2.3 THE B(3) FIELD IN CARTAN GEOMETRY.

The B(3) field is a consequence of the general expression for magnetic flux density in ECE theory:

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b . \quad -(54)$$

In general, summation over repeated indices means that:

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_{(1)} \times \mathbf{A}^{(1)} - \boldsymbol{\omega}^a{}_{(2)} \times \mathbf{A}^{(2)} - \boldsymbol{\omega}^a{}_{(3)} \times \mathbf{A}^{(3)} \quad -(55)$$

but this general expression can be simplified as discussed later in this book using the assumption:

$$\boldsymbol{\omega}^a{}_b = \epsilon^a{}_{bc}\, \boldsymbol{\omega}^c \quad -(56)$$

which is the expression for the duality of a tensor and vector. It can be shown using the vector form of the Cartan identity that the B(3) field is given by:

$$\mathbf{B}^{(3)} = \nabla \times \mathbf{A}^{(3)} - i\frac{k}{A^{(0)}}\, \mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \quad -(57)$$

where the potentials are related by the cyclic theorem:

$$\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} = i A^{(0)} \mathbf{A}^{(3)*} \quad -(58)$$

et cyclicum

For plane wave the potentials are as follows:

$$\mathbf{A}^{(1)} = \mathbf{A}^{(2)*} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} - i\mathbf{j})\, e^{i(\omega t - kz)}, \quad -(59)$$

$$\mathbf{A}^{(3)} = A^{(0)} \mathbf{k}, \quad -(60)$$

so the B(3) field is defined by:

$$\underline{B}^{(3)} = -\frac{i\kappa}{A^{(0)}} \underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(61)$$

Therefore B(3) is the result of general relativity, and does not exist in the Maxwell Heaviside field theory because the MH theory is a theory of special relativity without a geometrical connection. The B(3) field is a radiated longitudinal field that propagates in the (3) or Z axis. When it was inferred in Nov. 1991 it was a completely new concept, and it was gradually realized that it led to a higher topology electrodynamics which was identified with Cartan geometry in 2003. "Higher topology" in this sense means that a different differential geometry is needed to define electrodynamics. This can be seen through the fact that the field in U(1) gauge invariant electrodynamics is:

$$F = d \wedge A \quad -(62)$$

but in ECE theory it is:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \quad -(63)$$

with the presence of indices and spin connection. A choice of internal indices leads to O(3) electrodynamics as outlined above and explained in more detail later.

It was gradually realized that O(3) electrodynamics and ECE electrodynamics accurately reduce to the MH theory in certain limits, but also give much more information, an example being the inverse Faraday effect. The B(3) field led for the first time to an electrodynamics that is based on general covariance, and not Lorentz covariance, so it became easily possible to unify electromagnetism with gravitation.

It is important to realize that B(3) is not a static magnetic field, it interacts with material matter through the conjugate product $\underline{A}^{(1)} \times \underline{A}^{(2)}$ by which it is defined. So B(3) is intrinsically non linear in nature while a static magnetic field is not related to the conjugate product of non linear optics. The B(3) field needs for its definition a geometrical connection, and a different set of field equations from those that govern a static magnetic field. The latter is governed by the Gauss law of magnetism and the Ampere law. The static magnetic field does not propagate at c in the vacuum, but B(3) propagates in the vacuum along with A(1) and A(2) and when B(3) interacts with matter it produces a magnetization through a well defined hyperpolarizability in the inverse Faraday effect. The field equations needed to define B(3) must be obtained from Cartan geometry, and are not equations of Minkowski spacetime.

2.4. THE FIELD EQUATIONS OF ELECTROMAGNETISM.

These are based directly on the Cartan and Evans identities using the hypotheses (41) and (42) and give a richly structured theory summarized in the ECE Engineering Model on www.aias.us. Before proceeding to a description of the field equations a summary is given of the Cartan identity in vector notation. In a similar manner to the torsion, the second Cartan Maurer structure equation gives an orbital curvature and a spin curvature:

$$\underline{R}^a{}_b(\text{spin}) = \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b . \quad -(64)$$

As in UFT 254 consider now the Cartan identity:

$$d \wedge T^a + \omega^a{}_b \wedge T^b := R^a{}_b \wedge q^b . \quad -(65)$$

The space part of this identity can be written as:

$$\underline{\nabla} \cdot \underset{\sim}{T}^a + \underline{\omega}^a{}_b \cdot \underset{\sim}{T}^b = \underline{q}^b \cdot \left(\underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \right) \quad -(66)$$

Re arranging and using:

$$\underline{q}^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b = \underline{\omega}^a{}_b \cdot \underline{\omega}^a{}_c \times \underline{q}^c \quad -(67)$$

and
$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{q}^a = 0 \qquad -(68)$$

gives:
$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{q}^b = \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b - \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \qquad -(69)$$

i.e. gives the Cartan identity in vector notation, a very useful result that will be used later in this chapter and book. The self consistency and correctness of the result (69) is shown by the fact that it is an example of the well known vector identity:

$$\underline{\nabla} \cdot \underline{F} \times \underline{G} = \underline{G} \cdot \underline{\nabla} \times \underline{F} - \underline{F} \cdot \underline{\nabla} \times \underline{G}. \qquad -(70)$$

So it can be seen clearly that Cartan geometry generalizes well known geometry and vector identities.

For ECE electrodynamics Eq. (69) becomes:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b = \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{A}^b - \underline{A}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b. \qquad -(71)$$

The magnetic flux density is defined in ECE theory as:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \qquad -(72)$$

so:

$$\underline{\nabla} \cdot \underline{B}^a = -\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b \qquad -(73)$$

giving the Gauss law of magnetism in general relativity and ECE unified field theory.

As in UFT 256 the Cartan identity and the fundamental ECE hypotheses give the homogeneous field equations of electromagnetism in ECE theory:

$$\underline{\nabla} \cdot \underline{B}^a = \frac{\rho^m}{\epsilon_0 c} = \underline{\omega}^a{}_b \cdot \underline{B}^b - \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}) \qquad -(74)$$

and

$$\frac{\partial \underline{B}^a}{\partial t} + \underline{\nabla} \times \underline{E}^a = \underline{J}^m / \epsilon_0$$

$$= \underline{\omega}^a{}_b \times \underline{E}^b - c\omega_0 \underline{B}^a - c\left(\underline{A}^b \times \underline{R}^a{}_b(\text{orb}) - A_0^b \underline{R}^a{}_b(\text{spin})\right) \quad -(75)$$

in which the spin curvature is defined by Eq. (64) and the orbital curvature by:

$$\underline{R}^a{}_b(\text{orb}) = -\underline{\nabla}\, \omega^a{}_{0b} - \frac{1}{c}\frac{\partial \underline{\omega}^a{}_b}{\partial t} - \omega^a_{0}{}_c\, \underline{\omega}^c{}_b + \omega^c_{0b}\, \underline{\omega}^a{}_c . \quad -(76)$$

The right hand sides of these equations give respectively the magnetic charge density and the magnetic current density. The controversy over the existence of the magnetic charge current density has been going on for over a century, and the consensus seems to be that they do not exist. (If they are proven to be reproducible and repeatable the ECE theory can account for them as in the above equations.) If the magnetic charge current density vanishes then:

$$\underline{\omega}^a{}_b \cdot \underline{B}^b = \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}) \quad -(77)$$

and

$$\underline{\omega}^a{}_b \times \underline{E}^b - c\omega_0 \underline{B}^a = c\left(\underline{A}^b \times \underline{R}^a{}_b(\text{orb}) - A_0^b \underline{R}^a{}_b(\text{spin})\right) \quad -(78)$$

and they imply the Gauss law of magnetism in ECE theory:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(79)$$

and the Faraday law of induction:

$$\frac{\partial \underline{B}^a}{\partial t} + \underline{\nabla} \times \underline{E}^a = \underline{0} . \quad -(80)$$

The Evans identity gives

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 = \underline{\omega}^a{}_b \cdot \underline{E}^b - c\underline{A}^b \cdot \underline{R}^a{}_b(\text{orb}) \quad -(81)$$

and:

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a$$

$$= \underline{\omega}^a{}_b \times \underline{B}^b + \frac{\omega_0}{c} \underline{E}^b - A_0^b \underline{R}^a{}_b(\text{orb}) - \underline{A}^b \times \underline{R}^a{}_b(\text{spin}) \quad (82)$$

Eq. (81) defines the electric charge density:

$$\rho^a = \epsilon_0 \left(\underline{\omega}^a{}_b \cdot \underline{E}^b - c\underline{A}^b \cdot \underline{R}^a{}_b(\text{orb}) \right) - (83)$$

and Eq. (82) defines the electric current density:

$$\underline{J}^a = \frac{1}{\mu_0} \left(\underline{\omega}^a{}_b \times \underline{B}^b + \frac{\omega_0}{c} \underline{E}^b - \left(\underline{A}^b \times \underline{R}^a{}_b(\text{spin}) + A_0^b \underline{R}^a{}_b(\text{orb}) \right) \right) - (84)$$

With these definitions the inhomogeneous field equations become the Coulomb law:

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad - (85)$$

and the Ampère Maxwell law:

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad - (86)$$

2.5 THE FIELD EQUATIONS OF GRAVITATION.

As shown in the Engineering Model the field equations of gravitation are the two homogeneous field equations:

$$\underline{\nabla} \cdot \underline{h} = 4\pi G \rho_m \quad - (87)$$

and

$$\underline{\nabla} \times \underline{g} + \frac{1}{c} \frac{\partial \underline{h}}{\partial t} = \frac{4\pi G}{c} \underline{j}_m \quad - (88)$$

and the two inhomogeneous equations:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m \quad - (89)$$

and

$$\underline{\nabla} \times \underline{h} - \frac{1}{c}\frac{\partial \underline{g}}{\partial t} = \frac{4\pi G}{c}\underline{J}_m . \quad -(90)$$

Here $\underline{g}$ is the acceleration due to gravity, and $\underline{h}$ is the gravitomagnetic field, defined by the Cartan Maurer structure equations as:

$$\underline{g} = -\frac{\partial \underline{Q}}{\partial t} - \underline{\nabla}\,\underline{\Phi} - \omega_0 \underline{Q} + \underline{\Phi}\,\underline{\omega} \quad -(91)$$

and

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} - \underline{\omega} \times \underline{Q} . \quad -(92)$$

In the Newtonian physics only Eq. (89) exists, where G is the Newton constant and where ρ_m is the mass density. In the ECE equations there is a gravitomagnetic field $\underline{h}$ (developed in UFT 117 and UFT 118) and a Faraday law of gravitational induction, Eq. (88), developed in UFT 75. The latter paper describes the experimental evidence for the gravitational law of induction and UFT 117 and UFT 118 use the gravitomagnetic field to explain precession not explicable in the Newtonian theory.

It is likely that all the fields predicted by the ECE theory of gravitation will eventually be discovered because they are based on geometry as advocated by Kepler. During the course of the development of ECE there have been many advances in electromagnetism and gravitation. There has been space here for a short overview summary.