CHAPTER FIVE : THE UNIFICATION OF QUANTUM MECHANICS AND GENERAL RELATIVITY

The standard physics has completely failed to unify quantum mechanics and general relativity, notably because of indeterminacy, a non Baconian idea introduced at the Solvay Conference of 1927. The current attempts of the standard physics at unification revolve around hugely expensive particle colliders, and these attempts are limited to the unification of the electromagnetic and weak and strong nuclear fields, leaving out gravitation completely. So it is reasonable to infer that the standard physics will never be able to produce a unified field theory. In great contrast ECE theory has succeeded with unifying all four fundamental fields with a well known geometry due to Cartan as described in foregoing chapters of this book.

Towards the end of the nineteenth century the classical physics evolved gradually into special relativity and the old quantum theory. The experiments that led to this great paradigm shift in natural philosophy are very well known, so need only a brief description here. There were experiments on the nature of broadband (black body) radiation leading to the Rayleigh Jeans law, the Steffan Boltzmann distribution and similar. The failure of the Rayleigh Jeans law led to the Planck distribution and his inference of what was later named the photon. The photoelectric effect could not be explained using the classical physics, the Brownian motion needed a new type of stochastic physics indicating the existence of molecules, first proposed by Dalton. The specific heats of solids could not be explained adequately with classical nineteenth century physics. Atomic and molecular spectra could not be explained with classical methods, notably the anomalous Zeeman effect.

The Michelson Morley experiment gave results that could not be explained with the classical Newtonian physics, so that Fitzgerald in correspondence with Heaviside suggested a radically new physics that came to be known as special relativity. The mathematical framework for special relativity was very nearly inferred by Heaviside but was developed by Lorentz and Poincaré. Einstein later made contributions of his own. The subjects of special relativity and quantum theory began to develop rapidly. The many contributions of Sommerfeld are typically underestimated in the history of science, those of his students and post doctorals are better known. The old quantum theory evolved into the Schroedinger equation after the inference by de Broglie of wave particle dualism. Peter Debye asked his student Schroedinger to try to solve the puzzle posed by the fact that a particle could be a wave and vice versa, and during this era Compton gave an impetus to the idea of photon as particle by scattering high frequency electromagnetic radiation from a metal foil - Compton scattering.

The Schroedinger equation proved to be an accurate description of for example spectral phenomena in the non relativistic limit. In the simplest instance the Schroedinger equation quantizes the classical kinetic energy of the free particle, and does not attempt to incorporate special relativity into quantum mechanics. Sommerfeld had made earlier attempts but the main problem remained, how to quantize the Einstein energy equation of special relativity. The initial attempts by Klein and Gordon resulted in negative probability, so were abandoned for this reason. Pauli had applied his algebra to the Schroedinger equation, but none of these methods were successful in describing the g factor, Landé factor or Thomas precession in one unified framework of relativistic quantum mechanics.

Dirac famously solved the problem with the use of four by four matrices and Pauli algebra but in so doing ran in to the problem of negative energies. Dirac suggested tentatively that negative energies could be eliminated with the Dirac sea, but this introduced an unobservable, the Dirac sea still has not been observed experimentally. Unobservables began to proliferate in twentieth century physics, reducing it to dogma. However, Dirac was famously successful in explaining within one framework the g factor of the electron, the Landé factor, the Thomas factor and the Darwin term, and in producing a theory free of negative probabilities. The Dirac sea seemed to give rise to antiparticles which were observed. The Dirac sea itself cannot be observed, and the problem of negative energies was not solved by Dirac.

It is not clear whether Dirac ever accepted indeterminacy, a notion introduced by Bohr and Heisenberg and immediately rejected by Einstein, Schroedinger, de Broglie and others as anti Baconian and unphysical. The Dirac equation reduces to the Schroedinger and Heisenberg equations in well defined limits, but indeterminacy is pure dogma. It is easily disproven experimentally and has taken on a life of its own that cannot be described as science. Heisenberg described the Dirac equation as an all time low in physics, but many would describe indeterminacy in the same way. In this chapter, indeterminacy is disproven straightforwardly with the use of higher order commutators. Heisenberg's own methods are used to disprove the Heisenberg Uncertainty Principle, a source of infinite confusion for nearly ninety years. One of the major outcomes of ECE theory is the rejection of the Heisenberg Uncertainty Principle in favour of a quantum mechanics based on geometry.

The negative energy problem that plagued the Dirac equation is removed in this chapter by producing the fermion equation of relativistic quantum mechanics. This equation is not only Lorentz covariant but also generally covariant because it is derived from the tetrad postulate of a generally covariant geometry - Cartan geometry. All the equations of ECE theory are automatically generally covariant and Lorentz covariant in a well defined limit of general covariance. So the fermion equation is the first equation of quantum mechanics unified with general relativity. It has the major advantages of producing rigorously positive energy levels and of being able to express the theory in terms of two by two matrices. The fermion equation produces everything that the Dirac equation does, but with major advantages. So it should be viewed as an improvement on the deservedly famous Dirac equation, an improvement based on geometry and the ECE unified field theory.

The latter also produces the d'Alembert and Klein Gordon equations, and indeed all of the valid wave equations of physics. Some of these are discussed in this chapter.

5.1 THE FERMION EQUATION

The structure of ECE theory is the most fundamental one known in physics at present, simply because it is based directly on a rigorously correct geometry. The fermion equation can be expressed as in UFT 173 on www.aias.us in a succinct way:

$$\Pi_\mu \phi \sigma^\mu = mc \sigma^1 \phi \quad -(1)$$

where the fermion operator in covariant representation is defined as:

$$\Pi_\mu = (\Pi_0, \Pi_1, \Pi_2, \Pi_3). \quad -(2)$$

Here:

$$\Pi_0 = \sigma^0 p_0 , \quad \Pi_i = \sigma^3 p_i \quad -(3)$$

where p_μ is the energy momentum four vector:

$$p_\mu = (p_0, p_1, p_2, p_3). \quad -(4)$$

The Pauli matrices are defined by:

$$\sigma^{\mu} = (\sigma^0; \sigma^1, \sigma^2, \sigma^3) \quad - (5)$$

where:

$$\sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad - (6)$$

The eigenfunction of Eq. (1) is the tetrad {1 - 10}:

$$\psi = \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} \quad - (7)$$

whose entries are defined by the right and left Pauli spinors:

$$\phi^R = \begin{bmatrix} \psi_1^R \\ \psi_2^R \end{bmatrix}, \quad \phi^L = \begin{bmatrix} \psi_1^L \\ \psi_2^L \end{bmatrix}. \quad - (8)$$

This eigenfunction is referred to as "the fermion spinor".

The position representation of the fermion operator is defined by the symbol δ and is:

$$\delta_\mu = -\frac{i}{\hbar} \pi_\mu . \quad - (9)$$

Therefore the fermion equation is the first order differential equation:

$$i\hbar \, \delta_\mu \psi \sigma^\mu = mc\sigma^1 \psi . \quad - (10)$$

For purposes of comparison, the covariant format of the Dirac equation in chiral representation {13} is:

$$\gamma^\mu \partial_\mu \psi_D = mc\,\psi_D \qquad (11)$$

where:

$$\psi_D = \begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix} \qquad (12)$$

is a column vector with four entries, and where the Dirac matrices in chiral representation {13} are:

$$\gamma^\mu = (\gamma^0, \gamma^1, \gamma^2, \gamma^3). \qquad (13)$$

The complete details of the development of Eq. (1) are given in Note 172(8) accompanying UFT 172 on www.aias.us The ordering of terms in Eq. (1) is important because matrices do not commute and ψ is a 2 x 2 matrix. The energy eigenvalue of Eq. (1) is rigorously positive, never negative. The complex conjugate of the adjoint matrix of the fermion spinor is referred to as the "adjoint spinor" of the fermion equation, and is defined by:

$$\psi^+ = \begin{bmatrix} \psi_1^{R*} & \psi_1^{L*} \\ \psi_2^{R*} & \psi_2^{L*} \end{bmatrix}. \qquad (14)$$

The adjoint equation of Eq. (1) is defined as:

$$-i\hbar\,\partial_\mu \psi^+ \sigma^\mu = mc\,\sigma^1 \psi^+ \qquad (15)$$

where the complex conjugate of $i\hbar$ has been used. These equations have well known counterparts in the Dirac theory {1 - 10, 13} but in that theory the 4 x 4 gamma matrices are used and the definition of the adjoint spinor is more complicated.

The probability four-current of the fermion equation is defined as:

$$j^\mu = \frac{1}{2}\mathrm{Tr}\left(\psi\sigma^\mu\psi^+ + \psi^+\sigma^\mu\psi\right) \quad -(16)$$

and its Born probability is:

$$j^0 = \psi_1^R\psi_1^{R*} + \psi_2^R\psi_2^{R*} + \psi_1^L\psi_1^{L*} + \psi_2^L\psi_2^{L*} \quad -(17)$$

which is rigorously positive as required of a probability. It is the same as the Born probability of the chiral representation {1 - 10, 13} of the Dirac equation. In the latter the four current is defined as:

$$j^\mu_D = \bar{\psi}_D\gamma^\mu\psi_D \quad -(18)$$

and the adjoint Dirac spinor is a four entry row vector defined by:

$$\bar{\psi}_D = \psi_D^+\gamma^0. \quad -(19)$$

It is shown as follows that the probability four-current of the fermion equation is conserved:

$$\partial_\mu j^\mu = 0. \quad -(20)$$

To prove this result multiply both sides of Eq. (1) from the right with ψ^+:

$$i\hbar\partial_\mu\psi\sigma^\mu\psi^+ = mc\sigma^1\psi\psi^+. \quad -(21)$$

Multiply both sides of Eq. (15) from the right with ψ

$$-i\hbar\,\partial_\mu \phi^+ \sigma^\mu \phi = mc\sigma^1 \phi^+ \phi \quad -(22)$$

and subtract Eq. (22) form Eq. (21):

$$i\hbar\,\partial_\mu\left(\phi\sigma^\mu\phi^+ + \phi^+\sigma^\mu\phi\right) = mc\sigma^1\left(\phi\phi^+ - \phi^+\phi\right) \quad -(23)$$

By definition:

$$\phi\phi^+ - \phi^+\phi = \begin{bmatrix} \phi_1^R & \phi_2^R \\ \phi_1^L & \phi_2^L \end{bmatrix}\begin{bmatrix} \phi_1^{R*} & \phi_1^{L*} \\ \phi_2^{R*} & \phi_2^{L*} \end{bmatrix} - \begin{bmatrix} \phi_1^{R*} & \phi_1^{L*} \\ \phi_2^{R*} & \phi_2^{L*} \end{bmatrix}\begin{bmatrix} \phi_1^R & \phi_2^R \\ \phi_1^L & \phi_2^L \end{bmatrix} \quad -(24)$$

so

$$\mathrm{Trace}\left(\phi\phi^+ - \phi^+\phi\right) = 0. \quad -(25).$$

Therefore:

$$\mathrm{Trace}\left(\partial_\mu\left(\phi\sigma^\mu\phi^+ + \phi^+\sigma^\mu\phi\right)\right) = 0 \quad -(26)$$

and

$$\partial_\mu j^\mu = 0 \quad -(27)$$

Q. E. D.

The fermion equation (1) may be expanded into two simultaneous equations:

$$\left(E + c\underline{\sigma}\cdot\underline{p}\right)\phi^L = mc^2\phi^R \quad -(28)$$

$$\left(E - c\underline{\sigma}\cdot\underline{p}\right)\phi^R = mc^2\phi^L \quad -(29)$$

in which E and p are the operators of quantum mechanics:

$$E = i\hbar \frac{\partial}{\partial t}, \quad \underline{p} = -i\hbar \underline{\nabla} \quad -(30)$$

Eqs. (28) and (29) may be developed as:

$$(E - c\underline{\sigma}\cdot\underline{p})(E + c\underline{\sigma}\cdot\underline{p})\phi^L = m^2 c^4 \phi^L \quad -(31)$$

$$(E + c\underline{\sigma}\cdot\underline{p})(E - c\underline{\sigma}\cdot\underline{p})\phi^R = m^2 c^4 \phi^R \quad -(32)$$

from which there emerge equations such as:

$$\left(E^2 - c^2 \underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{p}\right)\phi^R = m^2 c^4 \phi^R \quad -(33)$$

Using the quantum postulates this becomes the wave equation:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\phi^R = 0 \quad -(34)$$

and it becomes clear that the fermion equation is a factorization of the EEC wave equation:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\phi = 0 \quad -(35)$$

whose eigenfunction is the tetrad (ϕ).

Therefore the fermion equation is obtained from the tetrad postulate and Cartan geometry. The tetrad is defined by:

$$\begin{bmatrix} V^R \\ V^L \end{bmatrix} = \begin{bmatrix} \phi_1^R & \phi_2^R \\ \phi_1^L & \phi_2^L \end{bmatrix} \begin{bmatrix} V^1 \\ V^2 \end{bmatrix} \quad -(36)$$

i.e. as a matrix relating two column vectors.