and becomes:
$$\left(\square + \left(\frac{mc}{\hbar}\right)^2\right) A^a_\mu = 0. \qquad -(123)$$

In ECE physics the conservation of charge current density is:

$$\partial_\mu J^{a\mu} = 0 \qquad -(124)$$

and is consistent with Eqs. (48) and (49).

In ECE physics the electric charge density is geometrical in origin and is:

$$\rho^a = \epsilon_0 \left(\boldsymbol{\omega}^a{}_b \cdot \boldsymbol{E}^b - c\boldsymbol{A}^b \cdot \boldsymbol{R}^a{}_b(\text{orb}) \right) \qquad -(125)$$

and the electric current density is:

$$\boldsymbol{J}^a = \frac{1}{\mu_0} \left(\boldsymbol{\omega}^a{}_b \times \boldsymbol{B}^b + \frac{\omega_0}{c} \boldsymbol{E}^b - \boldsymbol{A}^b \times \boldsymbol{R}^a{}_b(\text{spin}) - A_0^b \boldsymbol{R}^a{}_b(\text{orb}) \right) \qquad -(126)$$

Here $R^a_b(\text{spin})$ and $R^a_b(\text{orb})$ are the spin and orbital components of the curvature tensor {1 – 10}. So Eqs. (93), (125) and (126) give many new equations of physics which can be developed systematically in future work. In magnetostatics for example the relevant equations are:

$$\nabla \cdot \boldsymbol{B}^a = 0, \qquad -(127)$$
$$\nabla \times \boldsymbol{B}^a = \mu_0 \boldsymbol{J}^a, \qquad -(128)$$

and

$$\nabla \cdot \boldsymbol{J}^a = \nabla \cdot \nabla \times \boldsymbol{B}^a = 0 \qquad -(129)$$

so it follows from charge current conservation that:

$$\partial \rho^a / \partial t = 0. \qquad -(130)$$

If it is assumed that the scalar potential is zero in magnetostatics, the usual assumption, then:

$$\boldsymbol{J}^a = \frac{1}{\mu_0} \left(\boldsymbol{\omega}^a{}_b \times \boldsymbol{B}^b - \boldsymbol{A}^b \times \boldsymbol{R}^a{}_b(\text{spin}) \right) \qquad -(131)$$

because there is no electric field present. It follows from Eqs. (129) and (131) that

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{B}^b = \underline{\nabla} \cdot \underline{A}^b \times \underline{R}^a{}_b(\text{spin}) \quad -(132)$$

in ECE magnetostatics.

In UFT258 and immediately preceding papers of this series it has been shown that in the absence of a magnetic monopole:

$$\underline{\omega}^a{}_b \cdot \underline{B}^b = \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}) \quad -(133)$$

and that the space part of the Cartan identity in the absence of a magnetic monopole gives the two equations:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b = 0 \quad -(134)$$

and

$$\underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{A}^b = \underline{A}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b . \quad -(135)$$

In ECE physics the magnetic flux density is:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(136)$$

so the Beltrami equation gives:

$$\underline{\nabla} \times \underline{B}^a = \kappa \underline{B}^a = \kappa\left(\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b\right). \quad -(137)$$

Eq. (134) from the space part of the Cartan identity is also a Beltrami equation, as is any divergenceless equation:

$$\underline{\nabla} \times \left(\underline{\omega}^a{}_b \times \underline{A}^b\right) = \kappa\, \underline{\omega}^a{}_b \times \underline{A}^b \quad -(138)$$

From Eq. (137):

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) - \underline{\nabla} \times (\underline{\omega}^a{}_b \times \underline{A}^b) = \kappa(\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b) \quad -(139)$$

Using Eq. (138):

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) = \kappa \, \underline{\nabla} \times \underline{A}^a \quad -(140)$$

which implies that the vector potential is also defined in general by a Beltrami equation:

$$\underline{\nabla} \times \underline{A}^a = \kappa \underline{A}^a \quad -(141)$$

Q. E. D. This is a generally valid result of ECE physics which implies that:

$$\underline{\nabla} \cdot \underline{A}^a = 0. \quad -(142)$$

From Eq. (110) it follows that:

$$\partial \rho^a / \partial t = 0 \quad -(143)$$

is a general result of ECE physics.

From Eqs. (135) and (141):

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \kappa \underline{\omega}^a{}_b \quad -(144)$$

so the spin connection vector of ECE physics is also defined in general by a Beltrami equation. This important result can be cross checked for internal consistency using note 258(4) on www.aias.us, starting from Eq. (50) of this paper. Considering the X component for example:

$$\omega^a{}_{Xb} (\underline{\nabla} \times \underline{A}^b)_X = A^b_X (\underline{\nabla} \times \underline{\omega}^a{}_b)_X \quad -(145)$$

and it follows that: $$\frac{1}{A_X^{(1)}}\left(\underline{\nabla}\times\underline{A}^{(1)}\right)_X = \frac{1}{\omega^a{}_{X(1)}}\left(\underline{\nabla}\times\underline{\omega}^a{}_{(1)}\right)_X \quad -(146)$$

and similarly for the X and Z components. In order for this to be a Beltrami equation, Eqs. (141) and (144) must be true, Q. E. D.

In magnetostatics there are additional results which emerge as follows. From vector analysis:

$$\underline{\nabla}\cdot\underline{\omega}^a{}_b\times\underline{B}^b = \underline{B}^b\cdot\underline{\nabla}\times\underline{\omega}^a{}_b - \underline{\omega}^a{}_b\cdot\underline{\nabla}\times\underline{B}^b \quad -(147)$$

and

$$\underline{\nabla}\cdot\underline{A}^b\times\underline{R}^a{}_b(\text{spin}) = \underline{R}^a{}_b(\text{spin})\cdot\underline{\nabla}\times\underline{A}^a - \underline{A}^b\cdot\underline{\nabla}\times\underline{R}^a{}_b(\text{spin}) \quad -(148)$$

It is immediately clear that Eqs. (87) and (144) give Eq. (147) self consistently, Q. E. D. Eq. (148) gives

$$\underline{\nabla}\cdot\underline{\omega}^a{}_b\times\underline{B}^b = \underline{\nabla}\cdot\underline{A}^b\times\underline{R}^a{}_b(\text{spin}) = 0 \quad -(149)$$

and using Eq. (148):

$$\underline{\nabla}\times\underline{R}^a{}_b(\text{spin}) = \kappa\,\underline{R}^a{}_b(\text{spin}) \quad -(150)$$

so the spin curvature is defined by a Beltrami equation in magnetostatics. Also in magnetostatics:

$$\underline{\nabla}\times\underline{B}^a = \kappa\,\underline{B}^a = \mu_0\,\underline{J}^a \quad -(151)$$

so it follows that the current density of magnetostatics is also defined by a Beltrami equation:

$$\underline{\nabla} \times \underline{J}^a = \kappa \underline{J}^a \quad -(152)$$

All these Beltrami equations in general have intricate flow structures graphed following sections of this chapter and animated on www.aias.us. As discussed in Eqs. (31) to (35) of Note 258(5) on www.aias.us, plane wave structures and O(3) electrodynamics {1 – 10} are also defined by Beltrami equations. The latter give simple solutions for vacuum plane waves. In other cases the solutions become intricate. The B(3) field is defined by the simplest type of Beltrami equation

$$\underline{\nabla} \times \underline{B}^{(3)} = 0 \underline{B}^{(3)} . \quad -(153)$$

In photon mass theory therefore:

$$\underline{\nabla} \times \underline{A}^a = \kappa \underline{A}^a \quad -(154)$$

and

$$\left(\square + \left(\frac{mc}{\hbar} \right)^2 \right) \underline{A}^a = \underline{0} . \quad -(155)$$

It follows from Eq. (154) that:

$$\underline{\nabla} \cdot \underline{A}^a = 0 \quad -(156)$$

so:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) = \kappa \underline{\nabla} \times \underline{A}^a = \kappa^2 \underline{A}^a \quad -(157)$$

produces the Helmholtz wave equation:

$$\left(\nabla^2 + \kappa^2\right) \underline{A}^a = \underline{0} . \quad -(158)$$

Eq. (155) is

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \left(\frac{mc}{\hbar}\right)^2 \right) \underline{A}^a = \underline{0} \quad -(159)$$

so:

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \kappa^2 + \left(\frac{mc}{\hbar}\right)^2 \right) \underline{A}^a = \underline{0} . \quad -(160)$$

Now use:

$$\underline{p} = \hbar \underline{\kappa} \quad -(161)$$

and:

$$\frac{\partial^2}{\partial t^2} = -\frac{E^2}{\hbar^2} \quad -(162)$$

to find that Eq. (160) is the Einstein energy equation for the photon of mass m, so the analysis is rigorously self consistent, Q. E. D.

In ECE physics the Lorenz gauge is:

$$\partial_\mu A^{a\mu} = 0 \quad -(163)$$

i.e.

$$\frac{1}{c^2} \frac{\partial \phi^a}{\partial t} + \underline{\nabla} \cdot \underline{A}^a = 0 \quad -(164)$$

with the solution:

$$\frac{\partial \phi^a}{\partial t} = \underline{\nabla} \cdot \underline{A}^a = 0 . \quad -(165)$$

This is again a general result of ECE physics applicable under any circumstances. Also in ECE physics in general the spin connection vector has no divergence:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b = 0 \quad -(166)$$

because:

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \kappa\, \underline{\omega}^a{}_b . \quad -(167)$$

Another rigorous test for self consistency is given by the definition of the magnetic field in ECE physics:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(168)$$

so:

$$\underline{\nabla} \cdot \underline{B}^a = -\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b = 0 \quad -(169)$$

By vector analysis:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^a = \underline{A}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{A}^b \quad -(170)$$

$$= 0$$

because

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \kappa\, \underline{\omega}^a{}_b , \quad -(171)$$

$$\underline{\nabla} \times \underline{A}^b = \kappa\, \underline{A}^b , \quad -(172)$$

and:

$$\underline{\nabla} \cdot \underline{A}^b = 0 , \quad -(173)$$

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b = 0 . \quad -(174)$$

In the absence of a magnetic monopole Eq. (84) also follows from the space part of the Cartan identity. So the entire analysis is rigorously self consistent. The cross consistency of the Beltrami and ECE equations can be checked using:

$$\underline{B}^b = \kappa \underline{A}^b - \underline{\omega}^b{}_c \times \underline{A}^c \quad -(175)$$

as in note 258(1) on www.aias.us. Eq. (175) follows from Eqs. (168) and (172). Multiply Eq. (175) by $\underline{\omega}^a{}_b$ and use Eq. (133) to find:

$$\kappa \underline{\omega}^a{}_b \cdot \underline{A}^b - \underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \times \underline{A}^c = \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}). \quad -(176)$$

Now use:

$$\underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \times \underline{A}^c = \underline{A}^c \cdot \left(\underline{\omega}^a{}_b \times \underline{\omega}^b{}_c\right) \quad -(177)$$

and relabel summation indices to find:

$$\kappa \underline{\omega}^a{}_b \cdot \underline{A}^b - \underline{A}^b \cdot \left(\underline{\omega}^a{}_c \times \underline{\omega}^c{}_b\right) = \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}). \quad -(178).$$

It follows that:

$$\begin{aligned}\underline{R}^a{}_b(\text{spin}) &= \kappa \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \\ &= \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \end{aligned} \quad -(179)$$

Q. E. D. The analysis correctly and self consistently produces the correct definition of the spin curvature.

Finally, on the U(1) level for the sake of illustration, consider the Beltrami equations of note 258(3) on www.aias.us:

$$\underline{\nabla} \times \underline{A} = \kappa \underline{A} \quad -(180)$$

and

$$\underline{\nabla} \times \underline{B} = \kappa \underline{B}, \quad -(181)$$

in the Ampère Maxwell law

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial E}{\partial t} = \mu_0 \underline{J}. \quad -(182)$$

It follows that:

$$\kappa^2 \underline{A} = \underline{J} + \frac{1}{c^2}\frac{\partial E}{\partial t} \quad -(183)$$

where:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} \quad -(184)$$

Therefore

$$\kappa^2 \underline{A} = \mu_0 \underline{J} + \frac{1}{c^2}\frac{\partial}{\partial t}\left(-\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t}\right) \quad -(185)$$

and using the Lorenz condition:

$$\underline{\nabla} \cdot \underline{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0 \quad -(186)$$

it follows that:

$$\partial \phi / \partial t = 0. \quad -(187)$$

Using

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(188)$$

Eq. (185) becomes the d'Alembert equation in the presence of current density:

$$\Box \underline{A} = \mu_0 \underline{J}. \quad -(189)$$

The solutions of the d'Alembert equation (189) may be found from:

$$\underline{B} = \kappa \underline{A} \quad -(190)$$

showing in another way that as soon as the Beltrami equation (87) us used, U(1) gauge invariance is refuted.

3. 3 ELECROSTATICS, SPIN CONNECTION RESONANCE AND BELTRAMI STRUCTURES.

As argued already the first Cartan structure equation defines the electric field strength as:

$$\underline{E}^a = -c\,\underline{\nabla} A^a_0 - \frac{\partial \underline{A}^a}{\partial t} - c\,\omega^a_{0b}\underline{A}^b + c A^b_0 \underline{\omega}^a_b \quad -(191)$$

where the four potential of ECE electrodynamics is defined by:

$$A^a_\mu = (A^a_0, -\underline{A}^a) = \left(\frac{\phi^a}{c}, -\underline{A}^a\right). \quad -(192)$$

Here ϕ^a is the scalar potential. If it is assumed that the subject of electrostatics is defined by:

$$\underline{B}^a = \underline{0}, \quad \underline{A}^a = \underline{0}, \quad \underline{J}^a = \underline{0} \quad -(193)$$

then the Coulomb law in ECE theory is given by:

$$\underline{\nabla} \cdot \underline{E}^a = \underline{\omega}^a_b \cdot \underline{E}^b. \quad -(194)$$

The electric current in ECE theory is defined by:

$$\underline{J}^a = \epsilon_0 c\left(\omega^a_{0b}\underline{E}^b - cA^b_0 \underline{R}^a_b(\text{orb})\right) + c\,\underline{\omega}^a_b \times \underline{B}^b - c\,\underline{A}^b \times \underline{R}^a_b(\text{spin}) \quad -(195)$$

where $\underline{R}^a_b(\text{spin})$ is the spin part of the curvature vector and where $\underline{B}^b$ is the magnetic flux density. From Eqs. (193) and (195):

$$\underline{J}^a = \underline{0} = \epsilon_0 c\left(\omega^a_{0b}\underline{E}^b - cA^b_0 \underline{R}^a_b(\text{orb})\right) \quad -(196)$$

so in ECE electrostatics:

$$\omega^a{}_{0b}\,\underline{E}^b = c A^b{}_0 \underline{R}^a{}_b(\text{orb}) \quad -(197)$$

and

$$\underline{E}^a = -c\underline{\nabla} A^a{}_0 + c A^b{}_0 \underline{\omega}^a{}_b \quad -(198)$$

with

$$\underline{\nabla} \times \underline{E}^a = \underline{0}. \quad -(199)$$

From Eqs. (198) and (199)

$$\underline{\nabla} \times \underline{E}^a = c\,\underline{\nabla} \times \left(A^b{}_0\,\underline{\omega}^a{}_b\right) \quad -(200)$$

so we obtain the constraint:

$$\underline{\nabla} \times \left(A^b{}_0\,\underline{\omega}^a{}_b\right) = \underline{0}. \quad -(201)$$

The magnetic charge density in ECE theory is given by:

$$\rho^a_{mag} = \epsilon_0 c\left(\underline{\omega}^a{}_b \cdot \underline{B}^b - \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin})\right) \quad -(202)$$

and the magnetic current density by:

$$\underline{J}^a_{mag} = \epsilon_0\left(\underline{\omega}^a{}_b \times \underline{E}^b - c\,\omega^a{}_{0b}\,\underline{B}^a - c\left(\underline{A}^b \times \underline{R}^a{}_b(\text{orb}) - A^b{}_0\,\underline{R}^a{}_b(\text{spin})\right)\right) \quad -(203)$$

These are thought to vanish experimentally in electromagnetism, so:

$$\underline{\omega}^a{}_b \cdot \underline{B}^b = \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}) \quad -(204)$$

and

$$\underline{\omega}^a{}_b \times \underline{E}^a - c\,\omega^a{}_{0b}\,\underline{B}^b - c\,\underline{A}^b \times \underline{R}^a{}_b(orb) + c A^b{}_0\,\underline{R}^a{}_b(spin) = \underline{0} \quad -(205)$$

In ECE electrostatics Eq. (204) is true automatically because:

$$\underline{B}^b = 0, \quad \underline{A}^b = \underline{0} \quad -(206)$$

and Eq. (203) becomes:

$$\underline{\omega}^a{}_b \times \underline{E}^b + c A^b{}_0\,\underline{R}^a{}_b(spin) = \underline{0} \quad -(207)$$

So the equations of ECE electrostatics are:

$$\underline{\nabla} \cdot \underline{E}^a = \underline{\omega}^a{}_b \cdot \underline{E}^b, \quad -(208)$$

$$\omega^a{}_{0b}\,\underline{E}^b = \phi^b\,\underline{R}^a{}_b(orb) \quad -(209)$$

$$\underline{\omega}^a{}_b \times \underline{E}^b + \phi^b\,\underline{R}^a{}_b(spin) = \underline{0} \quad -(210)$$

$$\underline{E}^a = -\underline{\nabla}\phi^a + \phi^b\,\underline{\omega}^a{}_b \quad -(211)$$

Later on in this chapter it is shown that these equations lead to a solution in terms of Bessel functions, but not to Euler Bernoulli resonance.

In order to obtain spin connection resonance Eq. (208) must be extended to:

$$\underline{\nabla} \cdot \underline{E}^a = \underline{\omega}^a{}_b \cdot \underline{E}^b - c\,\underline{A}^b(vac) \cdot \underline{R}^a{}_b(orb) \quad -(212)$$

where $\underline{A}^b(vac)$ is the Eckardt Lindstrom vacuum potential {1 - 10}. The static electric field is defined by:

$$\underline{E}^a = -\underline{\nabla}\phi^a + \phi^b\,\underline{\omega}^a{}_b \quad -(213)$$

so from Eqs. (212) and (213):

$$\nabla^2 \phi^a + \left(\underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \right) \phi^c = \underline{\nabla} \cdot \left(\phi^b \underline{\omega}^a{}_b \right) + \underline{\omega}^a{}_b \cdot \underline{\nabla} \phi^b + c \underline{A}^b(\text{vac}) \cdot \underline{R}^a{}_b(\text{orb}). \quad -(214)$$

By the ECE antisymmetry law:

$$- \underline{\nabla} \phi^a = \phi^b \underline{\omega}^a{}_b \quad -(215)$$

leading to the Euler Bernoulli resonance equation:

$$\nabla^2 \phi^a + \left(\underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \right) \phi^c = \frac{1}{2} c \underline{A}^b(\text{vac}) \cdot \underline{R}^a{}_b(\text{orb}) \quad -(216).$$

and spin connection resonance {1 - 10}. The left hand side contains the Hooke law term and the right hand side the driving term originating in the vacuum potential. Denote:

$$\rho^a(\text{vac}) = \frac{\epsilon_0 c}{2} \underline{A}^b(\text{vac}) \cdot \underline{R}^a{}_b(\text{orb}) \quad -(217)$$

then the equation becomes:

$$\nabla^2 \phi^a + \left(\underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \right) \phi^c = \frac{\rho^a(\text{vac})}{\epsilon_0}. \quad -(218)$$

The left hand side of Eq. (218) is a field property and the right hand side a property of the ECE vacuum. In the simplest case:

$$\nabla^2 \phi + \omega_0^2 \phi = \frac{\rho(\text{vac})}{\epsilon_0} \quad -(219)$$

and produces undamped resonance if:

$$\rho(\text{vac}) = \epsilon_0 A \cos \omega t \quad -(220)$$

where A is a constant. The particular integral of Eq. (219) is:

$$\phi = \frac{A \cos \omega Z}{\omega_0^2 - \omega^2} \quad -(221)$$

and spin connection resonance occurs at:

$$\omega = \omega_0 \quad -(222)$$

when:

$$\phi \rightarrow \infty \quad -(223)$$

and there is a resonance peak of electric field strength from the vacuum.

Later in this chapter solutions of Eq. (218) are given in terms of a combination of Bessel functions, and also an analysis using the Eckardt Lindstrom vacuum potential as a driving term.

In the absence of a magnetic monopole the Cartan identity is, as argued already:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b = 0 \quad -(224)$$

which implies:

$$\underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{A}^b = \underline{A}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(225)$$

A possible solution of this equation is:

$$\underline{\omega}^a{}_b = \epsilon^a{}_{bc}\, \underline{\omega}^c \quad -(226)$$

leading as argued already to a rigorous justification for O(3) electrodynamics. The Cartan identity (224) is itself a Beltrami equation:

$$\underline{\nabla} \times (\underline{\omega}^a{}_b \times \underline{A}^b) = \kappa\, \underline{\omega}^a{}_b \times \underline{A}^b. \quad -(227)$$

From Eqs. (226) and (227):

$$\underline{\nabla} \times (\underline{A}^c \times \underline{A}^b) = \kappa\, \underline{A}^c \times \underline{A}^b. \quad -(228)$$

In the complex circular basis:

$$\underline{A}^{(1)} \times \underline{A}^{(2)} = i\, A^{(0)} \underline{A}^{(3)*} \quad -(229)$$

$$\text{et cyclicum}$$

so from Eqs. (228) and (229):

$$\underline{\nabla} \times \underline{A}^{(i)} = \kappa\, \underline{A}^{(i)}, \quad i = 1, 2, 3 \quad -(230)$$

which are Beltrami equations as argued earlier in this chapter.

This result can be obtained self consistently using the Gauss law:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(231)$$

which as argued already implies the Beltrami equation:

$$\underline{\nabla} \times \underline{B}^a = \kappa\, \underline{B}^a. \quad -(232)$$

From Eqs. (168) and (232):

$$\underline{\nabla} \times \underline{B}^a = \kappa\, \underline{B}^a = \kappa (\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b) \quad -(233)$$

so:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) - \underline{\nabla} \times (\underline{\omega}^a{}_b \times \underline{A}^b) = \kappa (\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b) \quad -(234)$$

Using Eq. (227) gives:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) = \kappa \underline{\nabla} \times \underline{A}^a \quad -(235)$$

which implies Eqs. (230a) to (230c) Q. E. D. As shown earlier in this chapter the Beltrami structure also governs the spin connection vector:

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \kappa \underline{\omega}^a{}_b . \quad -(236)$$

It follows that the equations:

$$\underline{\omega}^{(3)} = \frac{1}{2} \frac{\kappa}{A^{(0)}} \underline{A}^{(3)} \quad -(237)$$

and:

$$\underline{\omega}^{(2)} = \frac{1}{2} \frac{\kappa}{A^{(0)}} \underline{A}^{(2)} \quad -(238)$$

produce O(3) electrodynamics {1 - 10}:

$$\underline{B}^{(1)*} = \underline{\nabla} \times \underline{A}^{(1)*} - i \frac{\kappa}{A^{(0)}} \underline{A}^{(2)} \times \underline{A}^{(3)} \quad -(239)$$

et cyclicum

As shown in Note 259(3) on www.aias.us there are many inter-related equations of O(3) electrodynamics which all originate in geometry.

Later in this chapter it is argued a consequence of these conclusions is that the spin connection and orbital curvature vectors also obey a Beltrami structure.

The fact that ECE is a unified field theory also allows the development and interrelation of several basic equations, including the definition of B(3):

$$\underline{B}^{(3)} = \underline{\nabla} \times \underline{A}^{(3)} - i \frac{\kappa}{A^{(0)}} \underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(240)$$

It can be written as:

$$\underline{B} = -i\frac{e}{\hbar}\underline{A}\times\underline{A}^{*} = B^{(0)}\underline{k} = B_Z\,\underline{k} \quad -(241)$$

Although B(3) is a radiated and propagating field as is well known {1 - 10} Eq. (241) can be used as a general definition of the magnetic flux density for a choice of potentials. This is important for the subject of magnetostatics and the development {1 - 10} of the fermion equation with:

$$\underline{A} = \frac{1}{2}\underline{B}\times\underline{r} . \quad -(242)$$

Eq. (241) gives the transition from classical to quantum mechanics. In ECE electrodynamics $\underline{A}$ must always be a Beltrami field and this is the result of the Cartan identity as already argued. So it is necessary to solve the following equations simultaneously:

$$\underline{B} = -i\frac{e}{\hbar}\underline{A}\times\underline{A}^{*},$$
$$\underline{A} = \frac{1}{2}\underline{B}\times\underline{r} , \quad \underline{\nabla}\times\underline{A} = \kappa\underline{A} . \quad -(243)$$

This can be done using the principles of general relativity, so that the electromagnetic field is a rotating and translating frame of reference . The position vector is therefore:

$$\underline{r} = \underline{r}^{*} = \frac{r^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\phi} \quad -(244)$$

where:

$$\underline{r} = \underline{r}^{(1)} , \quad \underline{r}^{*} = \underline{r}^{(2)} , \quad \phi = \omega t - \kappa Z \quad -(245)$$

so:

$$\underline{r}^{(1)}\times\underline{r}^{(2)} = i r^{(0)}\underline{r}^{(3)*} \quad -(246)$$

et cyclicum

It follows that:

$$\underline{\nabla} \times \underline{r}^{(1)} = \kappa \underline{r}^{(1)} \quad -(247)$$
$$\underline{\nabla} \times \underline{r}^{(2)} = \kappa \underline{r}^{(2)} \quad -(248)$$
$$\underline{\nabla} \times \underline{r}^{(3)} = 0 \underline{r}^{(3)} \quad -(249)$$

The results (246) for plane waves can be generalized to any Beltrami solutions, so it follows that spacetime itself has a Beltrami structure.

From Eqs. (242) and (244):

$$\underline{A} = \underline{A}^{(1)} = \frac{B^{(0)} r^{(0)}}{2\sqrt{2}} (i\underline{i} + \underline{j}) e^{i\phi} = \frac{A^{(0)}}{\sqrt{2}} (i\underline{i} + \underline{j}) e^{i\phi} \quad -(250)$$

where:

$$A^{(0)} = \frac{1}{2} B^{(0)} r^{(0)} \quad -(251)$$

and from Eq. (250):

$$\underline{\nabla} \times \underline{A} = \kappa \underline{A} \quad -(252)$$

QED. Therefore it is always possible to write the vector potential in the form (242) provided that spacetime itself has a Beltrami structure. This conclusion ties together several branches of physics because Eq. (242) is used to produce the Landé factor, ESR, NMR and so on from the Dirac equation, which becomes the fermion equation {1 - 10} in ECE physics.

As argued already the tetrad postulate and ECE postulate give:

$$(\square + \kappa_0^2) \underline{A} = \underline{0} \quad -(253)$$

and the fermion or chiral Dirac equation is a factorization of Eq. (253). As shown in chapter one:

$$\kappa_0^2 = q^{\nu}_{a} \partial^{\mu} \left(\omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu} \right) \quad -(254)$$

where q^{ν}_{a} is the inverse tetrad, defined by :

$$q^{a}_{\nu} q^{\nu}_{a} = 1. \quad -(255)$$

In generally covariant format Eq. (253) is:

$$\left(\Box + \kappa_0^2 \right) A^{a}_{\mu} = 0 \quad -(256)$$

and with:

$$A^{a}_{\mu} = \left(A^{a}_{0}, -\underline{A}^{a} \right) \quad -(257)$$

it follows that:

$$\left(\Box + \kappa_0^2 \right) A_0 = 0, \quad -(258)$$

$$\left(\Box + \kappa_0^2 \right) \underline{A} = \underline{0}, \quad -(259)$$

which gives Eq. (253) Q.E.D. The d'Alembertian is defined by:

$$\Box = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2. \quad -(260)$$

The Beltrami condition:

$$\underline{\nabla} \times \underline{A} = \kappa \underline{A} \quad -(261)$$

gives the Helmholtz wave equation:

$$\left(\nabla^2 + \kappa^2 \right) \underline{A} = \underline{0} \quad -(262)$$

if:

$$\underline{\nabla} \cdot \underline{A} = 0. \quad -(263)$$

From Eq. (259):

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \kappa_0^2 \right) \underline{A} = \underline{0} \quad -(264)$$

so:

$$\frac{1}{c^2} \frac{\partial^2 \underline{A}}{\partial t^2} + \left(\kappa_0^2 + \kappa^2 \right) \underline{A} = \underline{0} \quad -(265)$$

which is the equation for the time dependence of $\underline{A}$. The Helmholtz and Beltrami equations are for the space dependence of $\underline{A}$. Eq. (267) is satisfied by:

$$\underline{A} = \underline{A}_0 \exp(i \omega t) \quad -(266)$$

where:

$$\frac{\omega^2}{c^2} = \kappa^2 + \kappa_0^2. \quad -(267)$$

Eq. (267) is a generalization of the Einstein energy equation for a free particle:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(268)$$

where:

$$E = \hbar \omega, \quad \underline{p} = \hbar \underline{\kappa} \quad -(269)$$

using:

$$\kappa_0^2 = \left(\frac{mc}{\hbar} \right)^2 = q_a^{\nu} \partial^{\mu} \left(\omega_{\mu\nu}^{a} - \Gamma_{\mu\nu}^{a} \right). \quad -(270)$$

So mass in ECE theory is defined by geometry.

The general solution of Eq (256) is therefore:

$$A^a_\mu = A^a_\mu(0)\exp\left(i(\omega t - \kappa Z)\right) \quad -(271)$$

where:

$$\omega^2 = c^2(\kappa^2 + \kappa_0^2). \quad -(272)$$

It follows that there exist the equations:

$$(\Box + \kappa_0^2)\phi^a = 0 \quad -(273)$$

and

$$(\nabla^2 + \kappa^2)\phi^a = 0 \quad -(274)$$

where ϕ^a is the scalar potential in ECE physics. For each a:

$$(\nabla^2 + \kappa^2)\phi = 0. \quad -(275)$$

Now write:

$$\kappa_0 = \frac{mc}{\hbar} \quad -(276)$$

where m is mass. The relativistic wave equation for each a is:

$$(\Box + \kappa_0^2)\phi = 0 \quad -(277)$$

which is the quantized format of:

$$E^2 = c^2p^2 + m^2c^4 \quad -(278)$$
$$= c^2p^2 + \hbar^2\kappa_0^2c^2$$

Eq. (278) is:

$$E = \gamma m c^2 \quad -(279)$$

where the Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(280)$$

and where the relativistic momentum is:

$$\underline{p} = \gamma m \underline{v} . \quad -(281)$$

Define the relativistic energy as:

$$T = E - mc^2 \quad -(282)$$

and it follows that:

$$T = (\gamma - 1) m c^2 \xrightarrow[v \ll c]{} \frac{1}{2} m v^2 \quad -(283)$$

which is the non relativistic limit of the kinetic energy, i.e.:

$$T = \frac{p^2}{2m} . \quad -(284)$$

Using:

$$T = i\hbar \frac{\partial}{\partial t} , \; \underline{p} = -i\hbar \underline{\nabla} \quad -(285)$$

Eq. (284) quantizes to the free particle Schroedinger equation:

$$-\frac{\hbar^2}{2m} \nabla^2 \phi = T \phi \quad -(286)$$

which is the Helmholtz equation:

$$\left(\nabla^2 + \frac{2mT}{\hbar^2}\right)\phi = 0. \quad -(287)$$

It follows that the free particle Schroedinger equation is a Beltrami equation but with the vector potential replaced by the scalar potential ϕ, which plays the role of the wavefunction. It also follows in the non relativistic limit that:

$$\left(\nabla^2 + \frac{2mT}{\hbar^2}\right)\underline{A} = \underline{0} \quad -(288)$$

so:

$$\kappa^2 = \frac{2mT}{\hbar^2}. \quad -(289)$$

The Helmholtz equation (287) can be written as:

$$\left(\nabla^2 + \kappa^2\right)\phi = 0 \quad -(290)$$

which is an Euler Bernoulli equation without a driving term on the right hand side. In the presence of potential energy V Eq. (286) becomes:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\phi = E\phi \quad -(291)$$

where H is the hamiltonian and E the total energy:

$$E = T + V \quad -(292)$$

Eq. (291) is:

$$\left(\nabla^2 + \kappa^2\right)\phi = \left(\frac{2mV}{\hbar^2}\right)\phi \quad -(293)$$

which is an inhomogeneous Helmholtz equation similar to an Euler Bernoulli resonance equation with a driving term on the right hand side. However Eq. (293) is an eigenequation rather than an Euler Bernoulli equation as conventionally defined, but Eq. (293) has very well known resonance solutions in quantum mechanics. Eq. (293) may be written as:

$$(\nabla^2 + \kappa_1^2)\phi = 0 \quad -(294)$$

where:

$$\kappa_1^2 = \frac{2m}{\hbar^2}(E - V) \quad -(295)$$

and in UFT226 ff. on www.aias.us was used in the theory of low energy nuclear reactors (LENR). Eq. (294) is well known to be a linear oscillator equation which can be used to define the structure of the atom and nucleus. It can be transformed into an Euler Bernoulli equation as follows:

$$(\nabla^2 + \kappa_1^2)\phi = A\cos(\kappa_2 Z) \quad -(296)$$

where the right hand side represents a vacuum potential. It is exactly the structure obtained from the ECE Coulomb law as argued already.

3.4 THE BELTRAMI EQUATION FOR LINEAR MOMENTUM

The free particle Schroedinger equation can be obtained from the Beltrami equation for momentum:

$$\underline{\nabla} \times \underline{p} = \kappa \underline{p} \quad -(297)$$

which can be developed into the Helmholtz equation:

$$(\nabla^2 + \kappa^2)\underline{p} = \underline{0} \quad -(298)$$

if it is assumed that:

$$\underline{\nabla} \cdot \underline{p} = 0 \quad -(299)$$

If $\underline{p}$ is a linear momentum in the classical straight line then:

$$\kappa = 0. \quad -(300)$$

In general however $\underline{p}$ has intricate Beltrami solutions, some of which are animated in UFT258 on www.aias.us and its animation section.

Now quantize Eq. (298):

$$\underline{p}\,\phi = -i\hbar\,\underline{\nabla}\phi \quad -(301)$$

so:

$$(\nabla^2 + \kappa^2)\,\underline{\nabla}\phi = \underline{0} \quad -(302)$$

Use:

$$\nabla^2\,\underline{\nabla}\phi = \underline{\nabla}\,\nabla^2\phi \quad -(303)$$

and:

$$\underline{\nabla}(\kappa^2\phi) = \kappa^2\,\underline{\nabla}\phi \quad -(304)$$

assuming that:

$$\underline{\nabla}\kappa^2 = \underline{0} \quad -(305)$$

to arrive at:

$$\underline{\nabla}\left((\nabla^2 + \kappa^2)\phi\right) = \underline{0} \quad -(306)$$

A possible solution is:

$$(\nabla^2 + \kappa^2)\phi = 0 \quad -(307)$$

which is the Helmholtz equation for the scalar ϕ, the wave function of quantum mechanics.

The Schroedinger equation for a free particle is obtained by applying Eq. (301) to:

$$E = \frac{p^2}{2m} \quad -(308)$$

so:

$$-\frac{\hbar^2}{2m} \nabla^2 \phi = E\phi \quad -(309)$$

and:

$$\left(\nabla^2 + \frac{2Em}{\hbar^2} \right) \phi = 0 \quad -(310)$$

Eqs. (307) and (310) are the same if:

$$\kappa^2 = \frac{2Em}{\hbar^2} \quad -(311)$$

Q. E. D. Using the de Broglie relation:

$$\underline{p} = \hbar \underline{\kappa} \quad -(312)$$

then:

$$p^2 = 2Em \quad -(313)$$

which is Eq. (308), Q. E. D. Therefore the free particle Schroedinger equation is the

Beltrami equation:

$$\underline{\nabla} \times \underline{p} = \left(\frac{2Em}{\hbar^2} \right)^{1/2} \underline{p} \quad -(314)$$

with:

$$\underline{p} \phi = -i\hbar \underline{\nabla} \phi . \quad -(315)$$

It can be argued that the Schroedinger equation originates in the Beltrami equation.