1)

CALCULATION OF THE EDDINGTON EFFECT IN GENERAL RELATIVITY AND NEWTONIAN DYNAMICS.

1) In Newtonian dynamics consider a mass m subjected to the gravitational pull of the sun, of mass M. The force between these two masses is:

$$\underline{F} = -\frac{mMG}{r^2}\underline{k}. \quad -(1)$$

Integrate eqn (1) to obtain:

$$T = \frac{1}{2}mv^2 = \frac{mMG}{r}. \quad -(2)$$

This gives:

$$v^2 = \frac{2MG}{r} \quad -(3)$$

and:

$$c^2 - v^2 = (c-v)(c+v)$$
$$= c^2\left(1 - \frac{2MG}{rc^2}\right). \quad -(4)$$

If $v \ll c$, then:

$$c - v \sim c\left(1 - \frac{2MG}{rc^2}\right)^{1/2}$$

$$\boxed{c - v \sim c\left(1 - \frac{MG}{rc^2}\right)}. \quad -(5)$$

2) In general relativity the same result is obtained from the Schwarzschild metric:

2)

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1}$$

$$- r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2. \quad -(6)$$

For any mass m the velocity v in the frame of the observer is:

$$v_0 = \frac{dr}{dt} = \frac{\left(1 - \frac{2GM}{rc^2}\right)^{1/2}}{\left(1 - \frac{2GM}{rc^2}\right)^{-1/2}} \frac{dr'}{dt'} \quad -(7)$$

$$v_0 = \left(1 - \frac{2GM}{rc^2}\right) v'. \quad -(8)$$

For a photon its ~~speed~~ appears to be slowed by

$$\boxed{v_0 = \left(1 - \frac{2GM}{rc^2}\right) c}. \quad -(9)$$

Eqn (9) is observed rather than eqn. (5).

To interpret eqn. (7) recall that the length contraction in special relativity is:

$$l = \gamma l' \quad -(10)$$

and that the relativistic momentum is:

$$p = \gamma p'. \quad -(11)$$

The time dilation is:

3)
$$t = \frac{1}{\gamma} t' \quad — (12)$$

The proper time is:
$$\tau = t' \quad — (13)$$

in the ~~observer~~ moving frame, so:
$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$
$$— (14)$$

is the Minkowski metric.

The Schwarzschild metric is ~~the~~ a generalization of the Minkowski metric. Eqn. (7) follows from ~~this~~.