SPIN CONNECTION RESONANCE IN GRAVITATIONAL GENERAL RELATIVITY*

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(Received August 16, 2006)

The equations of gravitational general relativity are developed with Cartan geometry using the second Cartan structure equation and the second Bianchi identity. These two equations combined result in a second order differential equation with resonant solutions. At resonance the force due to gravity is greatly amplified. When expressed in vector notation, one of the equations obtained from the Cartan geometry reduces to the Newton inverse square law. It is shown that the latter is always valid in the off resonance condition, but at resonance, the force due to gravity is greatly amplified even in the Newtonian limit. This is a direct consequence of Cartan geometry. The latter reduces to Riemann geometry when the Cartan torsion vanishes and when the spin connection becomes equivalent to the Christoffel connection.

PACS numbers: 95.30.Sf, 03.50.-z, 04.50.+h

1. Introduction

It is well known that the gravitational general relativity is based on Riemann geometry with a Christoffel connection. This type of geometry is a special case of Cartan geometry [1] when the torsion form is zero. Therefore gravitation general relativity can be expressed in terms of this limit of Cartan geometry. In this paper this procedure is shown to produce a second order differential equation with resonant solutions [2–20]. Off resonance the mathematical form of the Newton inverse square law is obtained from a well defined approximation to the complete theory, but the relation between the gravitational potential field and the gravitational force field is shown to contain the spin connection in general. At resonance the force field can be greatly amplified, or conversely decreased. This is shown in Section 2

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and given the appellation "spin connection resonance" (SCR). A short discussion is given of possible technological implications in the area of counter gravitation.

2. Gravitation and Cartan geometry

The existence of SCR in the theory of gravitation is shown by consideration of the second Cartan structure equation

$$R^a{}_b = D \wedge \omega^a{}_b \tag{1}$$

and the second Bianchi identity of Cartan geometry

$$D \wedge R^a{}_b := 0 . \tag{2}$$

Therefore

$$D \wedge (D \wedge \omega^a{}_b) = 0 \tag{3}$$

and this is a second order differential equation with resonant solutions [2–20]. In these equations $\omega^a{}_b$ is the spin connection form [1–20] and $R^a{}_b$ is the curvature or Riemann form. The symbol $D\wedge$ is the covariant exterior derivative of Cartan geometry and $\wedge$ represents the wedge product of Cartan geometry. If the torsion form T^a of Cartan geometry is zero

$$T^a = 0 . \tag{4}$$

Eqs. (1) to (3) reduce to Riemann geometry, and are fully equivalent to Riemann geometry [1-20]. It is well known that Riemann geometry was used by Einstein and independently by Hilbert to obtain the field equation of gravitational general relativity. From Eq. (3) it is shown in this paper that SCR exists in gravitational general theory within Einstein–Hilbert (EH) field theory. If Eqs. (1) and (2) are developed into tensor and vector equations they lose the basic simplicity of structure of Eqs. (1) and (2) and the existence of SCR is obscured. So the existence of this important resonance phenomenon has been missed for ninety years. Written out in full, Eqs. (1) and (2) are [1-20]

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{5}$$

and

$$d \wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b - R^a{}_c \wedge \omega^c{}_b := 0 , \tag{6}$$

where $d\wedge$ is the exterior derivative of Cartan geometry. Eq. (4) becomes

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b = 0 \tag{7}$$

so in Riemann geometry the following relation exists between the tetrad form q^a and the spin connection form

$$d \wedge q^a = q^b \wedge \omega^a{}_b . \tag{8}$$

EH theory relied on a second order description through the symmetric metric

$$g_{\mu\nu} = q^a{}_\mu q^b{}_\nu \eta_{ab} , \tag{9}$$

where η_{ab} is the Minkowski metric [1-20]. In Cartan geometry the description of gravitation is first order through the tetrad, and is simpler and more elegant. However the physical results of the two representations are the same. Eq. (6) can be rewritten as

$$d \wedge R^a{}_b = j^a{}_b , \tag{10}$$

$$d \wedge \widetilde{R}^a{}_b = \widetilde{j}^a{}_b , \tag{11}$$

where

$$j^a{}_b = R^a{}_c \wedge \omega^c{}_b - \omega^a{}_c \wedge R^c{}_b , \tag{12}$$

$$\widetilde{j}^a{}_b = \widetilde{R}^a{}_c \wedge \omega^c{}_b - \omega^a{}_c \wedge \widetilde{R}^c{}_b . \tag{13}$$

The tilde denotes the Hodge dual [1-20] of the tensor valued two-form

$$R^a{}_{b\mu\nu} = -R^a{}_{b\nu\mu} \tag{14}$$

in four-dimensions (time and three space dimensions). Eqs. (10) and (11) are directly analogous to the electrodynamic section field equations of Einstein–Cartan–Evans (ECE) field theory [2–20]

$$d \wedge F^a = \mu_0 j^a , \tag{15}$$

$$d \wedge \widetilde{F}^a = \mu_0 \widetilde{j}^a , \tag{16}$$

where F^a is the electromagnetic field form, μ_0 is the permeability in vacuo and j^a is the homogeneous current of ECE field theory. The ECE Ansatz defines

$$F^a = A^{(0)} T^a , \tag{17}$$

$$A^a = A^{(0)} q^a , \tag{18}$$

where $cA^{(0)}$ is the primordial or universal voltage of ECE theory and where A^a is the differential form that defines the electromagnetic potential. If there is interaction between electromagnetism and gravitation Eqs. (10) to (18) are inter-related by the first Bianchi identity of Cartan geometry [1–20]

$$d \wedge T^a + \omega^a{}_b \wedge T^b := R^a{}_b \wedge q^b \tag{19}$$

i.e.

$$d \wedge T^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \tag{20}$$

or

$$d \wedge F^a = A^{(0)} \left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \right) . \tag{21}$$

It is seen that

$$j^a = \frac{A^{(0)}}{\mu_0} (R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b) \tag{22}$$

and

$$\tilde{j}^a = \frac{A^{(0)}}{\mu_0} \left(\tilde{R}^a{}_b \wedge q^b - \omega^a{}_b \wedge \tilde{T}^b \right) . \tag{23}$$

Eq. (19) links the curvature form $R^a{}_b$ with the torsion form T^b. Eq. (17) defines the electromagnetic field F^a as the Cartan torsion T^a within a factor $A^{(0)}$ and Eq. (18) defines the electromagnetic potential A^a as the Cartan tetrad q^a within the same factor. In EH theory there is no torsion and the spin connection reduces to the Christoffel connection. This means that Eq. (19) reduces to

$$R^a{}_b \wedge q^b = 0 \tag{24}$$

which is the Ricci cyclic equation used in EH theory. In tensor notation Eq. (24) is the well known cyclic sum

$$R_{\sigma\mu\nu\rho} + R_{\sigma\rho\mu\nu} + R_{\sigma\nu\rho\mu} = 0 , \tag{25}$$

where

$$\begin{aligned} R_{\sigma\mu\nu\rho} &= g_{\sigma\kappa} R^{\kappa}{}_{\mu\nu\rho} , \\ R_{\sigma\rho\mu\nu} &= g_{\sigma\kappa} R^{\kappa}{}_{\rho\mu\nu} , \\ R_{\sigma\nu\rho\mu} &= g_{\sigma\kappa} R^{\kappa}{}_{\nu\rho\mu} . \end{aligned} \tag{26}$$

Here $R^{\kappa}{}_{\mu\nu\rho}$ is the Riemann tensor related to the Riemann form by

$$R^{\kappa}{}_{\mu\nu\rho} = q^{\kappa}{}_a q^b{}_\mu R^a{}_{b\nu\rho} . \tag{27}$$

Ricci inferred tensors in the late nineteenth century and Einstein used tensor notation from about 1906 to 1916 to develop the EH field equation from the second Bianchi identity (2) in tensor notation [1–20]. Cartan geometry was not available to Einstein until well after 1916, when Einstein and Cartan developed their well known correspondence. It was during this correspondence that Cartan suggested that the electromagnetic field be related to the torsion. The ECE Ansatz (17) was developed independently in early 2003 [2–20] without knowledge of Cartan's suggestion. The Ansatz is the simplest way in which F^a and T^a can be related by direct proportionality. The EH field equation is inferred from a comparison of this tensorial Bianchi identity of Riemann geometry with the Noether Theorem

$$\begin{aligned} D_\mu G^{\mu\nu} &= k D_\mu T^{\mu\nu} \\ &:= 0 . \end{aligned} \tag{28}$$

The EH field equation is a possible solution of Eq. (28) and is

$$G^{\mu\nu} = k T^{\mu\nu} . \tag{29}$$

Here $G^{\mu\nu}$ is the Einstein tensor, k the Einstein constant and $T^{\mu\nu}$ the canonical energy-momentum density in tensorial form. Eq. (29) is well known, but much less transparent than the equivalent Cartan equation

$$\begin{aligned} D \wedge \omega^a{}_b &= k D \wedge T^a{}_b \\ &:= 0 . \end{aligned} \tag{30}$$

The use of tensor notation obscured the existence of SCR for ninety years. From Eqs. (5) and (10) the basic SCR equation of EH theory is

$$d \wedge (d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b) = j^a{}_b \tag{31}$$

and its Hodge dual. It is shown in this section that Eq. (31) produces an infinite number of resonance peaks of infinite amplitude in the gravitational potential [2–20]. To show this numerically, Eq. (31) is developed in vector notation. Attention is restricted to the equivalent of the Coulomb Law in ECE gravitational theory. This equivalent is

$$\boldsymbol{\nabla} \cdot \boldsymbol{R}(\text{orbital}) = \boldsymbol{J}^0 , \tag{32}$$

where the orbital curvature vector [2–20] is defined from the Schwarzschild metric as

$$\boldsymbol{R}(\text{orbital}) = R^0{}_1{}^{01} \boldsymbol{i} + R^0{}_2{}^{02} \boldsymbol{j} + R^0{}_3{}^{03} \boldsymbol{k} . \tag{33}$$

The analogy of Eq. (32) in electrodynamics is the Coulomb Law [2–20]

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} , \tag{34}$$

where $\boldsymbol{E}$ is the electric field strength in volts per meter, ρ is the charge density in coulombs per cubic meter and ϵ_0 is the vacuum permittivity. In ECE theory

$$\boldsymbol{E} = -(\boldsymbol{\nabla} + \boldsymbol{\omega})\phi\,, \tag{35}$$

where $\boldsymbol{\omega}$ is the spin connection vector. Far off resonance [2–20]

$$\boldsymbol{\nabla}\phi = \boldsymbol{\omega}\phi\,. \tag{36}$$

There are direct gravitational analogues of Eqs. (34) to (36). These are found from the vector equivalent of the second Cartan structure equation, Eq. (1), which is

$$\boldsymbol{R}^a{}_b = -\frac{1}{c}\frac{\partial \boldsymbol{\omega}^a{}_b}{\partial t} - \boldsymbol{\nabla}\omega^{0a}{}_b - \omega^{0a}{}_c\boldsymbol{\omega}^c{}_b + \boldsymbol{\omega}^a{}_c\omega^{0c}{}_b \tag{37}$$

in vector notation. The spin connection form is the four-vector

$$\omega^a{}_{\mu b} = (\omega^a{}_{0b}, \boldsymbol{\omega^a{}_b}) \tag{38}$$

with time-like component $\omega^a{}_{0b}$ and space-like component $\boldsymbol{\omega^a{}_b}$). For each a and b the time-like component is a scalar and the space-like component is a vector in three dimensions. The electromagnetic analogue of Eq. (37) is [2–20]

$$\boldsymbol{E}^a = -\frac{\partial \boldsymbol{A}^a}{\partial t} - c\boldsymbol{\nabla}A^{0a} - c\omega^{0a}{}_b\boldsymbol{A}^b + cA^{0b}\boldsymbol{\omega}^a{}_b\,, \tag{39}$$

where $\boldsymbol{A}^a$ is the vector potential and

$$\phi^a = cA^{0a} \tag{40}$$

is the scalar potential. It is seen that $\boldsymbol{\omega}^a{}_b$ in gravitational theory plays the role of $\boldsymbol{A}^a$ in electromagnetic theory. The role of ϕ^a in electromagnetic theory is played by $\omega^{0a}{}_c$ in gravitational theory. The role of $\boldsymbol{E}^a$ in electromagnetic theory is played by $\boldsymbol{R}^a{}_b$ in gravitational theory. The vector $\boldsymbol{R}$ (orbital) is a particular case of $\boldsymbol{R}^a{}_b$ defined by Eq. (33). This particular case fixes the indices a and b.

Attention is now confined to the equivalent in gravitational theory of the electrostatic limit in electromagnetic theory, a limit in which $\boldsymbol{E}^a$ is expressed in terms of the scalar potential only

$$\boldsymbol{E}^a = -\boldsymbol{\nabla}\phi^a + \boldsymbol{\omega}^a{}_b\phi^b\,. \tag{41}$$

The equivalent of Eq. (41) in gravitational theory is

$$\boldsymbol{R}^a{}_b = -\boldsymbol{\nabla}\omega^{0a}{}_b + \boldsymbol{\omega}^a{}_c\omega^{0c}{}_b\,. \tag{42}$$

The gravitational scalar potential is therefore identified as the time-like part of the spin connection

$$\Phi^a_b = \omega^{0a}{}_b\,. \tag{43}$$

Thus

$$\boldsymbol{R}^a{}_b = -\boldsymbol{\nabla}\Phi^a_b + \omega^a{}_c\Phi^c_b\,. \tag{44}$$

It is convenient to use a negative sign for the vector part of the spin connection, so

$$\boldsymbol{R}^a{}_b := -\left(\boldsymbol{\nabla}\Phi^a_b + \omega^a{}_c\Phi^c_b\right)\,. \tag{45}$$

This is the direct gravitational analogue of the electromagnetic equation (35). The off resonance condition [2–20] is now defined by

$$\boldsymbol{\nabla}\Phi^a_b = \boldsymbol{\omega}^a{}_c\Phi^c_b\,. \tag{46}$$

This is the analogue of Eq. (36) in electrodynamics. Summation over repeated indices are implied, so

$$\boldsymbol{\omega}^a{}_c\Phi^c_b = \boldsymbol{\omega}^a{}_0\Phi^0_b + \cdots + \boldsymbol{\omega}^a{}_3\Phi^3_b\,. \tag{47}$$

Since a and b occur in the same way in all terms, Eq. (47) can be written as

$$(\boldsymbol{\omega}_c\Phi^c)^a{}_b = (\boldsymbol{\omega}_0\Phi^0 + \cdots + \boldsymbol{\omega}_3\Phi^3)^a{}_b\,. \tag{48}$$

The indices a and b are found by the use of the Schwarzschild metric in defining $\boldsymbol{R}$(orbital) in Eq. (33). For each a and b we define

$$\boldsymbol{\omega}\Phi := \boldsymbol{\omega}_0\Phi^0 + \cdots + \boldsymbol{\omega}_3\Phi^3\,. \tag{49}$$

With these definitions

$$\begin{aligned}\boldsymbol{R}(\text{orbital}) &= -(\boldsymbol{\nabla} + \boldsymbol{\omega})\Phi \\ &= R^0{}_1{}^{01}\boldsymbol{i} + R^0{}_2{}^{02}\boldsymbol{j} + R^0{}_3{}^{03}\boldsymbol{k}\,.\end{aligned} \tag{50}$$

From Eqs. (32) and (50)

$$\nabla^2\Phi - \boldsymbol{\nabla}\cdot(\boldsymbol{\omega}\Phi) = -J^0 \tag{51}$$

which is the gravitational analogue of the electromagnetic

$$\nabla^2\phi - \boldsymbol{\nabla}\cdot(\boldsymbol{\omega}\phi) = -\frac{\rho}{\epsilon_0}\,. \tag{52}$$

The Newton inverse square law is obtained from units analysis [2–20]

$$\boldsymbol{F} = m^2 G\boldsymbol{R}(\text{orbital})\,, \tag{53}$$

where $\boldsymbol{F}$ is force, m is mass and G is the Newton gravitational constant. Here $\boldsymbol{R}$(orbital) has the units of inverse square meters. The Newtonian limit is defined as the off resonance condition

$$\nabla \Phi = \omega \Phi \tag{54}$$

with gravitational potential

$$\Phi = -\frac{1}{r} \tag{55}$$

along the radial direction. Therefore the spin connection vector is found to be

$$\omega = -\frac{1}{r}\boldsymbol{e}_r \,, \tag{56}$$

where $\boldsymbol{e}_r$ is the radial unit vector in spherical polar coordinates. Therefore

$$\nabla \Phi = \omega \Phi = \frac{1}{r^2}\boldsymbol{e}_r \tag{57}$$

and

$$\boldsymbol{R}(\text{orbital}) = -\frac{2}{r^2}\boldsymbol{e}_r \,. \tag{58}$$

The Newton inverse square law emerges from Eqs. (53) and (58) by identifying

$$m^2 = m_1 m_2 \,, \tag{59}$$

where m_1 and m_2 are two gravitating masses, and by using half the value of $\boldsymbol{R}$(orbital) in Eq. (58). In the non-relativistic (classical) theory of Newtonian dynamics the spin connection is missing, and the classical relation of force and gravitational potential is

$$\boldsymbol{F} = -m_1 m_2 G \nabla \Phi \,. \tag{60}$$

In gravitational general relativity, gravitation is due to curvature, and the spin connection is always non-zero. This means that the force in general relativity is twice the force in the classical for a given potential Φ. The extra contribution is due to the spin connection as follows

$$\boldsymbol{F}(\text{classical}) = -m_1 m_2 G \nabla \Phi \,, \tag{61}$$

$$\boldsymbol{F}(\text{relativistic}) = -m_1 m_2 G (\nabla + \omega) \Phi \,. \tag{62}$$

Off resonance therefore the presence of the spin connection cannot be detected because it simply doubles the definition of the potential. The force is the quantity which is detected experimentally and as long as the force is expressed in terms of the potential as in Eq. (57) the inverse square law is not changed. Similar considerations apply [2–20] to the Coulomb law of ECE theory. The factor two also occurs in the original EH theory [1], but is arrived at via a different route in a much more complicated way. In the bending of light by gravity the relativistic result is now known to be twice the classical result to a precision of one part in 100,000. Again, the tensorial theory of this well known result is much more complicated than the simple route used here and elsewhere [2–20] in this series of papers. SCR emerges finally from the radial form of Eq. (51), which is

$$\frac{\partial^2 \Phi}{\partial r^2} - \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\Phi = -J^0 \,, \tag{63}$$

where it is assumed that there is an oscillatory driving term

$$J^0 = J^0(0)\cos(\kappa r) \,. \tag{64}$$

The electromagnetic analogue of Eq. (63) is

$$\frac{\partial^2 \phi}{\partial r^2} - \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\phi = -\frac{\rho(0)}{\epsilon_0}\cos(\kappa r) \tag{65}$$

which has been solved recently using analytical and numerical methods [2–20]. These solutions for ϕ and Φ show the presence of an infinite number of resonance peaks, each of which become infinite in amplitude at resonance.

The British Government is thanked for the award of a Civil List pension for distinguished services to science, and the staff of AIAS are thanked for many interesting discussions.

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