CLASSICAL ELECTRODYNAMICS WITHOUT THE LORENZ CONDITION: EXTRACTING ENERGY FROM THE VACUUM

ABSTRACT

It is shown that if the Lorenz condition is discarded, the Maxwell-Heaviside field equations become the Lehnert equations, indicating the presence of charge density and current density in the vacuum. The Lehnert equations are a subset of the O(3) Yang-Mills field equations. Charge and current density in the vacuum are defined straightforwardly in terms of the vector potential and scalar potential, and are conceptually similar to Maxwell's displacement current, which also occurs in the classical vacuum. The demonstration is made of the existence of a time dependent classical vacuum polarization which appears if the Lorenz condition is discarded. Vacuum charge and current appear phenomenologically in the Lehnert equations but fundamentally in the O(3) Yang-Mills theory of classical electrodynamics. The latter also allow for the possibility of the existence of vacuum topological magnetic charge density and topological magnetic current density. Both O(3) and Lehnert equations are superior to the Maxwell-Heaviside equations in being able to describe phenomena not amenable to the latter. In theory, devices can be made to extract the energy associated with vacuum charge and current.

INTRODUCTION

The Lorenz condition {1} is imposed arbitrarily in classical electrodynamics. If a pair of vector A' and scalar ϕ' potentials do not obey the Lorenz condition, a gauge transform is performed {1} in order to ensure that they do so. This operation is described in section 2. In section 3, a d'Alembert equation is derived without using the Lorenz condition from the vacuum Faraday equation and vacuum Ampère-Maxwell equation. The d'Alembert equation thus obtained contains a vacuum current density which is defined in terms of A and ϕ. This is precisely the result suggested phenomenologically by Lehnert {2-5} and Lehnert and Roy {6}. A comparison of the Lehnert equations and the original Maxwell-Heaviside equations without the Lorenz gauge leads to the relation:

$$\frac{\partial \boldsymbol{P}_A}{\partial t} = -\boldsymbol{j}_A \tag{1}$$

where $\boldsymbol{P}_A$ is a classical vacuum polarization and $\boldsymbol{j}_A$ is the classical vacuum current density. A fully relativistic treatment shows that discarding the Lorenz condition leads directly to the Lehnert equations, which contain vacuum charge density and vacuum current density.

In section 4, it is shown that the Lehnert equations are a subset of the O(3) Yang-Mills equations {6-12} of electrodynamics, in which vacuum charge density and current density occur fundamentally through the topology of the vacuum. The Lehnert and O(3) equations are successful in describing several phenomena not amenable to the Maxwell-Heaviside equations {6-12}. The fundamental O(3) equations also have the structure of the phenomenological Harmuth field equations {13-15} and are homomorphic with the Barrett field equations {16, 17}. Finally a short discussion is given of the possibility of extracting electric and magnetic energy from the vacuum.

THE LORENZ CONDITION

Jackson {1} discusses the Lorenz condition on p. 181 of the first edition. It is clear from his discussion that if initially:

$$\partial_\mu A^{\mu'} \neq 0 \quad (2)$$

a gauge transform is performed:

$$\partial_\mu A^{\mu'} \to \partial_\mu A^\mu + \Box\Lambda \quad (3)$$

such that

$$\partial_\mu A^{\mu'} = 0 \quad (4)$$

therefore forcing the condition

$$\partial_\mu A^\mu = -\Box\Lambda \quad (5)$$

as a special case. This procedure is used because it gives the d'Alembert equation in the vacuum and is convenient for special relativity. It has become habitual over the years to use this procedure as an intrinsic part of basic classical electrodynamics and quantum electrodynamics. It is however arbitrary, and discards valuable information as shown in the next section.

DISCARDING THE LORENZ CONDITION

In S.I. units, we define the potential four-vector by:

$$A^\mu \equiv (\phi, c\boldsymbol{A}) \quad (6)$$

and write in the vacuum the Faraday law of induction and the Ampère-Maxwell law:

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0} \quad (7)$$

$$\nabla \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \boldsymbol{0} \quad (8)$$

where $\boldsymbol{E}$ is the electric field strength and $\boldsymbol{B}$ the magnetic flux density. These are expressed using standard procedure in terms of the vector potential $\boldsymbol{A}$ and scalar potential ϕ:

$$\boldsymbol{B} = \nabla \times \mathrm{A}; \qquad \boldsymbol{E} = -\frac{\partial A}{\partial t} - \nabla\phi \quad (9)$$

This procedure results in the vacuum d'Alembert equation:

$$\Box \boldsymbol{A} = -\nabla\left(\nabla \cdot \mathrm{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t}\right) \equiv \mu_0 j_A \quad (10)$$

where μ_0 is the vacuum permeability and j_A is a vacuum current density defined by discarding the Lorenz condition. Therefore this current density comes from the vacuum in an analogous way to Maxwell's displacement current and this is precisely the result obtained by Lehnert and Roy {6} phenomenologically. They write the vacuum current as:

$$j_A^{\mu} = (0, \boldsymbol{j}_A) \tag{11}$$

and define a vacuum electric field strength $\boldsymbol{E}_A$ from this current density through a vacuum conductivity σ:

$$\boldsymbol{j}_A = \sigma \boldsymbol{E}_A \tag{12}$$

It is shown by Lehnert and Roy {6} that the equations of classical electrodynamics in the vacuum then become:

$$\nabla \cdot \mathrm{E} = 0; \qquad \nabla \times \mathrm{H} = \sigma \boldsymbol{E}_A + \frac{\partial \boldsymbol{D}}{\partial t} \tag{13}$$

$$\nabla \cdot \mathrm{B} = 0; \qquad \nabla \times \mathrm{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}. \tag{14}$$

However, we started from eqns. (7) and (8), which therefore must be the same as eqns . (13) and (14). To identify these equations, we write:

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}_A \tag{15}$$

where $\boldsymbol{P}_A$ is a classical vacuum polarization, and arrive at the result:

$$\frac{\partial \boldsymbol{P}_A}{\partial t} = -\boldsymbol{j}_A. \tag{16}$$

The concept of vacuum polarization already exists in quantum electrodynamics {6-12} and discarding the Lorenz condition gives its classical equivalent. If $\boldsymbol{P}_A$ varies more rapidly with time, then $\boldsymbol{j}_A$ increases in magnitude. We arrive at the conclusion that the Lorenz condition is equivalent to:

$$\frac{\partial \boldsymbol{P}_A}{\partial t} = \boldsymbol{0} \tag{17}$$

i.e. to a constant $\boldsymbol{P}_A$ which may or may not be zero. If the vacuum topology is such that $\frac{\partial \boldsymbol{P}_A}{\partial t}$ varies rapidly, then $\boldsymbol{j}_A$ becomes substantial in magnitude.

To make the development fully relativistic needs the introduction of a vacuum charge/current density four vector:

$$\rho_A^{\mu} = \left(\rho_A, \frac{1}{c} \boldsymbol{j}_A \right) \tag{18}$$

in which case the Lehnert equations {2-6} are obtained in the vacuum:

$$\nabla \cdot \boldsymbol{D} = \rho_A; \qquad \nabla \cdot \boldsymbol{B} = 0 \tag{19}$$

The vacuum charge density is:

$$\nabla \times \mathrm{H} = j_A + \frac{\partial \boldsymbol{D}}{\partial t}; \qquad \nabla \times \mathrm{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \mathbf{0}. \tag{20}$$

$$\rho_A = \frac{1}{\mu_0} \Box \phi \tag{21}$$

and the vacuum current density is:

$$j_A = \frac{1}{\mu_0} \Box A \tag{22}$$

Therefore the charge and current density introduced phenomenologically by Lehnert and Roy {2-6} are obtained simply by discarding the Lorenz condition. There appears to be no limit to the magnitude of $\frac{\partial P_A}{\partial t}$ and therefore to the magnitude of the vacuum charge density and current density. If devices can be made to utilize this energy, then sources of unlimited energy would be available from the structured vacuum.

THE O(3) YANG-MILLS EQUATIONS OF ELECTRODYNAMICS

These equations apply O(3) Yang-Mills gauge theory {6-12} rather than U(1) Yang-Mills gauge theory to electrodynamics, and give a set of equations which include a topological vacuum charge density and topological vacuum current density from fundamental arguments. The structure of the relevant O(3) equations are, in the vacuum:

$$\nabla \bullet \boldsymbol{D}^{(i)} = \rho_A; \qquad i = 1,2,3; \tag{23}$$

$$\nabla \times \mathrm{H}^{(i)} = \frac{\partial \boldsymbol{D}^{(i)}}{\partial t} + j_A^{(i)}; \quad i = 1,2,3 \tag{24}$$

in a complex basis ((1), (2), (3)) defined by circular polarization. Therefore the O(3) theory inherently breaks the Lorenz condition and gives rise to vacuum charge density and current density at a fundamental level in gauge theory. The O(3) theory also allows for the possibility of the existence of topological magnetic monopole and topological magnetic current. The O(3) equations are similar in structure to the phenomenological Harmuth equations {13-15} and are homomorphic with the fundamental SU(2) Yang-Mills gauge field equations of Barrett {16, 17}.

DISCUSSION

Both electric current density and electric charge density can be obtained from the structured vacuum, i.e. from the vector potential A and scalar potential ϕ. From the Aharonov-Bohm effect {16}, it is well known that potentials can exist in regions of the structured vacuum where there are no fields present. The Yang-Mills vacuum is infinitely degenerate {17}, and can support instantons in classical electrodynamics. Therefore, electric and magnetic fields are secondary quantities that can be generated from potential fluctuations; or alternatively, from the time dependence of classical vacuum polarization. Electric and magnetic fields do not necessarily emanate from the charge on the electron. Indeed, energy can be neither created nor destroyed, and charge of itself cannot give rise to energy. The structured vacuum contains energy, and some of this is converted into electrical energy in such a way that total energy in the universe is conserved and the laws of thermodynamics are obeyed. Therefore, there is no theoretical argument against the construction of devices by taking energy from the structured vacuum.

Asymmetrical regauging is any process that changes the potential energy of a system and also produces a net force in the process. It is the equivalent of discarding the Lorenz condition. Asymmetrical self-regauging (ASR) is any process whereby the system itself initiates its own asymmetrical regauging by processes ongoing in the system and involving its interchange with the active vacuum.

Several processes produce asymmetrical regauging. Any retro-reflection of a constant fraction of the escaping energy from a system otherwise in equilibrium with an input of energy to it, produces a classic positive feedback. So long as the system holds stable, the energy density of the system increases without bounds, as the number of iterative retro-reflections increases without bounds. In intensely scattering photo-reactive media, pumping and phase conjugate reflections combine to produce an extremely rapid increase in the energy density of the photo-reactive medium, approaching asymptotic infinity until deviations in geometry, the reflection and pumping processes, etc. occur in the medium. The pumping allows increasing of the emitted energy (phase conjugate reflection) by means of multi-wave interactions. The process rapidly ramps up the frequency of the emitted radiation to much higher level. However, after a phase lag behind the regauging rise, geometrical and process distortions occur and result in a very rapid and asymptotic quenching of the regauging process itself. The remaining energy density of the medium is greater than prior to ignition, due to the excess energy absorbed by the medium.

Bearden {18} has suggested several experimental demonstration mechanisms for test replication, including one after Sweet {19} based on the known induction of sustained self-oscillating fields in certain permanent magnet materials. {20} A suitable permanent magnet with its B-field in induced self-oscillation is surrounded by conductors in a circuit connected to a small resistive load. The waving B-field cuts the conductors, inducing alternating emf and current and powering the load, without external energy input by the experimenter. Vibration of nuclei at sonic frequencies is well known {37}. Sweet considered that self-oscillation in his barium ferrite magnet was due to multiple spin wave exchange forces providing self-pumping and feedback. Self-excited electrical oscillations from which electrical power is drawn have also been experimentally demonstrated in prototype systems {38} as has negative resistance {39, 40}.

The vacuum charge and current caused by discarding the Lorenz condition can be picked up by a receiver and used to generate electrical energy. This is similar to the interception of energy by a collecting device, and several of these have been mentioned in this paper. The challenge is to design such a device efficiently, so that an indefinite amount of energy can be collected from the structured vacuum. The "potential on the particle" is therefore an electromotive force produced by the potentials A and ϕ of the structured vacuum once the Lorenz condition is discarded. The latter process does not result in a violation of the principle of conservation of charge.

This is particularly important in iterative phase conjugate reflection processes with multiple photon and multiple wave interactions providing pumping, such as occur in intense nonlinear optical scattering media. The pumping results in gathering energy from multiple absorbed beams and steadily emitting increased energy in the average phase conjugate reflection beam. The iterative process within the system thus self-regauges the potential energy of the system by continually increasing the local energy density of space-time. Disruption of the physical geometry disrupts the pumping increase, resulting in a sharp quenching of the process.

A simple lasing without population inversion experiment by Lawandy {21} et al., and a corresponding patent by Lawandy {22} may also be considered an example of easily replicable ASR and extraction of electromagnetic energy from the vacuum. A colloidal suspension of TiO_2 particles in a beaker of laser dye fluid is illuminated by a single weak laser pulse fired into the fluid. The TiO_2 particles are previously screened so that their particle resonance frequency is in the vicinity of the laser frequency. A burst of much greater coherent energy is emitted from the apparatus. This latter phenomenon may be striking evidence of an often-overlooked feature inherent in the law that energy can neither be created nor destroyed. Rigorously, work is defined as changing the form of energy. When one joule of energy performs one joule of work, one joule of energy still remains, but in an altered form. If that remaining joule of energy has its form changed yet again, another joule of work has been done. And so on. The photo-refractive TiO_2 particles in the Lawandy colloidal suspension perform iterative phase conjugate reflections, so that the form of the energy iterates back and forth between wave energy and phase conjugate wave energy several times before escaping from the system. Each phase conjugate reflection changes the form of the energy, so each input joule of energy does iterative joules of work to increase the energy-collecting upon the particles. In effect, the system self-regauges its own energy density asymmetrically, thereby increasing its potential. This mechanism adds energy to the system from the vacuum, since any potential is a change to the vacuum potential. The emission of the added regauging energy provides the excess energy in the emitted burst of light.

Letokhov {23, 24} and others {25, 26} have also published extensively on similar "negative absorption" (which is excess emission) from the medium. Bohren {27} has reported a small particle energy absorption cross-section about 18 times the ostensible geometrical cross-section, with replication reported by Paul and Fischer {28}. Such negative absorption is experienced by metal particles at ultraviolet frequencies and by dielectric particles at infrared frequencies.

Nobelist Lee {29} has pointed out that such "vacuum engineering" is possible, but has not been specifically attempted by physicists. The well-known Casimir {30} effect is an example of electromagnetic force generated by the vacuum between two conducting plates. A beautiful experimental demonstration of the Casimir effect has been given by Lamoreaux.{31} A patent has been issued to Mead and Nachamkin {32} for a system of vacuum-energy extraction from the formation of electromagnetic energy between two Casimir plates.

Finally, there is no thermodynamic prohibition to extracting energy from the vacuum, as decisively demonstrated by Cole and Puthoff {33}.

Bearden {34} has also hypothesized asymmetrical self-regauging (ASR) as a possible mechanism for the gamma burster. Statistically, the mechanism could initiate in an exploding gas cloud, resulting in a rapid and asymptotic rise in regauging energy absorbed and re-radiated by the gas molecules and atoms (particles). This might be referred to as "ignition" of the process. Upon ignition, asymptotically rising acceleration forces appear upon the particles, producing a sharply increasing movement of the particles with a small phase lag behind the ASR rise. Physical movement disrupts the physical geometry of the particles, sharply quenching the regauging, but leaving an afterglow of increased kinetic energy and thermal radiation in the gas particles. Phenomena consistent with such ignition and quenching of a gamma burster, in an already exploding gas, were photographed by several satellites on Dec. 20, 1998. Among gamma burst events, variability of the average mass particle size involved would yield differing hysteresis times between ignition and quenching, thus accounting for observed differences in gamma burst lifetimes ranging from 10^{-3} seconds to 10^{3} seconds.

The ASR hypothesis would also account for the noted "no-host" problem {35} for other postulated mechanisms, in present gamma ray burst observations. At rare intervals, in an individual ignition process, Bearden speculates that the phase lag between ignition and quenching may be further lengthened by an increased mass of the particles. This could result in the ASR rise reaching an energy density that bursts the binding energy of a 4-space in its greater n-space, where $n > 4$. Quenching then occurs by rupture of the local 4-space and a rapidly expanding regauging process in n-space, forming a newly emerging 4-space universe. On the initiation side, the formation site of the process would appear as a continuing black hole, while in the emerging universe on the burst-out side, the site would appear as a "big bang" and an expanding 4-space.

An underlying relationship between gamma bursters and a big bang is in fact suggested from recent observations. For the first time, the Hubble Space Telescope on Jan. 23, 1999 caught an optical flash simultaneous with the initial gamma burst. Calculations showed that this burst was second in energy only to the big bang itself, generating some 3.4 x 10^{54} ergs – equivalent to converting two solar masses instantaneously into energy.{36} Further, there was no polarization of the afterglow, which has been taken as evidence of beaming, but such beaming indications could be due to the beaming involved in the intense iterative phase conjugate reflections of the ASR process.

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