COLD CURRENT WAVE EQUATIONS FOR THE ECE POTENTIAL

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ABSTRACT:
By applying constraints equivalent to the Lorenz gauge of classical electromagnetic theory, wave equations in scalar and magnetic potentials of ECE theory for each of the Maxwellian and the cold currents are shown to result. Solutions provided by this method are unaffected by the constraint and are shown to be generally valid. The set of wave equations is equivalent to the original equations of the ECE engineering model.

I. REDUCED COLD CURRENT MODEL

The ECE Theory of Electromagnetism has been developed at great length elsewhere [www.aias.us] and will not be reviewed here. The field equations are identical in form to Maxwell's equations, ie.

(1) $$\nabla \bullet \boldsymbol{B} = 0$$

(2) $$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0$$

(3) $$\nabla \bullet \boldsymbol{E} = \frac{\rho}{\varepsilon}$$

(4) $$\nabla \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_o \boldsymbol{J}$$

where ***E*** and ***B*** are the electric and magnetic fields respectively and c is the speed of light in the medium. ***J*** is the current density, ρ is the charge density, ε is the permittivity and μ_o is the permeability of the medium.

In ECE theory, the electric and magnetic induction field have new definitions differing from Maxwell's theory which incorporate a scalar (ω_o) and vector spin connection ($\boldsymbol{\omega}$), ie.

(5) $$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla \phi - \omega_o \boldsymbol{A} + \boldsymbol{\omega} \phi$$

(6) $$\boldsymbol{B} = \nabla \times \boldsymbol{A} - \boldsymbol{\omega} \times \boldsymbol{A}$$

where ***A*** is the magnetic vector potential and ϕ the scalar potential.

For the cold current model, it is postulated here that a second electric and magnetic induction field given by the spin connections terms is set up quasi-independently of the Maxwell field. New field variables are postulated to be in the following form.

(7) $$\boldsymbol{E} = \boldsymbol{E_o} + \boldsymbol{E_1}$$

(8) $$\boldsymbol{B} = \boldsymbol{B_o} + \boldsymbol{B_1}$$

where

(9) $$\boldsymbol{E_o} = -\frac{\partial \boldsymbol{A_o}}{\partial t} - \nabla \phi_o$$

(10) $$\boldsymbol{E_1} = -\omega_o \boldsymbol{A_o} + \boldsymbol{\omega} \phi_o$$

(11) $$\boldsymbol{B_o} = \nabla \times \boldsymbol{A_o}$$

(12) $$\boldsymbol{B_1} = -\boldsymbol{\omega} \times \boldsymbol{A_o}$$

We have separated the current and charge density into two components, the Maxwellian component $\boldsymbol{J_o}$, ρ_o and the "cold" component $\boldsymbol{J_1}$, ρ_1 as before [1], ie.

(13) $$\boldsymbol{J} = \boldsymbol{J_o} + \boldsymbol{J_1}$$

(14) $$\rho = \rho_o + \rho_1$$

The Maxwellian and the cold current electric and magnetic induction fields are $\boldsymbol{E_o}, \boldsymbol{E_o}, \boldsymbol{E_1}$ and $\boldsymbol{B_1}$ respectively.

If we substitute equations (7) and (8) into equations (1) through (6), we get

(16) $$\nabla \bullet (\boldsymbol{B_o} + \boldsymbol{B_1}) = 0$$

(17) $$\nabla \times (\boldsymbol{E_o} + \boldsymbol{E_1}) + \frac{\partial (\boldsymbol{B_o} + \boldsymbol{B_1})}{\partial t} = 0$$

(18) $$\nabla \bullet (\boldsymbol{E_o} + \boldsymbol{E_1}) = \frac{\rho_o + \rho_1}{\varepsilon}$$

(19) $$\nabla \times (\boldsymbol{B_o} + \boldsymbol{B_1}) - \frac{1}{c^2} \frac{\partial (\boldsymbol{E_o} + \boldsymbol{E_1})}{\partial t} = \mu_o (\boldsymbol{J_o} + \boldsymbol{J_1})$$

Separating these into Maxwellian and cold current pairs as in [1], we have

(20) $$\nabla \bullet (\boldsymbol{B_o}) = 0$$

(21) $$\nabla \times (\boldsymbol{E_o}) + \frac{\partial (\boldsymbol{B_o})}{\partial t} = 0$$

(22) $$\nabla \bullet (\boldsymbol{E_o}) = \frac{\rho_o}{\varepsilon}$$

(23) $$\nabla \times (\boldsymbol{B_o}) - \frac{1}{c^2}\frac{\partial(\boldsymbol{E_o})}{\partial t} = \mu_o \boldsymbol{J_o}$$ and

(24) $$\nabla \bullet (\boldsymbol{B_1}) = 0$$

(25) $$\nabla \times (\boldsymbol{E_1}) + \frac{\partial(\boldsymbol{B_1})}{\partial t} = 0$$

(26) $$\nabla \bullet (\boldsymbol{E_1}) = \frac{\rho_1}{\varepsilon}$$

(27) $$\nabla \times (\boldsymbol{B_1}) - \frac{1}{c^2}\frac{\partial(\boldsymbol{E_1})}{\partial t} = \mu_o \boldsymbol{J_1}$$

We can the rewrite equation (20) through (27) as the cold current equation set presented earlier [1] in a more elegant fashion by introducing new cold current scalar and vector potentials.

Substituting equations (9) and (10) into Maxwell equations (22) and (23) gives

(28) $$\nabla \times \nabla \times \boldsymbol{A_o} + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{A_o}}{\partial t^2} + \nabla\left(\frac{\partial \phi_o}{\partial t}\right)\right) = \mu_o \boldsymbol{J_o}$$

(29) $$\nabla \bullet \left(\frac{\partial \boldsymbol{A_o}}{\partial t} + \nabla \phi_o\right) = -\frac{\rho_o}{\varepsilon}$$

for the Maxwellian pair of equations.

Noting equation (24) we can introduce a cold current magnetic vector potential $\boldsymbol{A_1}$ where

(30) $$\boldsymbol{B_1} = \nabla \times \boldsymbol{A_1}$$

and upon substitution of this into equation (25) allows us to write in a completely general manner,

(31) $$\boldsymbol{E_1} = -\frac{\partial \boldsymbol{A_1}}{\partial t} - \nabla \phi_1$$

where ϕ_1 is a new cold current scalar potential.

Substitution of (30) and (31) into the remaining equations (26) and (27) gives

(32) $$\nabla \times \nabla \times \boldsymbol{A_1} + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{A_1}}{\partial t^2} + \nabla\left(\frac{\partial \phi_1}{\partial t}\right)\right) = \mu_o \boldsymbol{J_1}$$

(33) $$\nabla \bullet \left(\frac{\partial \boldsymbol{A_1}}{\partial t} + \nabla \phi_1\right) = -\frac{\rho_1}{\varepsilon}$$ for the cold current pair of equations.

But we also have from equations (10), (12), (30) and (31) that

(34) $$\nabla \times \boldsymbol{A_1} = -\boldsymbol{\omega} \times \boldsymbol{A_o}$$ and

(35) $$\nabla \phi_1 = -\frac{\partial \boldsymbol{A_1}}{\partial t} + \omega_o \boldsymbol{A_o} - \boldsymbol{\omega} \phi_o$$

These latter two equations represent the connectivity equations between the Maxwellian pair and the cold current pair.

We note that

(36) $$\phi = \phi_o + \phi_1$$

which is easily seen by substituting (9)and (31) into (17).

It is also relatively simple to see that

(37) $$\boldsymbol{A} = \boldsymbol{A_o} + \boldsymbol{A_1}$$

Adding equation (28) to (32) and utilizing equation (13) yields the original ECE Ampere – Maxwell electromagnetic equation [3].

The reduced cold current equations are presented in summary form in Table 1.

Table 1 Cold Current Equation Summary

Maxwell Equations	**Cold Current Equations**	**Connectivity Equations**
$\nabla \times \nabla \times \boldsymbol{A_o} + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{A_o}}{\partial t^2} + \nabla\left(\frac{\partial \phi_o}{\partial t}\right)\right) = \mu_o \boldsymbol{J_o}$	$\nabla \times \nabla \times \boldsymbol{A_1} + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{A_1}}{\partial t^2} + \nabla\left(\frac{\partial \phi_1}{\partial t}\right)\right) = \mu_o \boldsymbol{J_1}$	$\nabla \times \boldsymbol{A_1} = -\boldsymbol{\omega} \times \boldsymbol{A_o}$
$\nabla \bullet \left(\frac{\partial \boldsymbol{A_o}}{\partial t} + \nabla \phi_o\right) = -\frac{\rho_o}{\varepsilon}$	$\nabla \bullet \left(\frac{\partial \boldsymbol{A_1}}{\partial t} + \nabla \phi_1\right) = -\frac{\rho_1}{\varepsilon}$	$\nabla \phi_1 = -\frac{\partial \boldsymbol{A_1}}{\partial t} + \omega_o \boldsymbol{A_o} - \boldsymbol{\omega} \phi_o$

where $\boldsymbol{J} = \boldsymbol{J_o} + \boldsymbol{J_1}$ $\rho = \rho_o + \rho_1$ $\phi = \phi_o + \phi_1$ $\boldsymbol{A} = \boldsymbol{A_o} + \boldsymbol{A_1}$

It was demonstrated earlier [1] that each of these pair of curl and divergence equations is over specified. It is interesting to note that we have lost one pair of curl and divergence equations in this representation, having introduced that pair instead as the connectivity equations.

II. POTENTIAL BASED WAVE EQUATIONS

A wave equation for both Maxwell and the cold current can be derived as follows. Equation (32) can be written as

(38) $$-\nabla^2 \boldsymbol{A_o} + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{A_o}}{\partial t^2}\right) + \nabla(\nabla \bullet \boldsymbol{A_o}) + \frac{1}{c^2}\nabla\left(\frac{\partial \phi_o}{\partial t}\right) = \mu_o \boldsymbol{J_o}$$

If

(39) $$\nabla(\nabla \bullet \boldsymbol{A_o}) + \frac{1}{c^2}\nabla\left(\frac{\partial \phi_o}{\partial t}\right) = 0$$

then

(40) $$-\nabla^2 \boldsymbol{A_o} + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{A_o}}{\partial t^2}\right) = \mu_o \boldsymbol{J_o}$$

which is the accepted Maxwell wave equation for the magnetic vector potential.

Equation (39) is a generalization of the Lorenz gauge of traditional EM theory [2], ie.

(41) $$\nabla \bullet \boldsymbol{A_o} + \frac{1}{c^2}\left(\frac{\partial \phi_o}{\partial t}\right) = 0$$

According to [2], all potentials that satisfy Maxwell's equations satisfy the Lorenz constraint, so that application of the constraint is not limiting.

Equation (29) using equation equation (41) becomes

(42) $$-\nabla^2 \boldsymbol{\phi_o} + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{\phi_o}}{\partial t^2}\right) = \frac{\boldsymbol{\rho_o}}{\varepsilon}$$

which is the Maxwellian wave equation for the scalar or electric potential.

The same methods apply to the cold current equations. A wave equation for the cold current can be derived as follows. Equation (32) can be written as

(43) $$-\nabla^2 \boldsymbol{A_1} + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{A_1}}{\partial t^2}\right) + \nabla(\nabla \bullet \boldsymbol{A_1}) + \frac{1}{c^2}\nabla\left(\frac{\partial \phi_1}{\partial t}\right) = \mu_o \boldsymbol{J_1}$$

If

(44) $$\nabla(\nabla \bullet \boldsymbol{A_1}) + \frac{1}{c^2}\nabla\left(\frac{\partial \phi_1}{\partial t}\right) = 0$$

then

(45) $$-\nabla^2 \boldsymbol{A_1} + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{A_1}}{\partial t^2}\right) = \mu_o \boldsymbol{J_1}$$

which is the cold current wave equation expressed in terms of the cold current magnetic potential.

Equation (33) can be treated in a manner similar to equation (29) to give

(46) $$-\nabla^2 \phi_1 + \frac{1}{c^2}\left(\frac{\partial^2 \phi_1}{\partial t^2}\right) = \frac{\rho_1}{\varepsilon}$$

which is the cold current wave equation for the second or cold current scalar potential.

Note that

(47) $$\nabla \bullet \boldsymbol{A_1} + \frac{1}{c^2}\left(\frac{\partial \phi_1}{\partial t}\right) = 0$$

is a constraint identical in form to the Lorenz constraint but now applied to the cold current equations.

For the wave equations to be valid, the Lorenz constraint must be compatible with the connectivity equations (34) and (35). Taking the divergence of the second connectivity equation (35) and noting equations (10) and (26) gives

(48) $$\nabla^2 \phi_1 + \frac{\partial \nabla \bullet \boldsymbol{A_1}}{\partial t} = \nabla \bullet (\omega_o \boldsymbol{A_o} - \boldsymbol{\omega} \phi_o) = \frac{\rho_1}{\varepsilon}$$

which we recognize to be generally valid, so that the first connectivity equation is unchanged under the Lorenz constraint.

If one substitutes equations (12) and (31) into equation (25), one gets equation (34). This demonstrates that the second connectivity equation is unchanged under a Lorenz constraint.

It is straightforward to demonstrate that the two connectivity equations are interdependent. Taking the curl of equation (34) and the time derivative of equation (35) and eliminating $\boldsymbol{A_1}$ gives the cold current Faraday equation (25) demonstrating that the two connectivity equations are dependent upon each other and collapse to the Faraday equation.

The first connectivity equation can be reduced to a wave equation. Taking the curl of equation (34) gives

(49) $$\nabla \times \nabla \times \boldsymbol{A_1} = \nabla \times (-\boldsymbol{\omega} \times \boldsymbol{A_o})$$ but by virtue of equation (32) becomes

(50) $$\nabla \times (-\boldsymbol{\omega} \times \boldsymbol{A_o}) + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{A_1}}{\partial t^2} + \nabla\left(\frac{\partial \phi_1}{\partial t}\right)\right) = \mu_o \boldsymbol{J_1}$$

Using equation (35), this becomes

(51) $$\nabla \times (-\boldsymbol{\omega} \times \boldsymbol{A_o}) + \frac{1}{c^2}\frac{\partial(\omega_o \boldsymbol{A_o} - \boldsymbol{\omega} \phi_o)}{\partial t} = \mu_o \boldsymbol{J_1}$$

One could further take the curl of this equation to get

(52) $$-\nabla^2(-\boldsymbol{\omega}\times\boldsymbol{A_o})+\frac{1}{c^2}\nabla\times\frac{\partial(\omega_o\boldsymbol{A_o}-\boldsymbol{\omega}\phi_o)}{\partial t}=\mu_o\nabla\times\boldsymbol{J_1}$$

From equation (25), (10) and (12)

(53) $$\nabla\times(\omega_o\boldsymbol{A_o}-\boldsymbol{\omega}\phi_o)=-\frac{\partial(\boldsymbol{\omega}\times\boldsymbol{A_o})}{\partial t}$$

so that equation (52) becomes

(54) $$-\nabla^2(-\boldsymbol{\omega}\times\boldsymbol{A_o})+\frac{1}{c^2}\frac{\partial^2(-\boldsymbol{\omega}\times\boldsymbol{A_o})}{\partial t^2}=\mu_o\nabla\times\boldsymbol{J_1}$$

which is the third wave equation developed in an earlier paper [1].

The second connectivity equation (35) can be shown to be equivalent to equation (46) by taking the divergence of (35), substituting in equation (48) and noting equation (26). This yields no new information.

The final equation needed to complete the set of connectivity equations is a re-expression of equation (47), ie.

(55) $$\nabla\bullet(\omega_o\boldsymbol{A_o}-\boldsymbol{\omega}\phi_o)=\frac{\rho_1}{\varepsilon}$$

Although equation (55) is not independent of the rest, it allows the subsequent calculation of ω_o once the other field variables are known.

We have thus demonstrated that given the Lorenz conditions for the Maxwell and the cold current equations that the potential wave equations are completely general and can be used interchangeably with the general cold current equations of ECE EM theory. These equations are summarized in Table 2.

Table 2 Cold Current Wave Equation Summary

Maxwell Equations	**Cold Current Equations**	**Connectivity Equations**
$-\nabla^2\boldsymbol{A_o}+\frac{1}{c^2}\left(\frac{\partial^2\boldsymbol{A_o}}{\partial t^2}\right)=\mu_o\boldsymbol{J_o}$	$-\nabla^2\boldsymbol{A_1}+\frac{1}{c^2}\left(\frac{\partial^2\boldsymbol{A_1}}{\partial t^2}\right)=\mu_o\boldsymbol{J_1}$	$\nabla\times\boldsymbol{A_1}=-\boldsymbol{\omega}\times\boldsymbol{A_o}$ or $-\nabla^2(-\boldsymbol{\omega}\times\boldsymbol{A_o})+\frac{1}{c^2}\frac{\partial^2(-\boldsymbol{\omega}\times\boldsymbol{A_o})}{\partial t^2}=\mu_o\nabla\times\boldsymbol{J_1}$
$-\nabla^2\phi_o+\frac{1}{c^2}\left(\frac{\partial^2\phi_o}{\partial t^2}\right)=\frac{\rho_o}{\varepsilon}$	$-\nabla^2\phi_1+\frac{1}{c^2}\left(\frac{\partial^2\phi_1}{\partial t^2}\right)=\frac{\rho_1}{\varepsilon}$	$\nabla\bullet(\omega_o\boldsymbol{A_o}-\boldsymbol{\omega}\phi_o)=\frac{\rho_1}{\varepsilon}$

where $\boldsymbol{J}=\boldsymbol{J_o}+\boldsymbol{J_1}$ $\rho=\rho_o+\rho_1$ $\phi=\phi_o+\phi_1$ $\boldsymbol{A}=\boldsymbol{A_o}+\boldsymbol{A_1}$

The equation set consist of a vector and scalar wave equation for the Maxwellian and the cold current fields, and four connectivity equations, one of which can be represented as a vector wave equation.

III. BOUNDARY CONDITIONS

Solutions to the equation system presented in Table 2 require the specification of boundary conditions. One in general does not know anything about the spin connection values or the value of the cold current variables other than the assumption that they vanish at sufficient distances from the source of the disturbance.

Equation (12) and (24) gives one boundary condition. From the divergence theorem, this equation can be written

(56) $$\int \nabla \bullet (\boldsymbol{\omega} \times \boldsymbol{A_o}) \cdot dV = \oint (\boldsymbol{\omega} \times \boldsymbol{A_o}) \bullet \boldsymbol{n} \cdot ds = 0$$

This, as discussed in [4] represents the normal component of the flux of the quantity $(\boldsymbol{\omega} \times \boldsymbol{A_o})$ at a boundary and so gives a form of boundary condition on the problem that could be called a "flux boundary condition" to differentiate it from both the Dirichlet and the Neumann (and Robin) conditions.

Similarly, equation (55) can generate a boundary condition through the use of the Divergence theorem

(57) $$\int \nabla \bullet (-\boldsymbol{\omega} \phi_o + \omega_o \boldsymbol{A_o}) \cdot dV = \oint (-\boldsymbol{\omega} \phi_o + \omega_o \boldsymbol{A_o}) \bullet \boldsymbol{n} ds = \frac{\rho_1}{\varepsilon_o}$$

which represents the normal flux of the cold electric field $\boldsymbol{E_1} = -\boldsymbol{\omega} \phi_o + \omega_o \boldsymbol{A_o}$ at a boundary should the cold current charge distribution be known.

Boundary conditions on $\boldsymbol{A_o}$, ϕ_o, and the Maxwellian electric or magnetic field are generally known in enough detail to specify the problem. The question then remains as to whether equations (56) and (57), and the fact that we want the spin connection variables to vanish at sufficient distance from the source provides sufficient boundary information for a solution to be specified.

IV. CONSTITUTIVE RELATIONSHIPS

For the purposes of this discussion, we will assume that there are no electric or magnetic polarization in any of the materials. This restriction will be lifted in future variations of this theme.

A relationship generally exists in Maxwellian theory between the electric and magnetic inductive field and the current flowing in a material [2] ie.

(58) $$\boldsymbol{J_o} = \boldsymbol{F}(\boldsymbol{E_o}, \boldsymbol{B_o})$$

We postulate a similar relationship for the secondary current, ie.

(59) $$\boldsymbol{J_1} = \boldsymbol{F}(\boldsymbol{E_1}, \boldsymbol{B_1}).$$

For the case of a conductive material where the currents obey Ohm's Law, equations (58) and (59) reduce to,

(60) $$\boldsymbol{J_0} = \sigma_o \boldsymbol{E}_o$$

(61) $$\boldsymbol{J_1} = \sigma_1 \boldsymbol{E}_1$$

where σ_1 is a material property somewhat akin to conductivity σ_o in traditional Maxwell-Heaviside theory.

V. CONCLUSION

The Lorenz constraint of classical electromagnetic theory is shown to be valid for the cold current model of the ECE electromagnetic equations. Wave equations in scalar and magnetic potentials for each of the Maxwellian and the cold currents are shown to result, and to be generally valid under this constraint.

REFERENCES

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