This method can be extended to the general Schroedinger equation in which the potential energy V is present. Consider the momentum Beltrami equation (297) in the general case where κ depends on coordinates. Taking the curl of both sides of Eq. (297):

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{p}) = \underline{\nabla} \times (\kappa \underline{p}). \quad -(316)$$

By vector analysis Eq. (316) can be developed as:

$$\underline{\nabla} (\underline{\nabla} \cdot \underline{p}) - \nabla^2 \underline{p} = \kappa^2 \underline{p} + \underline{\nabla} \kappa \times \underline{p} \quad -(317)$$

so:

$$(\nabla^2 + \kappa^2) \underline{p} = \underline{\nabla} (\underline{\nabla} \cdot \underline{p}) - \underline{\nabla} \kappa \times \underline{p}. \quad -(318)$$

One possible solution is:

$$(\nabla^2 + \kappa^2) \underline{p} = \underline{0} \quad -(319)$$

and

$$\underline{\nabla} (\underline{\nabla} \cdot \underline{p}) = \underline{\nabla} \kappa \times \underline{p}. \quad -(320)$$

Eq. (320) implies

$$\underline{p} \cdot \underline{\nabla} (\underline{\nabla} \cdot \underline{p}) = \underline{p} \cdot \underline{\nabla} \kappa \times \underline{p} = 0. \quad -(321)$$

Two possible solutions of Eq. (321) are:

$$\underline{\nabla} \cdot \underline{p} = 0 \quad -(322)$$

and

$$\underline{\nabla}(\underline{\nabla}\cdot\underline{p}) = \underline{0}. \quad -(323)$$

Using the quantum postulate (301) in Eq. (319) gives:

$$(\nabla^2 + \kappa^2)\underline{\nabla}\phi = \underline{0} \quad -(324)$$

and the Schroedinger equation {1 - 10}:

$$(\nabla^2 + \kappa^2)\phi = 0. \quad -(325)$$

From Eq. (325)

$$\underline{\nabla}\left((\nabla^2 + \kappa^2)\phi\right) = \underline{0} \quad -(326).$$

i. e.

$$(\nabla^2 + \kappa^2)\underline{\nabla}\phi + \left(\underline{\nabla}(\nabla^2 + \kappa^2)\right)\phi = \underline{0}, \quad -(327)$$

a possible solution of which is:

$$(\nabla^2 + \kappa^2)\underline{\nabla}\phi = \underline{0} \quad -(328)$$

and

$$\left(\underline{\nabla}(\nabla^2 + \kappa^2)\right)\phi = \underline{0}. \quad -(329)$$

Eq. (329) is Eq. (324), Q. E. D. Eq. (329) can be written as:

$$\underline{\nabla}\nabla^2\phi + \underline{\nabla}\kappa^2\phi = \underline{0} \quad -(330)$$

i. e.

$$\underline{\nabla}\left(\nabla^2\phi + \kappa^2\phi\right) = \underline{0}. \quad -(331)$$

A possible solution of Eq. (331) is the Schroedinger equation:

$$\left(\nabla^2 + \kappa^2\right)\phi = 0. \quad -(332)$$

So the Schroedinger equation is compatible with Eq. (324).

Eq. (322) gives:

$$\nabla^2\phi = 0 \quad -(333)$$

which is consistent with Eq. (332) only if:

$$\kappa^2 = 0. \quad -(334)$$

Eq. (323) gives:

$$\underline{\nabla}\left(\nabla^2\phi\right) = \underline{0} \quad -(335)$$

where:

$$\nabla^2\phi = -\kappa^2\phi. \quad -(336)$$

Therefore:

$$\underline{\nabla}\left(\kappa^2\phi\right) = \left(\underline{\nabla}\kappa^2\right)\phi + \kappa^2\underline{\nabla}\phi \quad -(337)$$

and:

$$\underline{\nabla}\phi = -\left(\frac{\underline{\nabla}\kappa^2}{\kappa^2}\right)\phi \quad -(338)$$

Therefore:

$$\underline{\nabla} \cdot \underline{\nabla} \phi = \nabla^2 \phi = -\underline{\nabla} \cdot \left(\frac{\underline{\nabla} \kappa^2}{\kappa^2} \phi \right) \quad -(339)$$

$$= -\left(\left(\underline{\nabla} \cdot \left(\frac{\underline{\nabla} \kappa^2}{\kappa^2} \right) \right) \phi + \left(\frac{\underline{\nabla} \kappa^2}{\kappa^2} \right) \cdot \underline{\nabla} \phi \right)$$

From a comparison of Eqs. (332) and (339) we obtain the subsidiary condition:

$$\kappa^2 \nabla^2 \kappa^2 = \underline{\nabla} \kappa^2 \cdot \underline{\nabla} \kappa^2 + \kappa^6 \quad -(340)$$

where:

$$\kappa^2 = \frac{2m}{\hbar^2} (V - E). \quad -(341)$$

Therefore:

$$\underline{\nabla} \kappa^2 = \frac{2m}{\hbar^2} \underline{\nabla} V \quad -(342)$$

and

$$\nabla^2 \kappa^2 = \frac{2m}{\hbar^2} \nabla^2 V \quad -(343)$$

giving a cubic constraint in V - E :

$$(V - E)^3 - \frac{\hbar^2}{2m} \nabla^2 V (V - E) + \frac{\hbar^2}{2m} \left(\underline{\nabla} V \cdot \underline{\nabla} V \right) = 0 \quad -(344)$$

this can be written as a cubic equation in E, which is a constant. E is expressed in terms of V, $\underline{\nabla} V$, and $\nabla^2 V$. Using:

$$\underline{\nabla} E = \underline{0} \quad -(345)$$

gives a differential equation in V which can be solved numerically, giving an expression for V. Finally this expression for V is used in the Schroedinger equation:

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi = E \psi \quad -(346)$$

to find the energy levels of E and the wavefunctions ψ. These are energy levels and wavefunctions of the interior parton structure of an elementary particle such as an electron, proton or neutron. The well developed methods of computational quantum mechanics can be used to find the expectation values of any property and can be applied to scattering theory, notably deep inelastic electron electron, electron proton and electron neutron scattering. The data are claimed conventionally to provide evidence for quark structure, but the quark model depends on the validity of the U(1) and electroweak sectors of the standard model. In this book these sector theories are refuted in many ways.