1)

INHOMOGENEOUS LAWS AND THE SCHWARZSCHILD METRIC (SM)

The SM is a solution of the Einstein Hilbert field equation for a vacuum, i.e. of:

$$R_{\mu\nu} = 0 \qquad -(1)$$

From Birkhoff's Theorem the SM is the unique spherically symmetric solution of eqn (1), and is needed to define the spacetime around a gravitating object such as an electron, the earth, the sun, a pulsar or black hole. In the special relativistic limit the SM approaches the Minkowski metric. The latter is also spherically symmetric, so the description of Minkowski metric as "flat spacetime" is misleading.

There are in general eight non-zero elements of the Riemann tensor for the SM:

$$R^{0}{}_{101},\ R^{0}{}_{202},\ R^{0}{}_{303},\ R^{0}{}_{212}$$
$$R^{0}{}_{313},\ R^{1}{}_{212},\ R^{1}{}_{313},\ R^{2}{}_{323}$$

where:

$$R_{0101} = -R_{1001} \qquad -(2)$$

etc.

In the Coulomb Law of the inhomogeneous Evans field equation the non-vanishing elements are

$$R^{a}{}_{1}{}^{10},\ R^{a}{}_{2}{}^{20},\ R^{a}{}_{3}{}^{30}$$

where

$$R_{a110} = -R_{1a10} \qquad -(3)$$

etc.

2) Therefore it is seen that:

$$a = 0 \quad \left(\text{(Coulomb Law} \right) \quad - (4)$$

which is:

$$\boxed{\underline{\nabla} \cdot \underline{E}^{0} = -\phi^{(0)} \left(R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30} \right)}$$

$$- (5)$$

In the Ampère Maxwell law the non-vanishing elements are:

$$R^{a}{}_{0}{}^{10} \quad R^{a}{}_{2}{}^{12} \quad R^{a}{}_{3}{}^{13}$$

$$R^{a}{}_{0}{}^{20} \quad R^{a}{}_{1}{}^{21} \quad R^{a}{}_{3}{}^{23}$$

$$R^{a}{}_{0}{}^{30} \quad R^{a}{}_{1}{}^{31} \quad R^{a}{}_{2}{}^{32}$$

where

$$R_{a010} = -R_{0a10} \quad - (6)$$

and so on.

For the SM ~~is~~ it may be shown that

$$R^{0}{}_{212} = 0 \quad - (7)$$

$$R^{0}{}_{313} = 0.$$

and the only six non-vanishing elements are:

$$R^{0}{}_{101}, \quad R^{1}{}_{212}, \quad R^{1}{}_{313}$$

$$R^{2}{}_{323}, \quad R^{0}{}_{202}, \quad R^{0}{}_{303}$$

where:

$$R_{1212} = -R_{2112} \quad - (8)$$

etc.

It is seen from eqn (6) and the antisymmetry of the Riemann tensor in its first two indices that the Ampère Maxwell law is:

$$a = 1, 2, 3 \quad \text{(Ampère Maxwell)}. \quad -(9)$$

The Ampère Maxwell equations are therefore:

$$\underline{\nabla} \times \underline{B}^a = \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} + \mu_0 \underline{J}^a \quad -(10)$$

where:

$$\underline{J}^a = J^a_x \underline{i} + J^a_y \underline{j} + J^a_z \underline{k} \quad -(11)$$

and

$$J^a_x = -\frac{A^{(0)}}{\mu_0} \left(R^a{}_0{}^{10} + R^a{}_2{}^{12} + R^a{}_3{}^{13} \right) \quad -(12)$$

$$J^a_y = -\frac{A^{(0)}}{\mu_0} \left(R^a{}_0{}^{20} + R^a{}_1{}^{21} + R^a{}_3{}^{23} \right) \quad -(13)$$

$$J^a_z = -\frac{A^{(0)}}{\mu_0} \left(R^a{}_0{}^{30} + R^a{}_1{}^{31} + R^a{}_2{}^{32} \right). \quad -(14)$$

Using symmetry and eqn. (8) eqns (12) − (14) simplify to solutions such as:

$$J^1_x = -\frac{A^{(0)}}{\mu_0} \left(R^1{}_0{}^{10} + R^1{}_2{}^{12} + R^1{}_3{}^{13} \right) \quad -(15)$$

$$J^1_y = \cancel{-\frac{A^{(0)}}{\mu_0}} \; 0 \quad -(16)$$

$$J^1_z = 0 \quad -(17)$$

4)

$$J^2_x = \cancel{-\frac{A^{(0)}}{\mu_0}} \; 0 \qquad -(18)$$

$$J^2_y = -\frac{A^{(0)}}{\mu_0}\left(R^2{}_0{}^{20} + R^2{}_1{}^{21} + R^2{}_3{}^{23}\right) \qquad -(19)$$

$$J^2_z = 0 \qquad -(20)$$

$$J^3_x = 0 \qquad -(21)$$

$$J^3_y = 0 \qquad -(22)$$

$$J^3_z = -\frac{A^{(0)}}{\mu_0}\left(R^3{}_0{}^{30} + R^3{}_1{}^{31} + R^3{}_2{}^{32}\right) \qquad -(23)$$

Therefore there are only three non-vanishing components of current:

$$\boxed{\begin{aligned} J^1_x &= -\frac{A^{(0)}}{\mu_0}\left(R^1{}_0{}^{10} + R^1{}_2{}^{12} + R^1{}_3{}^{13}\right) \\ J^2_y &= -\frac{A^{(0)}}{\mu_0}\left(R^2{}_0{}^{20} + R^2{}_1{}^{21} + R^2{}_3{}^{23}\right) \\ J^3_z &= -\frac{A^{(0)}}{\mu_0}\left(R^3{}_0{}^{30} + R^3{}_1{}^{31} + R^3{}_2{}^{32}\right) \end{aligned}} \qquad -(24)$$

5) <u>Discussion</u>

It is seen that the Coulomb Law (5) and the Ampère Maxwell law (10) are defined in the SM by elements of the Riemann tensor. The indices $a = 1, 2$ and 3 of the Ampère Maxwell law are such that there are only three non-vanishing elements of current, defined by eqn (24). The only index that appears in the Coulomb law is $a = 0$, indicating that the Coulomb law is a function of time t in eqn (5). In an expanding cosmology the distance between two charges become progressively larger as the universe expands. The $a = 0$ index appears only for centrally directed fields such as a static electric field. The magnetic field is always a spin defined by the indices 1, 2 and 3 of space.

This proves that the IE and HE are fully compatible with the Schwarzschild metric.