1)

PROOF OF THE FREE SPACE CONDITION:

$$\omega^a{}_b = \kappa \epsilon^a{}_{bc} q^c$$

This fundamental condition is a solution of

$$\boxed{R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b} \quad \text{— (1)}$$

i.e.

$$(D \wedge \omega^a{}_b) \wedge q^b = \omega^a{}_b \wedge (D \wedge q^b) \quad \text{— (2)}$$

or

$$(d \wedge \omega^a{}_b) \wedge q^b + (\omega^a{}_c \wedge \omega^c{}_b) \wedge q^b$$
$$= \omega^a{}_b \wedge (d \wedge q^b) + \omega^a{}_b \wedge (\omega^b{}_c \wedge q^c).$$
$$\text{— (3)}$$

~~To~~ Prove

$$(d \wedge \omega^a{}_b) \wedge q^b = \omega^a{}_b \wedge (d \wedge q^b)$$
$$\text{— (4)}$$

Proof

For $a = 1$:

$$(d \wedge \omega^1{}_2) \wedge q^2 + (d \wedge \omega^1{}_3) \wedge q^3 \quad \text{— (5)}$$
$$= \omega^1{}_2 \wedge (d \wedge q^2) + \omega^1{}_3 \wedge (d \wedge q^3)$$

Eq. (5) is true if

$$\omega^1{}_2 = \kappa \epsilon^1{}_{23} q^3 = \kappa q^3 \quad \text{— (6)}$$

$$\omega^1{}_3 = \kappa \epsilon^1{}_{32} q^2 = -\kappa q^2 \quad \text{— (7)}$$

2) i.e.

$$(d \wedge q^3) \wedge q^2 - (d \wedge q^2) \wedge q^3 \qquad -(8)$$
$$= q^3 \wedge (d \wedge q^2) - q^2 \wedge (d \wedge q^3)$$
$$\Rightarrow \quad (d \wedge q^3) \wedge q^2 = -q^2 \wedge (d \wedge q^3)$$
$$-(d \wedge q^2) \wedge q^3 = q^3 \wedge (d \wedge q^2)$$
$$\qquad -(9)$$

Q.E.D.

To Prove

$$(\omega^a{}_c \wedge \omega^c{}_b) \wedge q^b = \omega^a{}_b \wedge (\omega^b{}_c \wedge q^c)$$
$$\qquad -(10)$$

Proof

For $a = 1$, $b = 2$, $c = 3$:

$$(\omega^1{}_3 \wedge \omega^3{}_2) \wedge q^2 = \omega^1{}_2 \wedge (\omega^2{}_3 \wedge q^3)$$
$$\omega^1{}_2 = K q^3 \; ; \; \omega^1{}_3 = -K q^2 \qquad -(11)$$
$$\omega^3{}_2 = -K q^1 \; ; \; \omega^2{}_3 = K q^1$$
$$\therefore \quad (q^2 \wedge q^1) \wedge q^2 = -q^3 \wedge (q^1 \wedge q^3)$$
$$\text{i.e.} \quad q^3 \wedge q^2 = -q^3 \wedge (-q^2)$$
$$= q^3 \wedge q^2 \qquad -(12)$$

Q.E.D

3) For o(3) electrodynamics we choose:

$$\omega^a{}_b = -\frac{1}{2}\kappa\,\epsilon^a{}_{bc}\,q^c \quad -(13)$$

in the structure relation:

$$D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \quad -(14)$$

<u>Proof</u> For $a = 1$:

$$D \wedge q^1 = d \wedge q^1 - \frac{1}{2}\left(\epsilon^1{}_{23}\,q^3 \wedge q^2 + \epsilon^1{}_{32}\,q^2 \wedge q^3\right) \quad -(15)$$

$$\boxed{D \wedge q^1 = d \wedge q^1 + \kappa\, q^2 \wedge q^3} \quad -(15a)$$

In the o(3) circular complex basis ~~this~~ gives o(3) electrodynamics.

Q.E.D.

This allows the tetrad of the free field to be identified as the potential, and also the spin connection. <u>o(3) electrodynamics</u> <u>is</u> <u>therefore</u> a <u>fundamental</u> <u>theory</u> <u>of</u> <u>general</u> <u>relativity</u>.

Conversely, O(3) e/m is the fundamental theory of electrodynamics, in which the spin connection and tetrad are duals:

$$\boxed{\omega^1{}_2 = -\frac{1}{2}\kappa q^3 \quad \text{et cyclicum}} \quad -(16)$$

and:

$$\boxed{\begin{aligned} \omega^0{}_1 &= \omega^2{}_3 \\ \omega^0{}_2 &= \omega^3{}_1 \\ \omega^0{}_3 &= \omega^1{}_2 \end{aligned}} \quad -(17)$$

Eqn (17) follows from the 4-D duality:

$$\omega^0{}_1 = \frac{1}{2}\left(\epsilon^0{}_{12}{}^3\omega^2{}_3 + \epsilon^0{}_{13}{}^2\omega^3{}_2\right)$$
$$= \omega^2{}_3 \quad -(18)$$

So it may be concluded that electromagnetism in free space is governed by the condition (1), the spin connection has the same symmetry as the field tensor, the tetrad has the same symmetry as the field components.

ELECTROMAGNETISM IS SPINNING SPACETIME