

178(2): First Approximation to the Fine Eigenvalues of the  
SR and NR Effects

The relevant approximation to the Dirac equation is:

$$\hat{H}_{rel} \psi = E_{rel} \psi \quad - (1)$$

i.e. which:

$$\hat{H}_{rel} = mc^2 + V + \frac{p^2}{2m} - \frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{B} \quad - (2)$$

Following Ryder, "Quantum Field Theory" (Cambridge Univ. Press, 1996) the non-relativistic limit is defined as:

$$(\hat{H}_{rel} - mc^2) \psi = (E_{rel} - mc^2) \psi \quad - (3)$$

i.e. 
$$\hat{H} \psi = E \psi \quad - (4)$$

where 
$$\hat{H} = \frac{p^2}{2m} + V - \frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{B} \quad - (5)$$

If  $\underline{B}$  is aligned in the  $z$  direction then:

$$\underline{\sigma} \cdot \underline{B} = \begin{bmatrix} B_z & 0 \\ 0 & -B_z \end{bmatrix} \quad - (6)$$

So: 
$$\hat{H}_+ = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V - \frac{e\hbar}{2m} B_z \quad - (7)$$

$$\hat{H}_- = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V + \frac{e\hbar}{2m} B_z \quad - (8)$$

2) In the first approximation, and for a weak magnetic field  $B_z$ , the wavefunction  $\psi$  can be taken to be those of the H atom, so there are two coupled equations:

$$(\hat{H}_+ - E) \frac{d\psi}{dc} = F_+ \psi \quad - (9)$$

and

$$(\hat{H}_- - E) \frac{d\psi}{dc} = F_- \psi \quad - (10)$$

so these equations can be evaluated by computer algebra to give  $F_+$  and  $F_-$  for every orbital.

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