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THE HOMOGENEOUS FIELD EQUATION

Barebones Notation

$$D \wedge F = R \wedge A \qquad -(1)$$

Tangent Bundle Notation

$$D \wedge F^a = R^a{}_b \wedge A^b \qquad -(2)$$

Complete Index Notation

$$(D \wedge F^a)_{\mu\nu\rho} = (R^a{}_b \wedge A^b)_{\mu\nu\rho} \qquad -(3)$$

Spin Connection Notation

$$(d \wedge F^a + \omega^a{}_b \wedge F^b)_{\mu\nu\rho} = (R^a{}_b \wedge A^b)_{\mu\nu\rho} \qquad -(4)$$

Tensor Notation

$$(d \wedge F)^a_{\mu\nu\rho} = \partial_\mu F^a_{\nu\rho} + \partial_\nu F^a_{\rho\mu} + \partial_\rho F^a_{\mu\nu} \qquad -(5a)$$

$$(\omega \wedge F)^a_{\mu\nu\rho} = \omega^a_{\mu b} F^b_{\nu\rho} + \omega^a_{\nu b} F^b_{\rho\mu} + \omega^a_{\rho b} F^b_{\mu\nu} \qquad -(5b)$$

$$(R \wedge A)^a_{\mu\nu\rho} = R^a{}_{b\mu\nu} A^b_\rho + R^a{}_{b\nu\rho} A^b_\mu + R^a{}_{b\rho\mu} A^b_\nu \qquad -(5c)$$

Eqs. (5) give the most complete expression of ~~R~~ homogeneous field equation, i.e. :

$$\partial_\mu F^a_{\nu\rho} + \partial_\nu F^a_{\rho\mu} + \partial_\rho F^a_{\mu\nu} + \omega^a_{\mu b} F^b_{\nu\rho} + \omega^a_{\nu b} F^b_{\rho\mu} + \omega^a_{\rho b} F^b_{\mu\nu}$$
$$= R^a{}_{b\mu\nu} A^b_\rho + R^a{}_{b\nu\rho} A^b_\mu + R^a{}_{b\rho\mu} A^b_\nu \quad -(6)$$

Maxwell-Heaviside Limit

The spin connection and Riemann forms vanish, so:

$$\partial_\mu F^a_{\nu\rho} + \partial_\nu F^a_{\rho\mu} + \partial_\rho F^a_{\mu\nu} = 0. \quad -(7)$$

The tangent bundle is undefined, so:

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0. \quad -(8)$$

Eqn. (8) is the homogeneous field equation of the MH theory. Eqn (8) cannot describe the effect of gravitation on electromagnetism and vice-versa, because it is a flat spacetime equation of special relativity.

Hodge Dual of the Homogeneous Field Equation.

The Hodge dual of the homogeneous field equation is a re-expression of the Bianchi identity and therefore contains the same information. In the general 4-D manifold the Hodge dual must be precisely defined as follows.

3) The general definition of the Hodge dual is given by Carroll:

$$*\tilde{X}_{\mu_1 \dots \mu_{n-p}} = \frac{1}{p!} \epsilon^{\nu_1 \dots \nu_p}{}_{\mu_1 \dots \mu_{n-p}} X_{\nu_1 \dots \nu_p} \quad -(9)$$

In a general n-dimensional manifold eq. (9) maps from a p-form of differential geometry to an (n-p)-form. The general Levi-Civita symbol is defined in any manifold to be:

$$\tilde{\epsilon}_{\mu_1 \mu_2 \dots \mu_n} = \begin{cases} 1 & \text{if } \mu_1 \mu_2 \dots \mu_n \text{ is an even permutation} \\ -1 & \text{" " " odd "} \\ 0 & \text{otherwise} \end{cases} \quad -(10)$$

The Levi-Civita tensor used in the definition of the Hodge dual is:

$$\epsilon_{\mu_1 \mu_2 \dots \mu_n} = (|g|)^{1/2} \tilde{\epsilon}_{\mu_1 \mu_2 \dots \mu_n} \quad -(11)$$

where $|g|$ is the numerical magnitude of the determinant of the metric. In a four-dimensional manifold a two-form is dual to a two-form:

$$*\tilde{X}_{\mu_1 \mu_2} = \frac{1}{2} \epsilon^{\nu_1 \nu_2}{}_{\mu_1 \mu_2} X_{\nu_1 \nu_2} \quad -(12)$$

Indices are raised and lowered on the Levi-Civita tensor by use of the metric tensor. The latter is normalized by:

$$g^{\mu\nu} g_{\mu\nu} = 4. \quad -(13)$$

4) Therefore we have results such as:

$$\widetilde{X}_{\mu\nu} = \frac{1}{2} \epsilon^{\rho\sigma}{}_{\mu\nu} X_{\rho\sigma} \quad -(14)$$

with, for example:

$$\epsilon_{\sigma\mu\nu\rho} = g_{\sigma\kappa} \epsilon^{\kappa}{}_{\mu\nu\rho}. \quad -(15)$$

We may ~~ther~~ rewrite eqn. (4) as:

$$\boxed{D^{\mu} \widetilde{F}^{a}_{\mu\nu} = \mu_0 \widetilde{j}^{a}_{\nu}} \quad -(16)$$

Note that eqns (4) and (16) contain the same information, they are both expressions of the homogeneous field equation of objective or generally covariant physics.

THE INHOMOGENEOUS FIELD EQUATION

The homogeneous field equation (4) or (16) is the generally covariant form of the Gauss Law applied to ~~the~~ magnetism and the Faraday Law of induction. The inhomogeneous field equation is deduced from eqn. (1) as follows:

$$\boxed{D \wedge \widetilde{F} = \widetilde{R} \wedge A} \quad -(17)$$

5) The complete description is therefore:

$$\boxed{\begin{aligned} D \wedge F &= R \wedge A = \mu_0 j \\ D \wedge \tilde{F} &= \tilde{R} \wedge A = \mu_0 J \end{aligned}} \quad -(18)$$

Eqs (18) are the generally covariant form of the four fundamental laws of electrodynamics.

In order to CAD / CAM a circuit working from the general 4-D manifold known as "Evans spacetime" eqs (18) must be solved simultaneously. The mathematical problem is one of solving simultaneous partial differential tensor equations with given initial and boundary conditions.

The Standard Model

The equivalents of eqs. (18) in the standard model are:

$$\boxed{\begin{aligned} d \wedge F &= 0 \\ d \wedge \tilde{F} &= \mu_0 J \end{aligned}} \quad -(19)$$

Eqs (19) are equations of a flat or Minkowski

6) spacetime. Eqns (19) are not objective equations of physics because they are not equations of general relativity. There is no indication in eqns (19) that J is derived from the wedge product of the Riemann and tetrad forms. The Evans field equations (18) ~~reinstate~~ show that

$$\boxed{J = \frac{1}{\mu_0} A^{(0)} \tilde{R} \wedge q} \quad -(20)$$

Eqn (20) shows that Evans spacetime is a source of electric charge / current density.

It is therefore very important to evaluate J numerically. For a given potential:

$$A = A^{(0)} q \quad -(21)$$

and given curvature:

$$R = D \wedge \omega \quad -(22)$$

we need to calculate J. In eqn. (22) ω is the spin connection, related to the Christoffel connection.

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SUMMARY OF THE UNIFIED FIELD THEORY

Any situation in field theory is described by:

$$D \wedge F^a = R^a{}_b \wedge A^b \quad -(23)$$

$$D \wedge \tilde{F}^a = \tilde{R}^a{}_b \wedge A^b \quad -(24)$$

where:

$$F^a = A^{(0)} T^a \quad -(25)$$

$$A^a = A^{(0)} q^a \quad -(26)$$

and:

$$D \wedge F^a = d \wedge F^a + \omega^a{}_b \wedge F^b. \quad -(27)$$

The fundamental charge-current three-forms are defined by:

$$j^a = \frac{1}{\mu_0} R^a{}_b \wedge A^b \quad -(28)$$

$$J^a = \frac{1}{\mu_0} \tilde{R}^a{}_b \wedge A^b. \quad -(29)$$

In eqs (23) to (29):

$A^{(0)}$ = fundamental potential magnitude in volts.

q^a = vector valued tetrad one-form

T^a = vector valued torsion two-form

$R^a{}_b$ = tensor valued curvature two-form.

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$\omega^a{}_b$ = spin-connection one-form

F^a = vector valued electromagnetic field two-form

A^a = vector valued electromagnetic potential one-form.

j^a = homogeneous current three-form.

J^a = inhomogeneous current three-form.

$d\wedge$ = exterior derivative

$D\wedge$ = covariant exterior derivative

μ_0 = permeability in vacuo (SI).

The above equations are in SI units.

$\tilde{F}^a$ is the Hodge dual of F^a in the general 4-D manifold (Evans spacetime)

$\tilde{R}^a{}_b$ is the Hodge dual of $R^a{}_b$ in Evans spacetime.

The following are well-known limiting forms of the Evans field theory.

Einstein Field Theory of Gravitation

This limit is defined by:

$$F^a = 0 \quad -(30)$$

$$R^a{}_b \wedge A^b = 0 \quad -(31)$$

$$\tilde{F}^a = 0 \quad -(32)$$

$$\tilde{R}^a{}_b \wedge A^b = 0 \quad -(33)$$

Eqn (31) implies that the Christoffel symbol is symmetric in its lower two indices. This self-consistently implies:

$$T^a(\text{Einstein}) = 0. \quad -(34)$$

Self-consistently ~~therefore~~, in the Einstein field theory of gravitation, there is no electromagnetic field present. In this theory the torsion tensor is zero because it is defined as the difference:

$$T^\kappa_{\mu\nu} = \Gamma^\kappa_{\mu\nu} - \Gamma^\kappa_{\nu\mu} = 0. \quad -(35)$$

Metrics that obey condition (35) cannot be used in a unified field theory, they can only be used to describe gravitation.

Maxwell Heaviside Field Theory of Electromagnetism

This is the limit described by:

$$d \wedge F = 0 \quad - (36)$$

$$d \wedge \tilde{F} = \mu_0 J \quad - (37)$$

Therefore:

$$R^a{}_b \wedge A^b = 0 \quad - (38)$$

$$\omega^a{}_b \wedge F^b = 0 \quad - (39)$$

$$\omega^a{}_b \wedge \tilde{F}^b = 0 \quad - (40)$$

The Evans spacetime reduces to Minkowski spacetime, so $D\wedge$ is replaced by $d\wedge$. In the original nineteenth century Maxwell Heaviside theory the field is an entity superimposed on a flat Euclidean 3-D space and the concept of time is distinct from that of space. The existence of the tangent bundle index a is not recognized, neither is that of the spin connection and curvature form $R^a{}_b$. The current J is essentially empirical in MH field theory.