REBUTTAL OF MISLEADING REMARKS BY BRUHN: THE B CYCLIC THEOREM

This is one in a long series of corrections of trivial or misleading claims by the known cyberstalker G. Bruhn, a retired academic apparently resident in Darmstadt. This one is in respect of the B Cyclic Theorem:

$$\underline{B}^{(1)} \times \underline{B}^{(2)} = i B^{(0)} \underline{B}^{(3)*} \quad -(1)$$

et cyclicum

where

$$\underline{B}^{(1)} = \underline{B}^{(2)*} = \frac{B^{(0)}}{\sqrt{2}} \left(\underline{i} - i \underline{j} \right) e^{i\phi} \quad -(2)$$

and where

$$\underline{B}^{(3)} = \underline{B}^{(3)*} = B^{(0)} \underline{k} . \quad -(3)$$

Here B denotes magnetic flux density and ϕ the phase of a plane wave. Define the unit vectors of the complex circular basis by:

$$\underline{e}^{(1)} = \underline{e}^{(2)*} = \left(\underline{i} - i \underline{j} \right) / \sqrt{2}, \quad -(4)$$

$$\underline{e}^{(3)} = \underline{e}^{(3)*} = \underline{k} . \quad -(5)$$

By simple algebra, Eq. (1) reduces to

$$\underline{e}^{(1)} \times \underline{e}^{(2)} = i \underline{e}^{(3)*} \quad -(6)$$

et cyclicum

which is the frame of reference itself. This frame is equivalent to the well known Cartesian frame:

$$\underline{i} \times \underline{j} = \underline{k} \quad -(7)$$

et cyclicum

made up of the unit vectors i, j and k. By definition the cyclic relations (7) are invariant under the general coordinate transformation, Q.E.D. Therefore the B Cyclic Theorem is invariant under the Lorentz transformation, Q.E.D.

The most general statement of this type of invariance is found in :

S. P. Carroll, "Spacetime and Geometry: an Introduction to General Relativity" (Addison Wesley 2004). In his 1997 notes this is on page 43. The complete vector field in any dimension and any spacetime is invariant under the general coordinate transformation:

$$V = V^{\mu} e_{\mu} = V^{\mu'} e_{\mu'}' \quad -(8)$$

where V^{μ} are its components and e_{μ} the elements of the basis set. For unit vectors:

$$V^{\mu} = e^{\mu} \quad -(10)$$

so:

$$V = e^{\mu} e_{\mu} = e^{\mu'} e_{\mu'} \quad -(11)$$

and the complete vector field is the covariant contravariant product of unit vectors. The contravariant unit vector transforms as:

$$e^{\mu'} = \Lambda^{\mu'}{}_{\mu} e^{\mu} \quad -(12)$$

where $\Lambda^{\mu'}{}_{\mu}$ is the Lorentz transform matrix, for example the Lorentz boost matrix:

$$\Lambda^{\mu'}{}_{\mu} = \begin{bmatrix} \cosh\phi & -\sinh\phi & 0 & 0 \\ -\sinh\phi & \cosh\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -(13)$$

and the covariant unit vector transforms as:

$$e_{\mu'} = \Lambda^{\mu}{}_{\mu'} e_{\mu} \quad -(14)$$

where $\Lambda^{\mu}{}_{\mu'}$ is the inverse Lorentz transform matrix, for example the inverse boost matrix:

$$\Lambda^{\mu}{}_{\mu'} = \begin{bmatrix} \cosh\phi & \sinh\phi & 0 & 0 \\ \sinh\phi & \cosh\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -(15)$$

So we obtain the results:

$$\begin{aligned} e^{0'} &= e^{0}\cosh\phi - e^{1}\sinh\phi, \quad & e^{2'} &= e^{2} \\ e^{1'} &= -e^{0}\sinh\phi + e^{1}\cosh\phi, \quad & e^{3'} &= e^{3} \end{aligned} \quad -(16)$$

and

$$\begin{aligned} e_{0}' &= e_{0}\cosh\phi + e_{1}\sinh\phi, \quad & e_{2}' &= e_{2}, \\ e_{1}' &= e_{0}\sinh\phi + e_{1}\cosh\phi, \quad & e_{3}' &= e_{3} \end{aligned} \quad -(17)$$

It is seen that:

$$e^{0\prime} = (\cosh\phi - \sinh\phi)\, e^{0} \quad -(18)$$

and:

$$e^{1\prime} = (\cosh\phi - \sinh\phi)\, e^{1}. \quad -(19)$$

In vector notation:

$$\underline{i}^{\prime} = (\cosh\phi - \sinh\phi)\, \underline{i}. \quad -(20)$$

Finally use:

$$\cosh\phi = \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}, \quad \sinh\phi = \frac{v}{c}\gamma, \quad -(21)$$

to find that the effect of the Lorentz boost is

$$\underline{i}^{\prime} = \left(\frac{1 - v/c}{1 + v/c}\right)^{1/2} \underline{i} \quad -(22)$$

Therefore:

$$\boxed{\underline{i}^{\prime} = \left(\frac{1 - v/c}{1 + v/c}\right)^{1/2} \underline{j}^{\prime} \times \underline{k}^{\prime}} \quad -(23)$$

and Eq. (7) is changed to Eq. (23) by the Lorentz boost. Therefore Eq. (7) is a Lorentz covariant equation because it is made up of Lorentz covariant quantities. This is an obvious result because a frame of reference is automatically an equation of physics and automatically Lorentz covariant. The B cyclic theorem is Lorentz covariant and is an equation of physics, QED.

Comments

It is known that G. Bruhn runs a website with trivially misleading claims. In this case his intent was to mislead colleagues into thinkink that the B Cyclci threom is somehow not Lorentz covariant. Two journals have published the misleading claim about the B Cyclic theorem by Bruhn: Physica Scripta and Foundations of Physics, and I request the editors responsible to retract the Bruhn papers.

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British Civil List
22nd May 2011.