REPRESENTATION OF THE VACUUM ELECTROMAGNETIC FIELD IN TERMS OF LONGITUDINAL AND TIME-LIKE POTENTIALS: CANONICAL QUANTIZATION

ABSTRACT

It is shown that the electromagnetic field in the vacuum can be represented by two scalar potential differences F and G. These are magnitudes of vectors f and g directed in the axis of propagation. Electromagnetic fields are derivable from F and G, which are governed by massless Klein-Gordon equations. Canonical quantization of this equation gives an ensemble of massless bosons (photons) without any of the difficulties associated with canonical quantization of the four potential. An experiment is suggested for the detection of longitudinal and time-like potentials of this type.

INTRODUCTION

The notable mathematical work of Whittaker {1, 2} showed that the electromagnetic field in the vacuum can be represented in terms of only two potential differences, F and G, which obey the Laplace and d'Alembert equations. These are the magnitudes of two longitudinally directed vector potentials f and g in the vacuum. In this paper, it is shown that the conventional vector potential $\boldsymbol{A}$ and the Stratton potential $\boldsymbol{S}$ can be expressed in terms of f and g, so that in general, the four potential A^{μ} has longitudinal and time-like components within the gauge invariant structure used by Whittaker, i.e. the Maxwell-Heaviside theory. The time-like component of A^{μ} can be expressed in terms of F and G, and obeys a massless Klein-Gordon equation. Upon canonical quantization, this gives straightforwardly an ensemble of massless bosons, thus avoiding the formidable difficulties inherent in the canonical quantization of the transverse part of A^{μ} in any gauge {3}. The energy of the electromagnetic field can be recovered from Noether's Theorem and expressed solely in terms of the Whittaker's scalar potentials, which are therefore physical in nature with an overall gauge invariant theory, the Maxwell Heaviside theory. However, f and g are longitudinally directed magnetic fluxes in vacuo and, for a finite beam radius, are accompanied by longitudinal magnetic flux densities in the vacuum. This is a logical extension of Whittaker's analysis, which led to the development of super-potential theory {3, 4}.

ELECTRIC AND MAGNETIC FIELDS IN TERMS OF g AND f.

If $\boldsymbol{k}$ is a unit vector in the propagation (longitudinal) axis of the beam, then f and g are defined as:

$$f = F\boldsymbol{k}; \qquad g = G\boldsymbol{k} \tag{1}$$

Whittaker showed {2} that in the vacuum,

$$\boldsymbol{E} = c\nabla \times (\nabla \times f) + \nabla \times \dot{g} \tag{2}$$

$$\boldsymbol{B} = \frac{1}{c}\nabla \times \dot{f} - \nabla \times (\nabla \times g) \tag{3}$$

where $\boldsymbol{E}$ is the electric field strength and $\boldsymbol{B}$ the magnetic flux density of the beam in S.I. units. The contemporary vector potential $\boldsymbol{A}$, and Stratton potential $\boldsymbol{S}$ {3, 4} are therefore defined by:

$$\boldsymbol{A} = -\nabla \times \boldsymbol{g} + \frac{1}{c}\dot{\boldsymbol{f}} \tag{4}$$

$$\boldsymbol{S} = -c\nabla \times \boldsymbol{f} - \dot{\boldsymbol{g}} \tag{5}$$

Therefore both $\boldsymbol{A}$ and $\boldsymbol{S}$ have longitudinal and transverse components in the vacuum, within the overall gauge invariant Maxwell-Heaviside structure. The transverse and longitudinal parts of $\boldsymbol{A}$ for example are respectively:

$$\boldsymbol{A}_T = -\nabla \times \boldsymbol{g} \tag{6}$$

$$\boldsymbol{A}_L = \frac{1}{c}\dot{\boldsymbol{f}}. \tag{7}$$

Therefore Whittaker showed that $\boldsymbol{A}$ is a combination of two more fundamental functions, $\boldsymbol{g}$ and $\boldsymbol{f}$, and that $\boldsymbol{g}$ and $\boldsymbol{f}$ can have physical effects in general. There exists in general a longitudinal four potential:

$$A_L^{\mu} = \left(-c\nabla \cdot \boldsymbol{f},\, \dot{\boldsymbol{f}}\right) \tag{8}$$

within the structure of the Maxwell-Heaviside theory on the classical level. By definition, the longitudinal four potential is given by:

$$A_L^{\mu} = \left(-c\frac{\partial F}{\partial Z}, \frac{\partial F}{\partial t}\boldsymbol{k}\right) \tag{9}$$

where F obeys the d'Alembert wave equation {1, 2} in the vacuum. In the special case where the transverse $\boldsymbol{A}_T$ consists of plane waves, it follows that:

$$F = iG. \tag{10}$$

Therefore the longitudinal $\dot{\boldsymbol{f}}$ and time-like $-c\nabla \cdot \boldsymbol{f}$ cannot be zero because, from eqns. (2) and (3), all $\boldsymbol{E}$ and $\boldsymbol{B}$ components would vanish. This rules out the use of the radiation gauge and the Coulomb gauge in the vacuum, and any canonical quantization procedure based on these gauges.

ANALYTICAL EXPRESSIONS FOR *F* AND *G*

The transverse part of $\boldsymbol{A}$ and the magnetic plane wave $\boldsymbol{B}$ are given from the scalar magnetic flux magnitude G

$$G = \frac{A^{(0)}}{\sqrt{2}}\left(X - iY\right)e^{i(\omega t - \kappa Z)} \tag{11}$$

which obeys the d'Alembert equation. We have:

$$\boldsymbol{A}_T = -\nabla \times \boldsymbol{g} = \frac{A^{(0)}}{\sqrt{2}}\left(i\boldsymbol{i} + \boldsymbol{j}\right)e^{i(\omega t - \kappa Z)} \tag{12}$$

$$B = \nabla \times A_T = \frac{B^{(0)}}{\sqrt{2}}(ii + j)e^{i(\omega t - \kappa Z)}. \tag{13}$$

The longitudinal part of A is given therefore by:

$$A_L = \frac{1}{c}\dot{G}k = -\kappa \frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)}k \tag{14}$$

and its time-like part by:

$$\phi_L = -\omega \frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)}. \tag{15}$$

The beam radius R is defined by the equation of the circle:

$$X^2 + Y^2 = R^2. \tag{16}$$

Therefore, by following Whittaker's logic, we find that there exist, in general, non-zero time-like and longitudinal parts of A^μ. These obey the Lorenz condition:

$$\partial_\mu A_L^\mu = \nabla \bullet A_L + \frac{1}{c^2}\frac{\partial \phi_L}{\partial t} = 0. \tag{17}$$

These potentials exist in the vacuum even though the longitudinal E_Z and B_Z are zero from eqns. (2) and (3). The question that arises therefore, is whether or not ϕ_L and A_L can have physically measurable effects.

CANONICAL QUANTIZATION

The Klein-Gordon equation $(\Box G = 0)$ for the time-like potential is regarded as an equation for a classical field, the real and physical part of ϕ_L. This procedure is well known and gives the positive definite Hamiltonian:

$$H_L = \frac{1}{\mu_0}\int B^{(0)2} dV \tag{18}$$

which reduces straightforwardly to:

$$H_L = \frac{1}{\mu_0 R^2}\int(\partial_0 \phi_L^* \partial_0 \phi_L + \nabla \phi_L \bullet \nabla \phi_L) dV. \tag{19}$$

The longitudinal four potential can be written in S.I. units as:

$$\phi_L = -\frac{A^{(0)}}{\sqrt{2}}\omega (X - iY)e^{i(\omega t - \kappa Z)} \tag{20}$$

$$A_L^\mu = (\phi_L, c\phi_L k) \tag{21}$$

and so there is an additional contribution to the Hamiltonian from the canonical quantization of the magnitude of the longitudinal vector part of A_L^μ. This contribution is identical with eqn. (18) and so the total contribution is:

$$H_{L(\mathrm{tot})} = \frac{2}{\mu_0}\int B^{(0)2}dV. \tag{22}$$

This is the electromagnetic energy in a volume of radiation V of the Maxwell-Heaviside theory.

As is well known, canonical quantization of $\square\phi_L = 0$ leads to the emergence of many different frequencies, and commutation relations emerge between the creation and annihilation operators:

$$\left[a(\kappa), a^+(\kappa')\right] = (2\pi)^3 2\omega_k \delta^3(\kappa - \kappa'). \tag{23}$$

The operator

$$N(\kappa) = a^+(\kappa)a(\kappa) \tag{24}$$

is that for the number of particles with momentum $\hbar\kappa$ and energy $\hbar\omega$. These particles are therefore Planck photons. The Hamiltonian, after quantization, is of the harmonic oscillator form:

$$H_L = \frac{1}{2}P^2(\kappa) + \frac{\omega_\kappa^2}{2}Q^2(\kappa) \tag{25}$$

where

$$P(\kappa) = \left(\frac{\omega_\kappa}{2}\right)^{1/2}\left[a(\kappa) + a^+(\kappa)\right] \tag{26}$$

$$Q(\kappa) = \frac{1}{(2\omega_\kappa)^{1/2}}\left[a(\kappa) - a^+(\kappa)\right] \tag{27}$$

and the potential ϕ_L becomes an infinite sum of oscillators after quantization. The operators a and a^+ are the annihilation and creation operators for the field quanta, and the energy of the field is positive definite as required. The particles obtained in this way obey Bose-Einstein statistics as required, but are scalar particles without spin.

The photons obtained in this way are however **physical**, because they contribute to the energy as just argued.

Transverse photons are obtained from the canonical quantization of the transverse part of the vector potential with two different circular polarizations. This is a very well known procedure and these massless photons have spin angular momentum. Therefore the logic of Whittaker's work leads to three classes of physical photons: time-like; longitudinal; and transverse. These are distinct objects: the transverse photons **with** spin may be absorbed, giving up their angular momentum to an atom, but the time-like and longitudinal photons without spin may not be absorbed, and thus be difficult to detect. Later in this paper, an experiment is proposed to detect the physical effect of time-like and longitudinal photons.

Note that the combination of longitudinal and transverse $A_L^\mu + A_T^\mu$ is fully, or manifestly, covariant. Canonical quantization in the radiation gauge is not manifestly covariant but otherwise satisfactory. However, canonical quantization in the Lorenz gauge is full of difficulties: 1) it is impossible to satisfy the covariant commutation relations; 2) a gauge fixing term has to be introduced; 3) the Lorenz condition can be used only as an operator identity; 4) the Hilbert space of particle states (Fock space) has an indefinite metric; 5) the Gupta-Bleuler method has to be used to result in the very dubious conclusion that the time-like and longitudinal photons cancel each other and are somehow physical only in combination. Although well accepted, this makes very little sense because it implies that linear momentum (longitudinal photon) and energy (time-like photon) are somehow physical only in combination and do not exist separately. This is clearly not the case in nature.

By separating the transverse and longitudinal parts of A^{μ} and applying canonical quantization separately, we arrive straightforwardly at the conclusion that there are two different types of photon, one corresponding to A_L^{μ} and the other to A_T^{μ}. The properties of the latter are well known, those of the former essentially unknown.

EXPERIMENT TO ISOLATE AND DETECT TIME-LIKE PHOTONS

The experimental design is very simple. Two dipole antennae are set up in close proximity so that the vector potentials from the two antennae cancel:

$$A_1 = -i\frac{\kappa e^{i\kappa r}}{4\pi c\varepsilon_0 r}p_1; \qquad A_2 = i\frac{\kappa e^{i\kappa r}}{4\pi c\varepsilon_0 r}p_2. \tag{28}$$

Here p_1 and p_2 are the dipole components of each antenna, κ is the wave-vector magnitude; r is the radius vector magnitude; ϵ_0 is the vacuum permittivity, and c is the vacuum speed of light. All vector quantities are zero:

$$A = A_1 + A_2 = 0; \qquad E = 0; \qquad B = 0 \tag{29}$$

and there are no radiated electric and magnetic fields. Additionally, Whittaker's f and g vector functions cancel:

$$g_1 = -g_2; \qquad f_1 = -f_2 \tag{30}$$

but their scalar magnitudes F and G are **additive**. In the radiation zone, where we can use plane waves, there is present the magnetic flux magnitude:

$$2G = \frac{2}{\sqrt{2}}A^{(0)}(X - iY)e^{i(\omega t - \kappa Z)} \tag{31}$$

which is obviously a scalar quantity. There are no vector electric or magnetic fields present, and no vector potentials. The only entity present is the time-like potential:

$$\phi_L = 2i\dot{G} \tag{32}$$

which obeys the massless Klein-Gordon equation:

$$\Box\phi_L = 0 \tag{33}$$

As shown, this leads on canonical quantization to the energy:

$$H_{Tot.} = \frac{2}{\mu_0}\int B^{(0)2}dV, \tag{34}$$

energy which can be detected as heat by a bolometer. This would indicate the existence of time-like photons. A receiver would detect nothing because there are no electric or magnetic fields present, and no vector potentials present. Therefore, there are no transverse or longitudinal photons present, just pure energy or power density in watts per square meter from physical time-like photons obtained from eqn. (32) by canonical quantization. This experiment would also show that the time-like photon is not accompanied by fields and potentials, and may not be absorbed by atoms, because it has no spin angular momentum. It is pure quantized energy.

EXPERIMENT TO DETECT SCALAR INTERFEROMETRY

The energy density of the time-like (scalar) beam designed in section 5 is:

$$\frac{En}{V} = 2\frac{GG^*}{R^4\mu_0} \tag{35}$$

where $B^{(0)}$ is the scalar magnetic flux density magnitude of the beam:

$$B^{(0)} = \frac{G}{\pi R^2} \tag{36}$$

where πR^2 is taken to be the (circular) beam area. The power density of the scalar beam is:

$$I = \frac{c}{\mu_0}\frac{En}{V} \tag{37}$$

where μ_0 is the vacuum permeability. If two such scalar beams (G_1 and G_2) interfere, the interference pattern is governed as usual by the product $(G_1 + G_2)(G_1^* + G_2^*)$, which can be rewritten as:

$$G = \frac{(G_1 + G_2)(G_1^* + G_2^*)}{G^0} \tag{38}$$

where

$$\Box G = B. \tag{39}$$

Therefore a scalar magnetic flux density magnitude B appears in the interference zone. It follows from the relation $E = cB$ in the vacuum, that a scalar electric field strength magnitude also appears. If the time-like potential corresponding to G is written as in eqn. (32), then eqn. (39) becomes:

$$\Box\phi_L = -\frac{\rho}{\varepsilon_0} \tag{40}$$

and there is charge density ρ present in the interference zone. In the interference zone therefore, there should be heat, detectible by a bolometer, and also an electromagnetic signal due to the charge density arriving at a receiver. This experiment would detect the phenomenon of scalar interferometry in a simple way.

CONCLUSION

Following the logic of Whittaker's well known analysis {1, 2}, it has been inferred that there exist physical transverse, time-like, and longitudinal photons. Two experiments have been suggested to verify or nullify these predictions.

REFERENCES

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{3} W. Heitler, "Quantum Theory of Radiation" (Oxford University press, New York, 1954).
{4} W.K.H. Panofsky and M. Phillips, "Classical Electricity and Magnetism" (Addison-Wesley, Reading, 1962).

FORMAL PROOF OF THE GAUGE INVARIANCE OF G, F, A AND ϕ

Whittaker shows that:

$$B = -\nabla \times (\nabla \times g) + \frac{1}{c} \nabla \times \dot{f} \tag{1}$$

$$E = c\nabla \times (\nabla \times f) + \nabla \times \dot{g} \tag{2}$$

Eqn. (1) is invariant under:

$$g \to g + \nabla a; \qquad \nabla \times g \to \nabla \times g + \nabla b \tag{3}$$

where a and b are arbitrary. This implies that:

$$\nabla \times g \to \nabla \times g + \nabla \times (\nabla a) = \nabla \times g \tag{4}$$

$$\nabla \times g \to \nabla \times g \tag{5}$$

Now use:

$$A = -\nabla \times g + \frac{1}{c} \dot{f} \tag{6}$$

The transverse part of A is:

$$A_T = -\nabla \times g \tag{7}$$

$$A_T \to A_T \tag{8}$$

The transverse vector potential is **physical**. This overturns Heaviside's assertion of 1895, and supports the point of view of Maxwell and Faraday.

Eqn. (2) is invariant under:

$$f \to f + \nabla c; \qquad \nabla \times f \to \nabla \times f + \nabla d \tag{9}$$

where c and d are arbitrary.

So:

$$\nabla \times f \to \nabla \times f \tag{10}$$

Now use:

$$S = -c\nabla \times f - \dot{g} \tag{11}$$

and the transverse part of Stratton's potential is **physical**.

$$S_T \to S_T \tag{12}$$

In the special case of plane waves:

$$S = icA \tag{13}$$

$$f = ig \tag{14}$$

$$\dot{f} = i\dot{g} \tag{15}$$

So:

$$E = ic\nabla \times (\nabla \times g) + \nabla \times \dot{g} \tag{16}$$

$$B = i\nabla \times (\nabla \times f) + \frac{1}{c}\nabla \times \dot{f} \tag{17}$$

If, however, $g \to g + \nabla a$

as in eqn. (3), then

$$\nabla \times g \to \nabla \times g; \qquad \nabla \times \dot{g} \to \nabla \times \dot{g}$$

So the longitudinal and transverse parts of both E and B are **physical**. This overturns the received view that only the transverse parts of E and B may be physical in vacuo.

The fields are given by:

$$E_X = \frac{\partial^2 F}{\partial X \partial Z} + \frac{1}{c}\frac{\partial^2 G}{\partial Y \partial t}; \qquad B_X = \frac{1}{c}\frac{\partial^2 F}{\partial Y \partial t} - \frac{\partial^2 G}{\partial X \partial Z};$$

$$E_Y = \frac{\partial^2 F}{\partial Y \partial Z} - \frac{1}{c}\frac{\partial^2 G}{\partial X \partial t}; \qquad B_Y = -\frac{1}{c}\frac{\partial^2 F}{\partial X \partial t} - \frac{\partial^2 G}{\partial Y \partial Z};$$

$$E_Z = \frac{\partial^2 F}{\partial Z^2} - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2}; \qquad B_Z = \frac{\partial^2 G}{\partial X^2} + \frac{\partial^2 G}{\partial Y^2}.$$

and if

$$g \to g + \nabla a; \qquad f \to f + \nabla c$$

the fields change in general, so the only possibility is

$$g \to g; \qquad f \to f$$

so g and f are **physical**.

This can be seen in another way from:

$$\Box G = 0; \qquad \Box F = 0$$

If

$$G \to G + \frac{\partial a}{\partial Z}$$

then

$$\Box G \neq 0$$

unless

$$\Box\left(\frac{\partial a}{\partial Z}\right) = 0. \qquad (18)$$

Therefore a is not a random function because it is given by eqn. (18).

CONDITION FOR VANISHING FIELDS

The condition for $\boldsymbol{E} = 0$; $\boldsymbol{B} = 0$ is, from eqns. (16) and (17):

$$\frac{\partial}{\partial t}(\nabla \times \boldsymbol{g}) = ic\nabla \times (\nabla \times \boldsymbol{g})$$

i.e.

$$\frac{\partial \mathrm{A}_T}{\partial t} = i\nabla \times \mathrm{A}_\mathrm{T}$$

$$\mathrm{A}_\mathrm{T} = \frac{A^{(0)}}{\sqrt{2}}(i\boldsymbol{i} + \boldsymbol{j})e^{i(\omega t - \kappa Z)}$$

This gives $\boldsymbol{E} = \boldsymbol{B} = \boldsymbol{0}$;

$$\mathrm{A}_\mathrm{L} = -\kappa \frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)}\boldsymbol{k}$$

$$\phi = -\omega \frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)}$$

$$G = \frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)}$$

$$F = i\frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)}$$

$$\boldsymbol{B}^{(3)} \neq \boldsymbol{0}$$

1) The only field present in vacuo is $\boldsymbol{B}^{(3)}$, which cannot be detected.

2) When interaction with matter occurs, the fields **re-appear**.