1)

TENSOR REPRESENTATION OF THE EVANS HOMOGENEOUS FIELD EQUATION (HE)

In tensor notation the HE is:

$$\partial_\mu F^a_{\nu\rho} + \partial_\nu F^a_{\rho\mu} + \partial_\rho F^a_{\mu\nu}$$

$$= R^a{}_{b\mu\nu} A^b_\rho + R^a{}_{b\nu\rho} A^b_\mu + R^a{}_{b\rho\mu} A^b_\nu \quad -(1)$$

$$- \omega^a_{\mu b} F^b_{\nu\rho} - \omega^a_{\nu b} F^b_{\rho\mu} - \omega^a_{\rho b} F^b_{\mu\nu}$$

This is equivalent to the vector notation:

$$d \wedge F = R \wedge A - \omega \wedge F. \quad -(2).$$

For the purpose of engineering eqn. (1) is, to an excellent approximation:

$$\partial_\mu F^a_{\nu\rho} + \partial_\nu F^a_{\rho\mu} + \partial_\rho F^a_{\mu\nu} = 0 \quad -(3)$$

Eqn (3) gives the Gauss law applied to magnetism:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(4)$$

and the Faraday Law of induction:

$$\underline{\nabla} \times \underline{B}^a + \frac{\partial \underline{E}^a}{\partial t} = \underline{0}, \quad -(5)$$

for all polarization indices a.

Notes

1) The influence of gravitation on electromagnetism and vice versa must be computed from the right hand side of eqn (1). This influence must lead to a violation of the well known laws (4) and (5).

2) Eqn (1) must in general be solved simultaneously with the inhomogeneous Evans field equation.

3) Eqn (1) can be written as:

$$\partial^\mu \tilde{F}^a_{\mu\nu} = \mu_0 \tilde{j}^a_\nu \sim 0 \quad -(6)$$

and for each index a this is the familiar Maxwell Heaviside field equation. In general $\tilde{F}^a_{\mu\nu}$ is the Hodge dual of $F^a_{\mu\nu}$ in Evans spacetime, and $\tilde{j}^a_\nu$ is the Hodge dual of the charge-current density three-form defined by the right hand side of eqn. (1), i.e.:

$$\partial_\mu F^a_{\nu\rho} + \partial_\nu F^a_{\rho\mu} + \partial_\rho F^a_{\mu\nu} = \mu_0 \left(j^a_{\mu\nu\rho} + j^a_{\nu\rho\mu} + j^a_{\rho\mu\nu} \right) \quad -(7)$$

where:

$$j^a_{\mu\nu\rho} = \frac{1}{\mu_0} \left(R^a{}_{b\mu\nu} A^b_\rho - \omega^a_{\mu b} F^b_{\nu\rho} \right) \quad -(8)$$

$\vdots$

Thus:

$$\tilde{j}^{a}_{\sigma} = \frac{1}{6}\epsilon^{\mu\nu\rho}{}_{\sigma}\, j^{a}_{\mu\nu\rho} = \frac{1}{6}\epsilon^{\nu\rho\mu}{}_{\sigma}\, j^{a}_{\nu\rho\mu} = \frac{1}{6}\epsilon^{\rho\mu\nu}{}_{\sigma}\, j^{a}_{\rho\mu\nu} \qquad (9)$$

and:

$$\tilde{F}^{a}_{\mu\sigma} = \frac{1}{2}\epsilon^{\nu\rho}{}_{\mu\sigma} F^{a}_{\nu\rho} \qquad (10)$$

$$\tilde{F}^{a}_{\nu\sigma} = \frac{1}{2}\epsilon^{\rho\mu}{}_{\nu\sigma} F^{a}_{\rho\mu} \qquad (11)$$

$$\tilde{F}^{a}_{\rho\sigma} = \frac{1}{2}\epsilon^{\mu\nu}{}_{\rho\sigma} F^{a}_{\mu\nu}. \qquad (12)$$

In computing these Hodge duals the correct general definition maps from a p-form of differential geometry to an (n − p)-form of differential geometry in a general n dimensional manifold. The general 4-D manifold is Evans spacetime. Thus:

$$\tilde{X}_{\mu_1\ldots\mu_{n-p}} = \frac{1}{p!}\epsilon^{\nu_1\ldots\nu_p}{}_{\mu_1\ldots\mu_{n-p}} X_{\nu_1\ldots\nu_p}. \qquad (13).$$

The general Levi-Civita symbol is:

$$\epsilon'_{\mu_1\mu_2\ldots\mu_n} = \begin{cases} 1 & \text{even} \\ -1 & \text{odd} \\ 0 & \text{otherwise.} \end{cases} \qquad (14)$$

4) The Levi-Civita tensor used in eq. (13) is:

$$\epsilon_{\mu_1 \mu_2 \ldots \mu_n} = \left(|g| \right)^{1/2} \epsilon'_{\mu_1 \mu_2 \ldots \mu_n} \quad - (15)$$

where $|g|$ is the numerical value of the determinant of the metric tensor $g_{\mu\nu}$.

4) The field tensor is defined by the torsion form:

$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu} \quad - (16)$$

and the potential is defined by the tetrad form:

$$A^a_\mu = A^{(0)} q^a_\mu . \quad - (17)$$

5) The metric is factorized into a dot product of tetrads:

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab} \quad - (18)$$

where η_{ab} is the diag $(1, -1, -1, -1)$ metric of the tangent bundle spacetime, a Minkowski, or "flat" spacetime.

6) The gamma and spin connection are related by the tetrad postulate (see eqn. (25)):

$$D_\mu q^a_\nu = 0 . \quad - (19)$$

5) 7) The first and second structure equations define the torsion and Riemann forms or tensors.

The torsion tensor is:

$$T^{\lambda}_{\mu\nu} = q^{\lambda}_{a} T^{a}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu} \quad -(20)$$

so vanishes for the Christoffel connection:

$$\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\nu\mu}. \quad -(21)$$

The Riemann tensor is:

$$R^{\sigma}_{\lambda\nu\mu} = q^{\sigma}_{a} q^{b}_{\lambda} R^{a}_{b\nu\mu}. \quad -(22)$$

Here:

$$R^{a}_{b\nu\mu} = \partial_{\nu}\omega^{a}_{\mu b} - \partial_{\mu}\omega^{a}_{\nu b} + \omega^{a}_{\nu c}\omega^{c}_{\mu b} - \omega^{a}_{\mu c}\omega^{c}_{\nu b} \quad -(23)$$

and

$$R^{\lambda}_{\sigma\nu\mu} = \partial_{\nu}\Gamma^{\sigma}_{\mu\lambda} - \partial_{\mu}\Gamma^{\sigma}_{\nu\lambda} + \Gamma^{\sigma}_{\nu\rho}\Gamma^{\rho}_{\mu\lambda} - \Gamma^{\sigma}_{\mu\rho}\Gamma^{\rho}_{\nu\lambda} \quad -(24)$$

where (tetrad postulate):

$$\partial_{\mu} q^{a}_{\nu} + \omega^{a}_{\mu b} q^{b}_{\nu} = \Gamma^{\lambda}_{\mu\nu} q^{a}_{\lambda}. \quad -(25)$$

The Evans spacetime is defined by ~~of~~ structure equations and Bianchi identities of differential geometry.