

Three Particle Problem w/ the New Universal Law of Gravitation

In three dimensions the new universal law of gravitation is:

$$U(\underline{r}) = -\frac{x^2 m M G}{|\underline{r} - \underline{r}_0|} + \frac{(x^2 - 1) L^2}{2m |\underline{r} - \underline{r}_0|^2} \quad - (1)$$

The Lagrangian for the problem of two planets orbiting the sun is:

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \left(\frac{m_1 M}{m_1 + M} \dot{r}_1^2 + \frac{m_2 M}{m_2 + M} \dot{r}_2^2 + \frac{m_1 m_2}{m_1 + m_2} \dot{r}_3^2 \right) \\ & - x_1^2 \frac{m_1 M G}{r_1} + \frac{(x_1^2 - 1) L_1^2}{2m_1 r_1^2} - x_2^2 \frac{m_2 M G}{r_2} + \frac{(x_2^2 - 1) L_2^2}{2m_2 r_2^2} \\ & - x_3^2 \frac{m_1 m_2 G}{r_3} + \frac{(x_3^2 - 1) L_3^2}{2m_1 r_3^2} \quad - (2) \end{aligned}$$

and the solution is:

$$r_1 = \frac{d_1}{1 + \epsilon_1 \cos(x_1 \theta)}, \quad r_2 = \frac{d_2}{1 + \epsilon_2 \cos(x_2 \theta)}, \quad r_3 = \frac{d_3}{1 + \epsilon_3 \cos(x_3 \theta)}$$

$$r_3^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos \alpha \quad - (3)$$

$$r_1^2 = r_2^2 + r_3^2 - 2r_2 r_3 \cos \beta \quad - (4)$$

$$r_2^2 = r_1^2 + r_3^2 - 2r_1 r_3 \cos \gamma$$