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DERIVATION OF A LOCALLY GAUGE INVARIANT PROCA EQUATION FROM U(1) AND O(3) GAUGE THEORY APPLIED TO VACUUM ELECTRODYNAMICS AND ACQUISITION OF PHOTON MASS AND REST ENERGY FROM THE VACUUM.

by

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ABSTRACT

It is shown that the vacuum four-current first introduced empirically by Lehnert { l-4) can be derived from a local gauge transformation. The novelty of this approach lies in the use of electromagnetic field components for the scalar field which is being subjected to a local gauge transformation. A Higgs mechanism is used to derive a locally gauge invariant Proca equation in a U(1) and O(3) invariant electrodynamics. The advantages of an O(3) over a U(1) invariant electrodynamics are discussed.

KEYWORDS: Locally gauge invariant Proca equation; derivation of vacuum charge / current density; photon mass; Higgs mechanism.

1. INTRODUCTION

In this paper the vacuum four-current first introduced by Lehnert { l-4) is derived by a novel method based on using electromagnetic field components for the internal gauge space. Two types of internal gauge space are considered: U(1) and O(3) invariant. The scalar components in the internal space are considered to be complex because the particle (photon) concomitant with the field is assumed to carry a novel topological charge g, which appears in the covariant derivative obtained by local gauge transformation. In Section 2, it is shown that this novel method produces the vacuum four-current first proposed empirically by Lehnert {1-4}, so the important conclusion is reached that the Lehnert four-current can be obtained rigorously from gauge theory. In section 3 a Higgs mechanism is used to show for the first time that the Proca equation is locally gauge invariant. In the received view {5, 6} the Proca equation is not locally gauge invariant because of the presence of an ad hoc mass term.

Finally the novel advantages of an O(3) over a U(1) invariant gauge theory applied to electrodynamics in this novel manner are discussed in detail.

2. NOVEL GLOBALLY INVARIANT LAGRANGIANS AND WAVE EQUATIONS IN THE INTERNAL SPACES OF THE U(1) AND O(3) INVARIANT GAUGE THEORIES.

The novelty of this theory is that the internal space is considered to be made up of scalar components of an electromagnetic field, a method which leads to the Lehnert four-current in the vacuum. This approach has not been used before because the internal space has always been considered to be made up of components of a matter field. Therefore the entire theory is developed in the vacuum, whereas gauge theory as usually applied is developed for field matter interaction. The important result is that the Lehnert four-current can be derived from rigorous gauge theory developed entirely in the vacuum.

In U(1) invariant form the internal space is made up of a complex scalar electromagnetic field component A and its complex conjugate A* :

$$A = \frac{1}{\sqrt{2}} (A_1 + iA_2)2 \qquad — (1)$$

$$A^* = \frac{1}{\sqrt{2}} (A_1 - iA_2) \qquad — (2)$$

Use of complex fields signal the existence of a topological charge g { 7, 8 }:

$$g = \frac{\kappa}{A^{(0)}} \qquad — (3)$$

and it is shown for the first time that g emerges from this method as part of a covariant derivative in the vacuum:

$$D_\mu = \partial_\mu + igA_\mu . \qquad — (4)$$

Standard methods { 5 } lead to the locally invariant wave equations:

$$\square A = \square A^* = 0 \qquad — (5)$$

in the internal space. These are novel equations because the internal space is usually associated with a matter field. The correctness of the approach can be assessed through the fact that eqns. (5) are d' Alembert equations for the scalar components A and A*. Using standard methods, local gauge transformation gives the locally invariant lagrangian:

$$\mathcal{L} = D_\mu A D^\mu A^* - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad -(6)$$

in the vacuum, THERE BEING NO MATTER FIELDS PRESENT. The locally invariant wave equations in the vacuum are:

$$D^\mu (D_\mu A) = 0 \quad -(7)$$

$$D_\mu (D^\mu A^*) = 0 \quad -(8)$$

and the field equation IN THE VACUUM is:

$$\partial_\nu F^{\mu\nu} = -igc \left(A^* D^\mu A - A D^\mu A^* \right) \quad -(9)$$

where the covariant derivatives are defined by:

$$D_\mu A = (\partial_\mu + igA_\mu) A \quad -(10)$$

$$D_\mu A^* = (\partial_\mu - igA_\mu) A^* \quad -(11)$$

and where the vacuum four-current:

$$J^\mu (vac) = -i\epsilon_0 gc \left(A^* D^\mu A - A D^\mu A^* \right) \quad -(12)$$

is the Lehnert four-current. Therefore the latter has been derived for the first time by using gauge theory based entirely on the electromagnetic field with no matter field present. Herein lies the novelty of our approach, one which leads to the important result that the Lehnert four-current can be derived rigorously from gauge theory.

If we use an O(3) invariant electrodynamics, which has several known advantages {9, 10} over U(1) invariant electrodynamics, we obtain the locally O(3) gauge invariant lagrangian:

$$\mathcal{L} = D_\mu \underline{A} \cdot D^\mu \underline{A}^* - \frac{1}{4} \underline{G}_{\mu\nu} \cdot \underline{G}^{\mu\nu} \quad - (13)$$

where A has three components in general and where $\underline{G}^{\mu\nu}$ is the O(3) invariant field tensor {10}. The field equation obtained from this novel approach (where no matter fields are used) is:

$$D_\nu \underline{G}^{\mu\nu} = -g\, D^\mu \underline{A}^* \times \underline{A} \quad - (14)$$

where the right hand side defines the O(3) invariant Lehnert vacuum four-current. It has therefore been shown for the first time that the Lehnert four current exists in an O(3) as well as in a U(1) invariant electrodynamics.

The locally O(3) invariant wave equations obtained from this theory are the novel vacuum wave equations:

$$(\partial_\mu \partial^\mu + ig A_\mu \partial^\mu - ig \partial_\mu A^\mu + g^2 A_\mu A^\mu) A^* = 0 \quad -(15)$$

$$(\partial_\mu \partial^\mu - ig A_\mu \partial^\mu + ig \partial_\mu A^\mu + g^2 A_\mu A^\mu) A = 0 \quad -(16)$$

which simplify to:

$$(\Box + (m_0 c / \hbar)^2) \underline{A}^* = \underline{0} \quad -(17)$$

$$(\Box + (m_0 c / \hbar)^2) \underline{A} = \underline{0} \quad -(18)$$

using the quantum ansatz $p_\mu = i\hbar \partial_\mu$. These are novel O(3) invariant Proca equations obtained using a novel method in which the internal space is assumed to be made up of electrodynamic components.

3. ACQUISITION OF PHOTON MASS FROM THE VACUUM

If on the U(1) level we extend the novel methods of the previous section to include a standard Higgs mechanism we obtain the U(1) field equation in the form:

$$\partial_\mu F^{\mu\nu} = -ig (A^* D^\mu A - A D^\mu A^*) - \frac{g^2 m^2}{\lambda} A^\mu + 2\sqrt{2} g^2 a A_1 A^\mu + \sqrt{2} a g \partial^\mu A_2 \quad -(19)$$

in which the term $-g \frac{m^a A^{\mu a}}{\lambda}$ implies that the electromagnetic four potential has acquired mass, which can be identified as photon mass. This verifies the approach used in the previous section. In general all four terms in the field equation (19) are vacuum four-currents and we reach the important result that each of them can produce ENERGY FROM THE VACUUM:

$$En(vac) = \int J^{\mu}(vac) A_{\mu} \, dV. \quad - (20)$$

Another novel and important result is that the minimum value of the locally gauge invariant lagrangian is:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + g^2 a^2 A_{\mu} A^{\mu} \quad - (21)$$

from which we obtain the Proca equation:

$$\partial_{\mu} F^{\mu\nu} + m_0^2 A^{\nu} = 0 \quad - (22)$$

if the mass of the photon is defined by:

$$m_0^2 = \frac{1}{2} g^2 |a^2| \quad - (23)$$

In an O(3) invariant theory the field equation obtained has the structure of a Yang Mills equation FOR THE VACUUM:

$$D_\nu \underline{G}^{\mu\nu} = -g D^\mu \underline{A}^* \times \underline{A} \quad -(24)$$

so the right hand side is the Lehnert vacuum four-current. At the Higgs minimum the Lehnert four-current is obtained from the symmetry broken vacuum and takes the form:

$$D_\nu \underline{G}^{\mu\nu} = -g^2 \underline{a}_0 \times (\underline{A}^\mu \times \underline{a}_0^*) - (25)$$

which is an O(3) invariant Proca equation.

SUMMARY

The novel methods and results used in this paper can be summarized as follows. A gauge theory has been used for the first time in which the internal space consists of components of an electromagnetic field, represented by potential components. This method on the U(1) level results in the Lehnert vacuum four-current {1 - 4}, which is known empirically to have several advantages over the Maxwell Heaviside view.

It has been shown for the first time that the Lehnert four-current also exists in an O(3) invariant gauge theory in which the internal space is electromagnetic in nature. The results of the theory are supported by the known advantages of the extended electrodynamics of Lehnert {4} over the Maxwell Heaviside electrodynamics. A gauge invariant Proca equation has been shown to exist for the first time using these novel theoretical methods, and the feasibility of extracting energy from the vacuum shown for the first time. It has been shown that electromagnetic energy from the vacuum is possible if there exists a Lehnert four-current.

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REFERENCES

{1} B. Lehnert, Optik, 99, 113 (1995).

{2} B. Lehnert, Phys. Scripta, 59, 204 (1996).

{3} B. Lehnert and S. Roy, "Extended Electromagnetic Theory" (World Scientific, Singapore, 1998).

{4} B. Lehnert in M. W. Evans (ed.), and the Royal Swedish Academy, "Contemporary Optics and Electrodynamics", a special topical issue in three parts of I. Progogine and S. A. Rice (series eds.), "Advances in Chemical Physics" (Wiley, New York, 2001, in prep.), second edition of M. W. Evans and S. Kielich (eds.), "Modern Nonlinear Optics", a special topical issue in three parts of I. Prigogine and S. A. Rice (series eds.), "Advances in Chemical Physics" (Wiley, New York, 1992, 1993, 1997 (paperback)), vol. 85.

{5} L. H. Ryder. "Quantum Field Theory" (Cambridge, 1987, 2nd. Ed.).

{6} L. O'Raigheartaigh, Rep. Prog. Phys., 42, 159 (1979).

{7} M. W. Evans, J.-P. Vigier, S. Roy and S. Jeffers, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 to 1999), vols. 1 to 5.

{8} M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the $\underline{B}^{(3)}$ Field" (World Scientific, Singapore, 2000).

{9} M. W. Evans et al., AIAS group paper, Phys. Scripta, 61, 79 (2000).

{10} M. W. Evans et al., AIAS Group paper, Found. Phys. Lett., 12, 579 (1999).