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MS in Preparation for "Optik"

U(1) VERSUS O(3) HOLONOMY IN THE SAGNAC EFFECT FOR ELECTROMAGNETIC WAVES AND MATTER WAVES.

by

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ABSTRACT

It is demonstrated that the Sagnac effect for electromagnetic and matter waves cannot be explained by a U(1) holonomy, which gives a null result. An O(3) holonomy is needed in both cases in order to reproduce the Sagnac and Michelson Gale effects to high accuracy.

KEYWORDS: Holonomy in the Sagnac effect; O(3) holonomy; non-Abelian Stokes Theorem.

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1. INTRODUCTION

It is well known that the Maxwell Heaviside field equations cannot describe the Sagnac effect, which is an extra phase shift observed when the platform of the Sagnac interferometer is rotated {1}. The same is true for the Michelson Gale experiment {2}, where the rotating platform is the diurnal rotation of the earth, and also for the ring laser gyro { 3 }. The basic reason for the null result from the Maxwell Heaviside equations is that they are metric invariant and frame invariant { 4 }, so are unaffected by rotation. Another way of seeing this is that the phase factor of received classical electrodynamics:

$$\gamma = \exp\left(i\left(\omega t - \underset{\sim}{\kappa} \cdot \underset{\sim}{r} + \alpha\right)\right) \quad - (1)$$

is invariant under motion reversal symmetry (T). Here ω is the angular frequency of the electromagnetic wave in free space at instant t and $\underline{\kappa}$ is its wave-vector at point r. The number α is random, because the phase in Maxwell Heaviside electrodynamics is defined only up to a random factor α { 1 }. One loop (e.g. clockwise, (C)) of the Sagnac interferometer is generated from the anticlockwise (A) loop by the operator T. This is true when the platform is at rest and so the Maxwell Heaviside phase cannot describe the phase shift observed in the Sagnac interferometer when the platform is at rest, because the Maxwell Heaviside phase is identical for A and C loops. It has been shown recently that this type of result is true in general in interferometry { 5 - 10

} . The Maxwell Heaviside theory does not describe interferometric phenomena.

The Maxwell Heaviside theory is generally accepted in received opinion as a gauge field theory invariant under local U(1) transformations of the lagrangian { 11 } of the internal space of the gauge theory, a scalar space. This view implies that the Sagnac effect must be described by a holonomy, or round trip in space-time, with parallel transport using U(1) covariant derivatives { 11 }. The holonomy must fail, however, to describe the Sagnac effect because it is equivalent to Maxwell Heaviside theory. In section 2 this conclusion is demonstrated in detail, starting from a generally applicable non-Abelian Stokes Theorem. In Section 3 it is shown how an O(3) invariant gauge field theory applied to electrodynamics reproduces the Sagnac and Michelson Gale effects, and the ring laser gyro, using a holonomy with O(3) covariant derivatives. In so doing it is shown that the Sagnac and all interferometric effects are O(3) invariant, and depend on a phase factor which is an area integral over the $B^{(3)}$ field { 5 - 10 }. The latter is therefore the cause of all interferometry and related physical optical effects. This result is also true in the Sagnac effect for matter waves, such as electron waves, i.e. an O(3) holonomy is needed for the recent observation of the Sagnac effect {12} in electrons. This result is demonstrated in Section 4. Finally in Section 5 it is shown that an O(3) holonomy is compatible with the explanation recently offered by Vigier {13} of the Sagnac effect using finite photon mass.

2. NULL RESULT OF THE U(1) HOLONOMY IN THE SAGNAC EFFECT WITH PLATFORM AT REST AND IN MOTION.

For all gauge group symmetries the holonomy results in the non-Abelian Stokes Theorem {14}:

$$\oint D_\mu dx^\mu := -\frac{1}{2}\int [D_\mu, D_\nu]\, d\sigma^{\mu\nu} \qquad (2)$$

consisting of an identity between properties of covariant derivatives D_μ for any gauge group symmetry, including U(1) and O(3). Eqn. (2) is equivalent for all gauge group symmetries to the Jacobi identity {11}:

$$\sum_{\text{cyclic}} [D_\rho\ [D_\mu\ D\ ]] \qquad 0 \qquad (\ )$$

In general the covariant derivative can be written as:

$$D_\mu = \partial_\mu - igA_\mu \qquad -(4)$$

where g is a topological charge {10} and where:

$$A_\mu = M^a A_\mu^a . \qquad -(5)$$

Here M^a are the group rotation generators {11}. For U(1) we use the scalar M = -1 {11}. Interferometry is described {10} by the phase factor identity equivalent to eqn. (2), i.e.:

$$\gamma = \exp\left(ig\oint D_\mu dx^\mu\right) := \exp\left(-\frac{ig}{2}\int [D_\mu, D_\nu]\, d\sigma^{\mu\nu}\right). \qquad -(6)$$

This expression simplifies to:

$$\gamma = \exp\left(ig\oint A_\mu dx^\mu\right) := \exp\left(-\frac{ig}{2}\int G_{\mu\nu}\, d\sigma^{\mu\nu}\right) \qquad -(7)$$

where $G_{\mu\nu}$ is the field tensor for any gauge group symmetry:

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \qquad -(8)$$

If the gauge group is U(1) the field tensor simplifies to:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad -(9)$$

and the expression on the left hand side of eqn. (7) simplifies to:

$$\gamma = \exp\left(ig \oint \underline{A} \cdot d\underline{r} \right). \quad -(10)$$

The U(1) holonomy for the Sagnac effect with platform at rest therefore

produces the phase factor:

$$\gamma = \exp\left(ig \oint \underline{A} \cdot d\underline{r} \right) := \exp\left(ig \int \underline{B}^{*} \cdot d\underline{Ar} \right). \quad -(11)$$

However, in the Maxwell Heaviside theory to which this U(1) holonomy corresponds, both $\underline{A}$ and $\underline{B}$ are transverse plane waves in free space. Therefore A is perpendicular to the path r. Therefore:

$$\oint \underline{A} \cdot dr = 0 \quad -(12)$$

both for the A and C loops, and there is no observed phase shift when the platform of the Sagnac interferometer is at rest, contrary to observation {1 - 3

}. The phase factor (11) is invariant under U(1) gauge transformation and is metric invariant, and so cannot describe what is usually referred to as the Sagnac effect, the extra phase shift observed when the platform is rotated {1 - 3}.

3. DESCRIPTION OF THE SAGNAC EFFECT WITH AN O(3) HOLONOMY.

The O(3) holonomy is a round trip in space-time using parallel transport with O(3) covariant derivatives. The internal space of the gauge field theory is the physical three dimensional space described by the rotation group O(3) using a complex basis ((1), (2), (3)) {5 - 10}. The latter is based on the empirical existence of circular polarization: the indices (1) and (2) correspond to left and right handed circularly polarized waves and (3) is defined {5 - 10} by the unit vector algebra:

$$\underline{e}^{(1)} \times \underline{e}^{(2)} = i\, \underline{e}^{(3)*} \quad - (13)$$

in cyclic permutation. In the Sagnac effect the O(3) holonomy results in the phase factors:

$$\gamma = \exp\left(ig \oint \underline{A}^{(i)} \cdot d\underline{r}\right) := \exp\left(ig \int \underline{B}^{(i)} \cdot d\underline{Ar}\right)$$

$$i = 1, 2, 3 \quad - (14)$$

one for each index (1), (2) and (3). However, by definition:

$$\oint \underline{A}^{(i)} \cdot d\underline{r} = 0 \; ; \qquad -(15)$$

$$i = 1 \text{ and } 2$$

so the Sagnac effect when the platform is at rest is described by the phase factor:

$$\gamma = \exp\left(ig \oint \underline{A}^{(3)} \cdot d\underline{r}\right) := \exp\left(ig \int \underline{B}^{(3)} \cdot d\underline{Ar}\right) \qquad -(16)$$

in which $A^{(3)}$ is parallel to the path. The topological charge is defined {5 - 10} by:

$$g = \frac{\kappa}{A^{(0)}} \qquad -(17)$$

where $A^{(0)}$ is the scalar magnitude of $\underline{A}^{(3)}$. Therefore eqn. (16) becomes:

$$\gamma = \exp\left(i \oint \underline{\kappa}^{(3)} \cdot d\underline{r}\right) := \exp\left(ig \int \underline{B}^{(3)} \cdot d\underline{Ar}\right) \qquad -(18)$$

in

the left hand side of this equation is a line integral which changes sign on application of T. The empirically observed phase shift when the platform is at rest is therefore given precisely by eqn. (18) to be:

$$\phi = 2\underline{\kappa}^{(3)} \cdot \underline{r} . \qquad -(19)$$

The $\underline{B}^{(3)}$ field component on the right hand side of eqn. (18) is generated from parallel transport with O(3) covariant derivatives, specifically from the commutator {5 - 10} :

$$\underline{B}^{(3)*} := -ig\,\underline{A}^{(1)} \times \underline{A}^{(2)} . \qquad -(20)$$

In general gauge field theory a gauge transformation is a rotation in the internal gauge space, a rotation defined by:

$$S = \exp\left(i M^a \Lambda^a (x^\mu)\right) \qquad -(21)$$

where M^a are the group rotation generators and Λ^a are angles. In the O(3) gauge field theory with a physical internal space with basis ((1), (2), (3)) the rotation in the internal space is a physical rotation, corresponding to the rotation of the Sagnac platform. This rotation, or gauge transformation, results in {14} :

$$A_\mu^{(3)} \rightarrow A_\mu^{(3)} + \frac{1}{g} \partial_\mu \Lambda^{(3)} \quad -(22)$$

whose time-like part is:

$$\omega \rightarrow \omega + \Omega \quad -(23)$$

where Ω is the angular frequency of rotation of the platform. Using eqns. (17) and (20) the right hand side of eqn. (18) canbe written as:

$$\gamma = \exp\left(i \frac{\omega g^3}{c} Ar \right) \quad (24)$$

and eqn. (23) produces {14} the correct phase shift observed empirically {1 - 3} in the Sagnac effect when the platform is rotated:

$$\Delta\phi = \frac{4\omega\Omega Ar}{c^2}. \quad -(25)$$

The Sagnac effect is therefore correctly described by the O(3) holonomy and is essentially the frequency shift, eqn. (23). The frequency shift is independent of whether the observer is on or off the platform, as observed { 1 – 3 }, and the area A_r can be any shape, as observed { 1 – 3 }. The correct description emerges from the fact that the internal gauge space is a physical space and therefore a gauge transformation is a physical rotation resulting in eqn. (23), a frequency shift.

4. SAGNAC EFFECT IN MATTER WAVES WITH AN O(3) HOLONOMY.

Recently Hasselbach et al. { 18a } have observed the Sagnac effect with electron waves. The phase shift caused by the rotating platform is the same as that observed with electromagnetic waves, and is eqn. (25). In this section this significant result is explained using the same theoretical structure as in section three, i.e. using an O(3) holonomy. The theoretical explanation and experimental result { 18a } suggests that the Sagnac effect is independent of the property of the wave being used for its observation, and should be the same for all matter waves, for example neutron, atomic and molecular matter waves. This result implies that the Sagnac effect is determined only by the rate at which the platform is rotated, i.e. is determined by the angular frequency Ω .

~~The effect is therefore compatible with waves corresponding~~

The theoretical explanation of the effect observed by Hasselbach et al. {15a} starts with the O(3) invariant energy momentum tensor:

$$\underline{p}^{\mu} = p^{\mu(1)}\underline{e}^{(1)} + p^{\mu(2)}\underline{e}^{(2)} + p^{\mu(3)}\underline{e}^{(3)}$$
$$= \hbar\left(\kappa^{\mu(1)}\underline{e}^{(1)} + \kappa^{\mu(2)}\underline{e}^{(2)} + \kappa^{\mu(3)}\underline{e}^{(3)}\right) \quad - (26)$$

where

$$\omega^2 = c^2\kappa^2 + \frac{m_0^2 c^4}{\hbar^2} . \quad - (27)$$

Here $\hbar$ is the Dirac constant, and m_0 the rest mass of the particle concomitant with the wave. In condensed notation both p_μ and κ_μ are governed by a gauge transformation:

$$p_\mu \rightarrow S p_\mu S^{-1} - i\left(\partial_\mu S\right) S^{-1} \quad - (28)$$

and similarly for κ_μ. For rotation about the Z axis we obtain the result:

$$\kappa^{0(3)} \rightarrow \kappa^{0(3)} \pm \partial^0 \alpha \quad - (29)$$

which is the same as:

$$\omega \rightarrow \omega \pm \Omega \quad - (30)$$

The holonomy difference with platform at rest for anticlockwise (A) and clockwise (C) loops is:

$$\Delta\gamma = \exp\left(2i\kappa^2 A_r\right) \quad — (31)$$

where, from eqn. (27),

$$\kappa^2 = \frac{\omega^2}{c^2} - \frac{m_0^2 c^4}{\hbar^2} \quad — (32)$$

The extra holonomy difference due to the rotating platform is, from eqn. (30):

$$\Delta\Delta\gamma = \exp\left(\frac{4i\omega\Omega A_r}{c^2}\right) \quad — (33)$$

giving again eqn. (25), as observed.

5. SAGNAC EFFECT FOR PHOTON WITH REST MASS.

The explanation of the Sagnac effect for the photon with rest mass is the same as that for the electron in section four and is compatible with the explanation given by Vigier { 13) of the Sagnac effect and Langevin paradox using finite photon mass. However, Vigier did not use gauge theory but rather a kinematic explanation. The gauge theory used in this paper has the advantage of demonstrating the compatibility of photon mass with the $\underline{B}^{(3)}$ field. Both can be used as an explanation of the Sagnac effect. The $B^{(3)}$ field has the additional advantage of being the touchstone for the explanation of all interferometric effects through the O(3) invariant equation (16); and also of providing an O(3) invariant explanation of the Aharonov Bohm effect { 15 }.

ACKNOWLEDGMENTS

Funding for individual member laboratories of AIAS is acknowledged together with funding from the U.S. Department of Energy in the form of supercomputer time and the website:

http://www.ott.doe.gov/electromagnetic/

which contains material on papers and review articles produced by AIAS.

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