PROOF OF THE TETRAD POSTULATE FROM FIRST PRINCIPLES

Consider the following basic properties of the tetrad:

$$q^b_\nu q^\nu_b = 1 \qquad -(1)$$

$$q^a_\mu q^\mu_a = 1 \qquad -(2)$$

$$q^\mu_a q^a_\nu = \delta^\mu_\nu \qquad -(3)$$

$$q^a_\mu q^\mu_b = \delta^a_b \qquad -(4)$$

where δ^μ_ν and δ^a_b are Kronecker deltas. We now differentiate eqns (1) to (4) covariantly using the Leibnitz Theorem:

$$q^\nu_b D_\rho q^b_\nu + q^b_\nu D_\rho q^\nu_b = 0 \qquad -(5)$$

$$q^a_\mu D_\rho q^\mu_a + q^\mu_a D_\rho q^a_\mu = 0 \qquad -(6)$$

$$q^\mu_a D_\rho q^a_\nu + q^a_\nu D_\rho q^\mu_a = 0 \qquad -(7)$$

$$q^a_\mu D_\rho q^\mu_b + q^\mu_b D_\rho q^a_\mu = 0. \qquad -(8)$$

Rearranging dummy indices in eqn (5) $(a \rightarrow b, \mu \rightarrow \nu)$

$$q^\mu_a D_\rho q^a_\mu + q^b_\nu D_\rho q^\nu_b = 0. \qquad -(9)$$

Rearranging dummy indices in eqn (8) $(\mu \rightarrow \nu)$:

$$q^\mu_b D_\rho q^a_\mu + q^a_\nu D_\rho q^\nu_b = 0. \qquad -(10)$$

Multiply eqn (9) by q^a_μ:

$$D_\rho q^a_\mu + q^a_\mu q^b_\nu D_\rho q^\nu_b = 0 \qquad -(11)$$

2) multiply eqn (10) by q^{μ}_{b}:

$$D_\rho q^a_\mu + q^b_\mu q^a_\nu D_\rho q^\nu_b = 0 \quad - (12)$$

It is seen that eqn (11) is of the form:

$$x + ay = 0 \quad - (13)$$

and eqn (12) is of the form:

$$x + by = 0 \quad - (14)$$

where:

$$a \neq b. \quad - (15)$$

The only possible solution is:

$$x = 0 \quad - (16)$$

$$y = 0. \quad - (17)$$

This means:

$$\boxed{D_\rho q^a_\mu = 0} \quad - (18)$$

$$D_\rho q^\nu_b = 0. \quad - (19)$$

Eqn (18) is the tetrad postulate of Cartan. It is true for all connections because no restriction on the connection is used in deriving it.