THE HOMOGENEOUS AND INHOMOGENEOUS EVANS FIELD EQUATIONS.

by

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ABSTRACT

The homogeneous (HE) and inhomogeneous (IE) Evans unified field equations are deduced from differential geometry and the Hodge dual in the general four dimensional manifold (Evans spacetime) of unified field theory. The HE is the first Bianchi identity of differential geometry multiplied on both sides by the fundamental voltage $A^{(0)}$, a scalar valued electromagnetic potential magnitude. The IE is deduced by evaluating the Hodge dual of the Riemann form and the Hodge dual of the torsion form in the first Bianchi identity, then multiplying both sides of the resultant equation by $A^{(0)}$. This procedure generalizes the well known Hodge dual relation between the anti-symmetric electromagnetic field tensors of the homogeneous and inhomogeneous Maxwell Heaviside field equations of the standard model. The resulting HE and IE equations are correctly objective, or generally covariant, whereas the MH equations are valid only in the Minkowski spacetime of special relativity, and so are not generally covariant or objective equations of physics. For this reason the HE and IE field equations are able to analyze the mutual effects of gravitation on electromagnetism and vice versa, whereas the MH equations fail qualitatively in this objective.

Keywords: Homogeneous and inhomogeneous Evans field equations; Hodge dual; Evans unified field theory.

1. INTRODUCTION.

It is well known that the homogeneous and inhomogeneous Maxwell Heaviside (MH) field equations of the standard model are respectively {1, 2}:

$$d \wedge F = 0 \qquad -(1)$$

$$d \wedge \tilde{F} = \mu_0 J \qquad -(2)$$

in differential geometry. Here d ^ is the exterior derivative, F is the scalar valued electromagnetic field two-form; $\tilde{F}$ is the Hodge dual {1} of F in Minkowski spacetime and so is another scalar valued two-form, and J is the scalar valued charge-current density three-form. Eqs. (1) and (2) are written in S.I. units and μ_0 is the S.I. permeability in vacuo. In Section 2 the MH equations of the standard model are made objective or generally covariant equations of the Evans unified field theory {3-32}. Eq. (1) is developed into the homogeneous Evans field equation (HE) and Eq. (2) is developed into the inhomogeneous Evans field equation (IE). The resulting equations are written in the general four dimensional manifold known as Evans spacetime and are:

$$d \wedge F^a = R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b = \mu_0 j^a \qquad -(3)$$

$$d \wedge \tilde{F}^a = \tilde{R}^a{}_b \wedge A^b - \omega^a{}_b \wedge \tilde{F}^b = \mu_0 J^a \qquad -(4)$$

Here

$$D \wedge F^a = d \wedge F^a + \omega^a{}_b \wedge F^b \qquad -(5)$$

is the covariant exterior derivative where $\omega^a{}_b$ is the spin connection {1} of differential geometry. In Eqs (3) and (4) F^a is the vector valued electromagnetic field two-form; $\tilde{F}^a$ is its Hodge dual {1} in Evans spacetime, $R^a{}_b$ is the tensor valued Riemann or curvature two-form; $\tilde{R}^a{}_b$ is the Hodge dual of $R^a{}_b$ in Evans spacetime; A^a is the vector-valued

$\tilde{R}^a{}_b$

electromagnetic potential one-form. Eq. (3) is the first Bianchi identity of differential geometry {1} mutiplied on both sides by the fundamental voltage $A^{(0)}$ {3-32}; a scalar valued electromagnetic potential magnitude in volts. Thus the HE equation is:

$$A^{(0)}(D \wedge T^a) = A^{(0)}(R^a{}_b \wedge q^b). \quad -(6)$$

Similarly the IE equation is:

$$A^{(0)}(D \wedge \tilde{T}^a) = A^{(0)}(\tilde{R}^a{}_b \wedge q^b). \quad -(7)$$

In Eqs. (6) and (7) T^a is the vector-valued torsion two-form of differential geometry and q^b is the vector valued tetrad one-form of differential geometry. Thus {3-32}:

$$F^a = A^{(0)} T^a \quad -(8)$$

$$A^a = A^{(0)} q^a \quad -(9)$$

Eqs. (8) and (9) convert differential geometry to the unified field theory.

In Section 2 the current terms j^a and J^a are developed in terms of the fundamental differential forms of geometry. In section 3 the mutual effects of gravitation and electromagnetism are discussed in some detail and suggestions made for numerical solutions of the HE and IE field equations. Essentially this work shows that all of physics originates in spacetime geodynamics {33}.

2.

2. DEVELOPMENT OF THE CURRENT TERMS j and J.

It is convenient to summarize the notation used in the unified field theory as follows, first for the HE and then for the IE field equations.

The HE equation in "barebones notation", with all indices suppressed is:

$$D \wedge F = R \wedge A. \quad -(10)$$

The tangent bundle indices are first restored to give:

$$D \wedge F^a = R^a{}_b \wedge A^b. \quad -(11)$$

Secondly the indices of the base manifold are restored to give:

$$(D \wedge F^a)_{\mu\nu\rho} = (R^a{}_b \wedge A^b)_{\mu\nu\rho}. \quad -(12)$$

In tensor notation, Eq (12) is developed as follows:

$$\partial_\mu F^a_{\nu\rho} + \partial_\nu F^a_{\rho\mu} + \partial_\rho F^a_{\mu\nu} + \omega^a_{\mu b} F^b_{\nu\rho} + \omega^a_{\nu b} F^b_{\rho\mu} + \omega^a_{\rho b} F^b_{\mu\nu}$$
$$= R^a{}_{b\mu\nu} A^b_\rho + R^a{}_{b\nu\rho} A^b_\mu + R^a{}_{b\rho\mu} A^b_\nu. \quad -(13)$$

It can therefore be seen that the condensed notation of Eq (10) is equivalent to solving simultaneous partial differential equations for given initial and boundary conditions. This problem can be addressed numerically and the result will be an estimate of the effect of gravitation on electromagnetism for a given experimental situation. Analytical methods of approximation and experimental data where available should always be used as guidelines to the numerical solution.

The homogeneous MH equation of the standard model loses a lot of information in comparison with the HE equation. This can be seen as follows. In barebones notation the equivalent of Eq. (10) in the standard model is:

$$d \wedge F = 0. \qquad -(14)$$

There are no tangent bundle indices in equation (14) because it is an equation of Minkowski spacetime in which the electromagnetic field is thought of as a separate entity, an entity superimposed on the frame (the Minkowski spacetime). The Evans field theory on the other hand is a unified field theory of general relativity in which the unified field is the Evans spacetime, or general four dimensional manifold. Tangent bundles {1} are defined on this base manifold, and so tangent bundle indices appear in Eq. (11). These indices define states of polarization of the electromagnetic field. The indices of the base manifold can be restored in Eq. (14) to give:

$$(d \wedge F)_{\mu\nu} = 0 \qquad -(15)$$

an equation which when written out in full is:

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0. \qquad -(16)$$

In comparison with the HE equation it is seen that there is no spin connection and no Riemann form in Eq. (16). The latter cannot therefore describe the effect of gravitation on electromagnetism and vice versa. This is an obvious and well known failure of the standard model, meaning that the Evans field theory should be preferred to the standard model. The latter is not an objective theory of nature, while the Evans field theory is objective and thus the first generally covariant theory of ALL natural philosophy - the first workable unified field theory.

The charge-current density of the HE equation is the vector valued three-form:

$$j = \frac{1}{\mu_0} \left(R \wedge A - \omega \wedge F \right) \qquad - (17)$$

in barebones notation. It is therefore a balance of terms. Eq. (3) describes the Gauss Law of magnetism and the Faraday Law of induction in the unified field theory. Experimentally it is known that these laws are obeyed to high precision, for example in standards laboratories, implying:

$$d \wedge F^a = 0. \qquad - (18)$$

Therefore:

$$j^a = \frac{1}{\mu_0} \left(R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b \right) \sim 0, \qquad - (19)$$

i.e. j is very small experimentally in laboratory experiments. In a cosmological context however it may become possible to detect j experimentally, for example in electromagnetic radiation grazing an intensely gravitating object (Eddington type experiment) or in anomalous redshift data or similar. If j vanishes identically it follows from Eqs. (6) and (19) that:

$$R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b. \qquad - (20)$$

Using the structure equations of differential geometry:

$$T^b = D \wedge q^b \qquad - (21)$$

$$R^a{}_b = D \wedge \omega^a{}_b \qquad - (22)$$

Eq. (20) implies:

$$(D \wedge \omega^a{}_b) \wedge q^b = \omega^a{}_b \wedge (D \wedge q^b) \qquad - (23)$$

One possible solution of Eq. (23) is:

$$\omega^a{}_b = \epsilon^a{}_{bc}\, q^c \qquad -(24)$$

and this is true in circular polarization {3-32}.

The first Bianchi identity of differential geometry, Eq. (6), allows for:

$$R^a{}_b \wedge q^b \neq \omega^a{}_b \wedge T^b, \qquad -(25)$$

and so j can be non-zero mathematically. For j to be non-zero the gamma connection of Riemann geometry {1} must be asymmetric in its lower two indices, and when this condition is true gravitation and electromagnetism are mutually influential {3-32}. Initially circularly polarized electromagnetic radiation is changed by intense gravitation into elliptical polarization when j is non-zero. In the laboratory this is expected to be a very small effect, because Eq. (18) is true experimentally to high precision in the laboratory, but in a cosmological context j may become observable as discussed already. In the standard model j is always zero. So this type of experiment may be used to test the difference between Evans field theory and the standard model. Numerical methods would be needed to simulate and design such an experiment and to estimate j for a given R and given spin connection in the presence of gravitation. "The presence of gravitation" means that there is a contribution to the complete (in general asymmetric) gamma connection from a component of the gamma connection which is symmetric in its lower two indices (the well known Christoffel, Levi-Civita or Riemannian connection {1}). "The absence of gravitation" means that the gamma connection is anti-symmetric in its lower two indices. This is the gamma connection of electromagnetism uninfluenced by gravitation. "The absence of electromagnetism" means that the gamma connection is symmetric in its lower two indices, and that the torsion form vanishes by definition {1}. In all three cases note carefully that the Riemann tensor or Riemann form is non-zero. The Riemann form of electromagnetism is therefore non-zero in the absence of gravitation but is undefined in the standard model. On the other hand the torsion form vanishes by definition when the gamma connection is symmetric in its lower two indices. If the Riemann form and connection form are both zero, the spacetime is Minkowski spacetime, and gravitation and electromagnetism are not defined because the unified Evans field is not defined in a Minkowski spacetime. In the self-inconsistent and incomplete standard model electromagnetism is a separate philosophical entity superimposed on the frame of Minkowski spacetime, and gravitation is in essence the Christoffel connection.

The charge-current density J of the IE equation can become much larger than j and is of much greater practical importance in the acquisition of electric power from Evans spacetime. The practically important current J is defined as:

$$J = \frac{1}{\mu_0}\left(\tilde{R} \wedge A - \omega \wedge \tilde{F}\right) \quad -(26)$$

in barebones notation. Reinstating indices:

$$J^a_{\mu\nu\rho} = \frac{1}{\mu_0}\left(\tilde{R}^a{}_b \wedge A^b - \omega^a{}_b \wedge \tilde{F}^b\right)_{\mu\nu\rho} \quad -(27)$$

and when written out in tensor notation we obtain:

$$\begin{aligned} &J^a_{\mu\nu\rho} + J^a_{\nu\rho\mu} + J^a_{\rho\mu\nu} \\ &= \frac{1}{\mu_0}\left(\tilde{R}^a{}_{b\mu\nu} A^b_\rho + \tilde{R}^a{}_{b\nu\rho} A^b_\mu + \tilde{R}^a{}_{b\rho\mu} A^b_\nu \right. \\ &\left. - \omega^a{}_{b\mu} \tilde{F}^b_{\nu\rho} - \omega^a{}_{b\nu} \tilde{F}^b_{\rho\mu} - \omega^a{}_{b\rho} \tilde{F}^b_{\mu\nu}\right). \end{aligned} \quad -(28)$$

The IE equation is obtained in analogy with the inhomogeneous MH equation (2), in which the scalar valued {1} electromagnetic field two-form F is replaced by its Hodge dual {1} $\tilde{F}$ in Minkowski spacetime. Therefore in order to obtain the IE equation from the HE equation the Hodge duals are defined of F^a and $R^a{}_b$ in Evans spacetime. Here F^a is a vector valued {1} two-form of the Evans (i.e. unified) field, and $R^a{}_b$ is a tensor valued two-form of the Evans field. A differential two-form is antisymmetric in the indices of the base manifold. Thus:

$$F^a{}_{\mu\nu} = -F^a{}_{\nu\mu} \qquad -(29)$$

$$R^a{}_{b\mu\nu} = -R^a{}_{b\nu\mu}. \qquad -(30)$$

The Hodge dual {1} of a vector valued two-form in the general four-dimensional manifold is another vector valued two-form, another antisymmetric tensor with respect to the base manifold. Similarly the Hodge dual of a tensor valued two-form is another tensor valued two-form. Thus by anti-symmetry (Eqs. (29) and (30)) we obtain the IE from the HE.

In so doing care must be taken to define the Hodge dual correctly. The general definition of the Hodge dual for any differential form in any manifold is {1}:

$$\tilde{X}_{\mu_1 \cdots \mu_{n-p}} = \frac{1}{p!} \epsilon^{\nu_1 \cdots \nu_p}{}_{\mu_1 \cdots \mu_{n-p}} X_{\nu_1 \cdots \nu_p}. \qquad -(31)$$

Eq. (31) maps from a p-form to an (n - p)-form the general n dimensional manifold. The Levi-Civita symbol in the general manifold is defined {1} as:

$$\epsilon'_{\mu_1\mu_2\cdots\mu_n} = \begin{cases} 1 & \text{if } \mu_1\mu_2\cdots\mu_n \text{ is even} \\ -1 & \text{" \quad " \quad " odd} \\ 0 & \text{otherwise.} \end{cases} \qquad -(32)$$

The Levi-Civita symbol used in the definition of the Hodge dual is {1}:

$$\epsilon_{\mu_1\mu_2\cdots\mu_n} = (|g|)^{1/2} \epsilon'_{\mu_1\mu_2\cdots\mu_n} \qquad -(33)$$

where $|g|$ is the numerical magnitude of the determinant of the metric. In the general four-dimensional manifold (Evans spacetime), a two-form is dual to a two-form:

$$\tilde{X}_{\mu_1\mu_2} = \frac{1}{2}\epsilon^{\nu_1\nu_2}{}_{\mu_1\mu_2} X_{\nu_1\nu_2}. \quad -(34)$$

Indices are raised and lowered in the Levi-Civita tensor by use of the metric tensor. The latter is normalized by:

$$g^{\mu\nu} g_{\mu\nu} = 4. \quad -(35)$$

Therefore:

$$\tilde{X}_{\mu\nu} = \frac{1}{2}\epsilon^{\rho\sigma}{}_{\mu\nu} X_{\rho\sigma}. \quad -(36)$$

This is the correct definition of the Hodge dual of a differential two-form in Evans spacetime. For numerical solutions this definition must be coded correctly in order to define the IE equation and the current $J^a_{\mu\nu\rho}$. The definition (36) is true both for a vector valued and tensor valued two-form of Evans spacetime.

None of these fundamentally important concepts of differential geometry appear in the standard model but they are of basic importance for the engineering of electric power form Evans spacetime through the current $J^a_{\mu\nu\rho}$.