1)

THE COULOMB AND AMPÈRE-MAXWELL LAWS IN THE EVANS FIELD THEORY

These important laws are derived in the Evans field theory from the Bianchi identity:

$$D \wedge T^a = R^a{}_b \wedge q^b \quad -(1)$$

i.e.:

$$d \wedge T^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \quad -(2)$$

by taking the Hodge duals of T^a and $R^a{}_b$.

Thus:

$$d \wedge \tilde{T}^a = \tilde{R}^a{}_b \wedge q^b - \omega^a{}_b \wedge \tilde{T}^b \quad -(3).$$

The inhomogeneous field equation is therefore:

$$\boxed{\begin{aligned} d \wedge \tilde{F}^a &= \mu_0 J^a \\ &= \tilde{R}^a{}_b \wedge A^b - \omega^a{}_b \wedge \tilde{F}^b \end{aligned}} \quad -(4)$$

The inhomogeneous charge-current density is

$$\boxed{J^a = \frac{1}{\mu_0}\left(\tilde{R}^a{}_b \wedge A^b - \omega^a{}_b \wedge \tilde{F}^b\right)} \quad -(5)$$

For each polarization index a eqn. (4) has the mathematical structure of the well

2) known inhomogeneous Maxwell - Heaviside field equation:

$$d \wedge \tilde{F} = \mu_0 J . \quad - (6)$$

Eqn. (6) is a combination of the Coulomb Law:

$$\boxed{\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\epsilon_0}} \quad - (7)$$

and the Ampère-Maxwell Law:

$$\boxed{\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} + \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t}} \quad - (8)$$

Here ρ is charge density and $\underline{J}$ is current density. In eqns (7) and (8) the usual vector notation of electrical engineering has been used.

From eqn (5) the Evans field theory shows the origin of ρ and $\underline{J}$ in the Evans spacetime, the general four-dimensional manifold of differential geometry. In eqn (8) $\partial \underline{E} / \partial t$ is the Maxwell displacement current.

3) The Hodge duals used in eqn. (5) are defined as follows. First define the metric tensors using the tetrads:

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab} \quad - (9)$$

$$g^c_{\mu\nu} = q^a_\mu \wedge q^b_\nu \quad - (10)$$

$$g^{ab}_{\mu\nu} = q^a_\mu q^b_\nu . \quad - (11)$$

These are respectively the inner, wedge, and outer products of tetrads. Note that in Einstein's field theory of gravitation, only eqn (9) is used. This is the inner, or dot, product of two tetrads.

Next construct the determinants:

$$g_{grav} = |g_{\mu\nu}| \quad - (12)$$

$$g^{(c)}_{em} = |g^c_{\mu\nu}| . \quad - (13)$$

In eqn. (12) g_{grav} is a number, (i.e. scalar), evaluated from the determinant of the matrix $g_{\mu\nu}$.
In eqn. (13) there are determinants for each index c.

4) Now consider the Riemann form $R^a{}_b$ for Einsteinian gravitation. This is defined by $g_{\mu\nu}$ of eqn. (9). Its dual is therefore:

$$\tilde{R}^a{}_b{}_{grav} = \frac{1}{2} |g_{grav}|^{1/2} \epsilon R^a{}_b{}_{grav} \quad - (14)$$

where $|g_{grav}|$ is the modulus or positive value of g_{grav}. In eqn. (14) ϵ is the Levi Civita symbol (with indices suppressed).

Next consider the torsion and Riemann forms for electromagnetism. These are defined by the antisymmetric metric (10) of spinning spacetime. Their duals are therefore defined by:

$$\tilde{R}^a{}_{b\,em} = \frac{1}{2} |g^{(c)}_{em}|^{1/2} \epsilon R^a{}_{b\,em} \quad - (15)$$

$$\tilde{T}^a{}_{em} = \frac{1}{2} |g^{(c)}_{em}|^{1/2} \epsilon T^a{}_{em} \quad - (16)$$

From the Gauss Law applied to electromagnetism:

$$\boxed{\underline{\nabla} \cdot \underline{B} = 0} \quad - (17)$$

and the Faraday Law of induction:

$$\boxed{\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0}} \quad - (18)$$

5) it follows that:

$$R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b. \quad -(19)$$

From eqns (15), (16) and (19):

$$\tilde{R}^a{}_{b\,em} \wedge q^b = \omega^a{}_b \wedge \tilde{T}^b_{em}, \quad -(20)$$

from which it follows that the inhomogeneous field equation is:

$$\boxed{\begin{aligned} d \wedge \tilde{F}^a &= \frac{1}{2} |g_{grav}|^{1/2} \epsilon R^a{}_b \wedge A^b \\ &= \mu_0 J^a \end{aligned}}$$

$$-(21)$$

The inhomogeneous current is therefore:

$$\boxed{J^a = \frac{1}{\mu_0} \cdot \frac{1}{2} |g_{grav}|^{1/2} \epsilon R^a{}_b \wedge A^b}$$

$$-(22)$$

Finally the charge density ρ and the current density $\underline{J}$ are defined by:

$$\tilde{J}^a_\mu = (\rho^a, -c\underline{J}^a). \quad -(23)$$

6) The Coulomb Law in the Evans field theory is therefore:

$$\boxed{\underline{\nabla} \cdot \underline{E}^a = \frac{\rho^a}{\epsilon_0}} \quad -(24)$$

and the Ampère-Maxwell law in the Evans field theory is:

$$\boxed{\underline{\nabla} \times \underline{B}^a = \mu_0 \underline{J}^a + \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t}} \quad -(25).$$

Notes

1) It can be seen from eqn. (22) that ρ^a and $\underline{J}^a$ are proportional to the product of $|g_{\mu\nu}|^{1/2}$ and $R^a{}_b$. This shows that charge density and current density appear in the presence of mass. For a point mass the curvature is infinite.

2) Extra polarization, or tangent bundle, indices a appear in eqns. (24) and (25), showing longitudinal and transverse spatial and timelike polarizations.

MWE.