RIEMANNian Geometry of the Space-Time

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Investigating the properties of diverse manifolds, DONALDSON [2] as well as FREEDMAN [3] determined special properties of the four-manifold. In that, DONALDSON's work is based on the work of a group around ATIYAH (DONALDSON [4]), who include the YANG-MILLS equations. These involve "exotic smooth structures" of the four-manifold.

This step has been done with the belief in the physical relevance of quantum field theory. Physicists take the YANG-MILLS equations as generalizations of MAXWELL's equations. However, the author could demonstrate [5], that MAXWELL's equations go into the system of source-free EINSTEIN-MAXWELL equations

$$R_{ik} = \kappa \cdot (\frac{1}{4}\, g_{ik} F_{ab} F^{ab} - F_{ia} F_k{}^a) \qquad , \tag{1}$$

$$F^{ia}{}_{;a} = 0 \qquad , \tag{2}$$

$$F_{ik} = A_{i,k} - A_{k,i} \qquad . \tag{3}$$

Physicists call that an electrovacuum. As well, the material quantities like mass, spin, charge, magnetic momentum are integration constants of above system of PDE. Physicists always claim that this step would disregard the quantum phenomena. Strangely enough, the values of the integration constants for most stable solutions from numerical simulations are just the known particle numbers [5].

Equ. (1) to (3) involve a special kind of RIEMANNian geometry of the four-manifold of signature (+,+,+,–), what is explained as follows. As well,

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†from a news article [1]

the derivation follows in general that of the RICCI main directions as done by EISENHART [6]. Unlike all other manifolds, the results for the four-manifold involve two main surfaces instead of four main directions.

The RICCI main directions (written in terms according to EISENHART) follow from

$$\mathsf{det}|R_{ik} + \rho g_{ik}| = 0 \tag{4}$$

with the solutions

$$\rho_{|1} = \rho_{|4} = +\rho_o \qquad , \qquad \rho_{|2} = \rho_{|3} = -\rho_o \tag{5}$$

with

$$\rho_o{}^2 = R_1{}^a R^1{}_a = R_2{}^a R^2{}_a = R_3{}^a R^3{}_a = R_4{}^a R^4{}_a \qquad . \tag{6}$$

Characteristical are the two double-roots, that means: There are two dual surfaces of the congruences $e_{|1}{}^i e_{|4}{}^k - e_{|1}{}^k e_{|4}{}^i$ and $e_{|2}{}^i e_{|3}{}^k - e_{|2}{}^k e_{|3}{}^i$ with extreme mean RIEMANNian curvature. $e_{|1}...e_{|4}$ are the vectors of an orthogonal quadrupel in those "main surfaces".

With it we get

$$g_{ik} = e_{|1\_i}e_{|1\_k} + e_{|2\_i}e_{|2\_k} + e_{|3\_i}e_{|3\_k} - e_{|4\_i}e_{|4\_k} \qquad , \tag{7}$$

$$\frac{R_{ik}}{\rho_o} = -e_{|1\_i}e_{|1\_k} + e_{|2\_i}e_{|2\_k} + e_{|3\_i}e_{|3\_k} + e_{|4\_i}e_{|4\_k} \qquad . \tag{8}$$

If we set

$$c_{|ik} = -c_{|ki} = F_{ab} e_{|i}{}^a e_{|k}{}^b \tag{9}$$

follows

$$-\kappa\Big((c_{|23})^2 + (c_{|14})^2\Big) = 2\rho_o \qquad , \tag{10}$$

$$c_{|12} = c_{|34} = c_{|13} = c_{|24} = 0 \qquad . \tag{11}$$

With it, the field tensor

$$F_{ik} = -c_{|14}(e_{|1\_i}e_{|4\_k} - e_{|1\_k}e_{|4\_i}) + c_{|23}(e_{|2\_i}e_{|3\_k} - e_{|2\_k}e_{|3\_i}) \tag{12}$$

is performed from the main surfaces !

With it, these surfaces depict the electromagnetic fields same way like the curvature vector does it with acceleration + gravitation. Both quantities are parameters of the curve as described by each body in the space-time. They accompany this curve.

It were a great task for mathematicians to discuss the properties of this special manifold. We have to take notice of geometric boundaries.

References

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