$$\tilde{X}_{\mu_1 \dots \mu_{n-p}} = \frac{1}{p!} \epsilon^{\tilde{\nu}_1 \dots \tilde{\nu}_p} X_{\tilde{\nu}_1 \dots \tilde{\nu}_p} \quad - (47)$$

The general Levi-Civita symbol is:

$$\epsilon'_{\mu_1 \mu_2 \dots \mu_n} = \begin{cases} 1 & \text{even permutation} \\ -1 & \text{odd " } \\ 0 & \text{otherwise " } \end{cases} \quad - (48)$$

and the Levi-Civita tensor used in Eq. (47) is:

$$\epsilon_{\mu_1 \mu_2 \dots \mu_n} = (|g|)^{1/2} \epsilon'_{\mu_1 \mu_2 \dots \mu_n} \quad - (49)$$

where $|g|$ is the numerical value (i.e. a number) of the determinant of the metric tensor

$g_{\mu\nu}$. The field tensor is defined by the torsion tensor:

$$F_{\mu\nu}^a = A^{(0)} T_{\mu\nu}^a \quad - (50)$$

and the potential is defined by the tetrad:

$$A_{\mu}^a = A^{(0)} e_{\mu}^a \quad - (51)$$

The metric is factorized into a dot product of tetrads:

$$g_{\mu\nu} = e_{\mu}^a e_{\nu}^b \eta_{ab} \quad - (52)$$

where:

$$\eta_{ab} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad - (53)$$

is the diagonal metric tensor of the tangent bundle spacetime, a Minkowski or flat spacetime.

The gamma and spin connections are related by the tetrad postulate:

$$D_{\mu} e_{\nu}^a = \partial_{\mu} e_{\nu}^a + \omega_{\mu b}^a e_{\nu}^b - \Gamma_{\mu\nu}^{\lambda} e_{\lambda}^a = 0. \quad - (54)$$

The torsion and curvature (or Riemann) tensors are defined by the Maurer Cartan structure

relations of differential geometry. The torsion tensor is the covariant derivative of the tetrad

and is:

$$T_{\mu\nu}^{\lambda} = e_{\lambda}^a T_{\mu\nu}^a = \Gamma_{\mu\nu}^{\lambda} - \Gamma_{\nu\mu}^{\lambda}. \quad - (55)$$

It therefore vanishes for the Christoffel connection of Einstein's gravitational theory:

$$\Gamma_{\mu\nu}^{\lambda} = \Gamma_{\nu\mu}^{\lambda}. \quad - (56)$$

The curvature tensor is the covariant derivative of the spin connection and is:

$$R_{\lambda\nu\mu}^{\sigma} = e_{\lambda}^a e_{\nu}^b R_{b\nu\mu}^a. \quad - (57)$$

It follows that:

$$R^a{}_{b\sim\mu} = \partial_{\sim} \omega_{\mu}^a{}^b - \partial_{\mu} \omega_{\sim}^a{}^b + \omega_{\sim c}^a \omega_{\mu}^c{}^b - \omega_{\mu c}^a \omega_{\sim}^c{}^b \quad - (58)$$

and

$$R^{\lambda}{}_{\sigma\sim\mu} = \partial_{\sim} \Gamma_{\mu\lambda}^{\sigma} - \partial_{\mu} \Gamma_{\sim\lambda}^{\sigma} + \Gamma_{\sim\rho}^{\sigma} \Gamma_{\mu\lambda}^{\rho} - \Gamma_{\mu\rho}^{\sigma} \Gamma_{\sim\lambda}^{\rho} \quad - (59)$$

The Evans spacetime is therefore completely defined by the structure relations. The homogeneous field equation is the first Bianchi identity of differential geometry within the C negative factor A. The second Bianchi identity leads to the Noether Theorem of Evans field theory, and states that the covariant derivative of the curvature tensor vanishes identically.

Note that Eq. (39) is equivalent to the use of Riemann normal coordinates and a locally flat spacetime, because it is an equation in ordinary derivatives, not covariant derivatives. In general, $|g|$, the modulus of the determinant of the metric, in Eq. (49) is a function of x^{μ} , but at a point p in the Evans spacetime or manifold M it is always possible to define the Riemann normal coordinate system so the metric is in canonical form, and:

$$\partial_{\mu} g = 0 \quad - (60)$$

This defines the locally flat spacetime. Note carefully that the general homogeneous equation (37) cannot be expressed as a Hodge dual equation of type (42), so the appropriate equation for numerical solution must always be Eq. (37). Eq. (42) is mentioned only because of the traditional method of expressing the homogeneous Maxwell Heaviside equation of Minkowski spacetime (HME) as the Hodge dual equation. The correct form of the HME is the following Bianchi identity of Minkowski spacetime:

$$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0, \quad - (61)$$

a spacetime in which the Hodge dual is definable as:

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}. \quad - (62)$$

It may then be proven that Eq. (61) is the same equation as:

$$\partial_\mu \tilde{F}^{\mu\nu} = 0. \quad - (63)$$

The proof is as follows. From Eq. (63):

$$\partial_\lambda \tilde{F}^{\lambda\rho} + \partial_\mu \tilde{F}^{\mu\rho} + \partial_\nu \tilde{F}^{\nu\rho} = 0, \quad - (64)$$

and using Eq. (62):

$$\frac{1}{2} \left(\partial_\lambda (\epsilon^{\lambda\rho\mu\nu} F_{\mu\nu}) + \partial_\mu (\epsilon^{\mu\rho\nu\lambda} F_{\nu\lambda}) + \partial_\nu (\epsilon^{\nu\rho\lambda\mu} F_{\lambda\mu}) \right) = 0 \quad - (65)$$

Using the Leibniz Theorem and the constancy of $\epsilon^{\mu\nu\rho\sigma}$ in Minkowski spacetime, Eq.

(65) becomes:

$$\epsilon^{\lambda\rho\mu\nu} \partial_\lambda F_{\mu\nu} + \epsilon^{\mu\rho\nu\lambda} \partial_\mu F_{\nu\lambda} + \epsilon^{\nu\rho\lambda\mu} \partial_\nu F_{\lambda\mu} = 0 \quad - (66)$$

We may add individual indices of Eq. (66), to give, for example:

$$\epsilon^{1\rho 23} \partial_1 F_{23} + \epsilon^{2\rho 31} \partial_2 F_{31} + \epsilon^{3\rho 12} \partial_3 F_{12} + \dots = 0 \quad - (67)$$

which is

$$\partial_1 F_{23} + \partial_2 F_{31} + \partial_3 F_{12} + \dots = 0 \quad - (68)$$

upon using:

$$\epsilon^{1023} = \epsilon^{2031} = \epsilon^{3012} = -1, \quad - (69)$$

Proceeding in this way we see that Eq. (61) is the same as Eq. (63). Note carefully that

this proof is not true in general for Evans spacetime, because in that spacetime we obtain

results such as:

$$\partial^\mu \tilde{F}_{\mu\sigma}^a = \frac{1}{2} \partial^\mu (\epsilon^{\nu\rho}{}_{\mu\sigma} F_{\nu\rho}^a) = \frac{1}{2} (\epsilon^{\nu\rho}{}_{\mu\sigma} \partial^\mu F_{\nu\rho}^a + (\partial^\mu \epsilon^{\nu\rho}{}_{\mu\sigma}) F_{\nu\rho}^a) \quad - (70)$$

and since $\epsilon^{\nu\rho}{}_{\mu\sigma}$ depends on x^μ , one obtains:

$$\partial^\mu \epsilon^{\nu\rho}{}_{\mu\sigma} \neq 0 \quad - (71)$$

and there is an extra term which does not appear in Eq. (63) of Minkowski spacetime.

Only in the special case of Eq. (62) do we obtain:

$$\partial^\mu \epsilon^{\nu\rho}{}_{\mu\sigma} = 0. \quad - (72)$$

Therefore in general, the homogeneous Evans equation (37) must be solved simultaneously with the inhomogeneous Evans equation. The latter is an expression for the covariant exterior derivative of the Hodge dual of $F_{\mu\nu}^a$. In the minimalist or barebones notation of differential geometry this expression is:

$$d \wedge \tilde{F} \sim \tilde{R} \wedge A - \omega \wedge \tilde{F}, \quad - (73)$$

$$= \mu_0 \mathcal{J}$$

where \tilde{R} denoted the Hodge dual of the curvature form. Eq. (73) is the objective or

generally covariant expression in unified field theory of the inhomogeneous Maxwell

Heaviside equation (IMH) of special relativity:

$$d \wedge \tilde{F} = \mu_0 \mathcal{J}. \quad - (74)$$

It is seen by comparison of Eqs. (73) and (74) that in the unified field theory d^\wedge has been replaced by D^\wedge as required. Since both F and R in Eq. (38) are two-forms it follows by symmetry that if one takes the Hodge dual of F on the left hand side, one must take the Hodge dual of R on the right hand side. The reason is that the Hodge dual of any two-form in Evans spacetime is always another two-form. It is convenient to rewrite Eq. (73) as:

$$D \wedge \tilde{F} = \tilde{R} \wedge A \quad - (75)$$

and in analogy with Eq. (37), the tensorial expression for Eq. (73) is:

$$\begin{aligned} & d_\mu \tilde{F}_{\nu\rho}^a + d_\nu \tilde{F}_{\rho\mu}^a + d_\rho \tilde{F}_{\mu\nu}^a \\ &= \tilde{R}^a{}_{b\mu\nu} A_\rho^b + \tilde{R}^a{}_{b\nu\rho} A_\mu^b + \tilde{R}^a{}_{b\rho\mu} A_\nu^b \\ &\quad - \omega_{\mu\nu}^a \tilde{F}_{\rho}^b - \omega_{\nu\rho}^a \tilde{F}_{\mu}^b - \omega_{\rho\mu}^a \tilde{F}_{\nu}^b. \quad - (76) \end{aligned}$$

Therefore the general computational task is to solve Eqs. (37) and (76)

simultaneously for given initial and boundary conditions. In the last analysis this is a problem in simultaneous partial differential equations. We may now define the inhomogeneous current

$$J_{\mu\nu\rho} = \frac{1}{\mu_0} \left(\tilde{R}^a{}_{b\mu\nu} A_\rho^b - \omega_{\mu\nu}^a \tilde{F}_{\rho}^b \right) \quad - (77)$$

and we may note that $J_{\mu\nu\rho}$ is in general much larger than the homogeneous current $j_{\mu\nu\rho}$.

Therefore $J_{\mu\nu\rho}$ is of great practical importance for the acquisition of electric power from Evans spacetime and for counter gravitational technology in the aerospace industry.

Finally a convenient form of the inhomogeneous field equation may be obtained from the following considerations. We first construct the following Hodge duals:

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \quad - (78)$$

in the MH limit:

$$d \wedge F = 0 \quad - (79)$$

$$d \wedge \tilde{F} = \mu_0 J \quad - (80)$$

and note that:

$$(d \wedge \tilde{F})^{\mu\nu} \neq \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} (d \wedge F)_{\rho\sigma} \quad - (81)$$

In convenient shorthand notation this result may be written as:

$$d \wedge \tilde{F} \neq \frac{1}{2} \epsilon' d \wedge F, \quad - (82)$$

a result which becomes:

$$d \wedge \tilde{F} \neq \frac{1}{2} |g|^{1/2} \epsilon' d \wedge F \quad - (83)$$

in Evans spacetime. This is a key result because it shows that J^a is not zero if j^a is zero or almost zero, and it is J^a that is the important current term for the acquisition of electric power from Evans spacetime.

For the purposes of computation a systematic method of constructing the inhomogeneous Evans field equation (IE) is needed, where every term needed for coding is defined precisely. In order to do this begin with the fundamental definitions of differential geometry, the two Maurer Cartan structure equations:

$$T^a = D \wedge v^a, \quad - (84)$$

$$R^a_b = D \wedge \omega^a_b, \quad - (85)$$

and the two Bianchi identities:

$$D \wedge T^a = R^a_b \wedge v^b, \quad - (86)$$

$$D \wedge R^a_b = 0. \quad - (87)$$

In order to correctly construct the Hodge duals of T^a and R^a_b appearing in the IE the determinant of the metric must be defined correctly in each case (see Eq. (49)). In order to proceed consider the limit of the Evans unified field theory that gives Einstein's field theory of gravitation uninfluenced by electromagnetism. In the Einstein limit the metric tensor is symmetric and is defined by the inner or dot product of two tetrads:

$$g_{\mu\nu}^{(s)} = v_{\mu}^a(s) v_{\nu}^b(s) \eta_{ab}. \quad - (88)$$

The differential geometry appropriate to the Einstein theory is then:

$$T^{a(s)} = 0, \quad - (89)$$

$$d \wedge v^{a(s)} = -\omega^a_b(s) \wedge v^b(s), \quad - (90)$$

$$R^a_b(s) \wedge v^b(s) = 0. \quad - (91)$$

The determinant of the metric is defined in this limit by:

$$g^{(s)} = |g_{\mu\nu}^{(s)}|, \quad - (92)$$

and so the Hodge dual of the Riemann form is defined in Einstein's theory of gravitation by:

$$\tilde{R}^a_b(s) = \frac{1}{2} |g^{(s)}|^{1/2} \epsilon' R^a_b(s). \quad - (93)$$

Consider next the limit of the Evans field theory that gives the free electromagnetic field

when there is no field matter interaction, matter being defined by the presence of non-zero

mass. The differential geometry that defines this limit is:

$$T^a(A) = D \wedge \vartheta^a(A), \quad - (94)$$

$$R^a_b(A) = D \wedge \omega^a_b(A), \quad - (95)$$

$$g_{\mu\nu}^{(A)} = \vartheta^a_{\mu}(A) \wedge \vartheta^b_{\nu}(A), \quad - (96)$$

and the determinant of the metric is:

$$g^{(A)} = |g_{\mu\nu}^{(A)}|. \quad - (97)$$

The Hodge duals of the torsion and Riemann forms for the free electromagnetic field are

therefore:

$$\tilde{T}^a(A) = \frac{1}{2} |g^{(A)}|^{1/2} \epsilon' T^a(A), \quad - (98)$$

$$\tilde{R}^a_b(A) = \frac{1}{2} |g^{(A)}|^{1/2} \epsilon' R^a_b(A). \quad - (99)$$

Thirdly, when, the electromagnetic field interacts with matter, as in the IE, the

appropriate differential geometry is:

$$T^a = D \wedge \vartheta^a, \quad - (100)$$

$$R^a_b = D \wedge \omega^a_b, \quad - (101)$$

$$g_{\mu\nu}^{ab} = \vartheta^a_{\mu} \vartheta^b_{\nu}, \quad - (102)$$

and the determinant of the metric tensor is now:

$$g = |g_{\mu\nu}^{ab}|. \quad - (103)$$

The metric tensor itself is the sum of symmetric and anti-symmetric component metric

tensors:

$$g_{\mu}^a g_{\nu}^b = \frac{1}{2} \left(\left(g_{\mu}^a g_{\nu}^b \right)^{(S)} + \left(g_{\mu}^a g_{\nu}^b \right)^{(A)} \right).$$

The Hodge dual of the Riemann form in the IE is therefore:

$$\tilde{R}^a_b = \frac{1}{2} |g|^{1/2} \epsilon^{\dots} R^a_b, \quad - (105)$$

because in general both the symmetric and antisymmetric metrics contribute to the Riemann tensor or Riemann form when there is field matter interaction.

However, the Hodge dual of the torsion form in the IE is:

$$\tilde{T}^a = \frac{1}{2} |g^{(A)}|^{1/2} \epsilon^{\dots} T^a, \quad - (106)$$

because only the antisymmetric metric contributes to the torsion tensor or torsion form from Eq. (94).

Therefore the computational algorithm is fully defined, and the computational task in general is to solve the HE and IE SIMULTANEOUSLY for given initial and boundary conditions. The HE and IE contain more information than the equivalents in Maxwell Heaviside field theory, and the engineering task is to CAD / CAM a circuit taking electric power from Evans spacetime, defined by the general four dimensional manifold in which the Riemann and torsion tensors are both non-zero. In the Minkowski spacetime of Maxwell Heaviside field theory both tensors are zero. We must use the computer to define this extra source of power and to optimise circuits which utilise this extra source of power in practical devices.

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